Exact solution and Monte-Carlo simulations of a energy-enstrophy theory for geo/astro - physical flows - Venus Super-rotating atmosphere.

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Summary. The applications of equilibrium statistical mechanics to macroscopic systems - in the sense that the fundamental particles of these systems are not atoms and molecules - started almost simultaneously in the three subfields of this conference, namely, galactic systems (Lynden-Bell), turbulent fluid flows (Onsager, Feynman, Kraichnan) and electron plasmas (Montgomery, Williams, Lundgren, Pointin). Many of these applications have a quasi-2D nature which in the case of fluid flows result in inverse energy cascades, that is, the nonequilibrium flux of energy from small and intermediate to large scales. This phenomenon has striking experimental consequences (van Heijst et al, Sommeria et al, Swinney et al) in real fluids, namely the persistence of non-uniform large scale energetic structures in quasi-2D systems long after similar initial conditions in a 3D flow have dissipated into a uniform quiescent state. Equivalent observations are relevant in galactic systems and electron plasmas (Hasagewa-Miwa model).

There are currently many statistical equilibrium models in the literature - vortex gas, spectral energy-enstrophy, spin-lattice models and variations based on conserving different invariants - partly because experiments sensitive enough to distinguish between these theories have only been done in the last decade or so. In this presentation, I will focus on those systems where damped or forced-damped nearly inviscid flows are observed in quasi-2D settings with complex boundaries such as no-slip boundaries in a rectangular box and in particular barotropic flows coupled to a massive rotating sphere. In particular, the fluid component of these coupled systems are not governed by Hamilton's equations even if one ignores the small effects of viscosity in the interior of geophysical flows, and their angular momentum and kinetic energy are not fixed, but rather exchanged statistically with reservoirs residing in the infinitely massive sphere - a convenient theoretical construct. In addition, none of the vorticity moments with the exception of total circulation are conserved by this coupled system in particular and by quasi-2D flows with complex boundaries in general.

This implies directly that the energy and/or angular momentum should be modelled by Gibbs canonical ensembles - that is not only natural but the alternative microcanonical ensemble in the energy are intractable. The experimental or observational setting further eliminates many choices - the vortex gas models and the

Miller-Robert models are not usable because they conserve an infinite number of vorticity moments or enstrophies. Only the Kraichnan energy-enstrophy theories are left. But Kraichnan's absolute equilibrium theory based on Gibbs ensemble doubly canonical in energy and enstrophy are not well-defined at small absolute values of temperature because they are Gaussian models.

With this background and history, I will therefore present a energy-enstrophy model based on Gibbs canonical ensemble for the energy but which uses a microcanonical constraint on enstrophy designed specifically to eliminate the low temperature defects of Kraichnan's theory. Usually microcanonical constraints in statistical mechanics lead to technical dead-ends but here - because fixing the enstrophy is equal to fixing the sum of squares of local vorticity or lattice spins in a convergent family of finite dimensional approximations - this microcanonical enstrophy constraint leads to an exactly-solvable or completely integrable partition function for a spherical model. The approach introduced by Kac and Berlin is based on the method of steepest descent or saddle points which gives exact expressions for the partition function and free energy in the nonextensive thermodynamic limit.

Exact solutions then give critical temperatures for Bose-Einstein condensation of random initial vorticity into super-rotating equilibrium states which have different angular momentum than the initial data. In other words, the complex torque coupling between fluid and solid sphere is a necessary part of the BEC phase transition to super-rotation because transfer of angular momentum acts like a catalyst for the inverse cascade of energy from small random eddies into a large-scale self-organized flow state. More importantly the exact solution predict phase transitions on the basis of a few key physical parameters - rate of spin of sphere and fixed relative enstrophy of flow after normalizing to a unit sphere and unit area density of fluid.

Monte-Carlo simulations confirm the phase transitions predicted exactly by the microcanonical enstrophy theory.