

**ENGR 1600 Materials Science for Engineers Section 7.**  
**5th quiz, Wednesday, March 1, 2006.**

\_\_\_\_\_ (Print)  
LAST NAME, FIRST NAME

1. Plastic deformation

(a) (20) A fcc single crystal metal was subjected to a tensile stress in [001] direction. It exhibited a slip on (111) plane in  $[0\bar{1}1]$  direction when the tensile stress was 34.3 MPa. Calculate the shear stress, which caused the slip. This is called the critical resolved shear stress.

(b) (20) A single crystal zinc with hexagonal close packed structure is oriented with the normal to the basal plane making an angle of  $60^\circ$  with the tensile axis, and the three slip directions making angles of  $38^\circ$ ,  $45^\circ$  and  $84^\circ$  with the tensile axis. If the critical resolved shear stress for zinc is 53.2 MPa, what would be the minimum tensile stress needed to cause plastic deformation in this specimen. (Hint: In hcp structure, slip takes place on the basal plane and there are three equivalent slip directions on the plane.)

$$\tau = \sigma \cos \phi \cos \lambda.$$

## 2. Brittle fracture

(a) (20) One theory of brittle fracture is based upon the idea of stress concentration.

When the concentrated stress at the crack tip of a brittle material reached the theoretical strength,  $\sigma_{th}$ , the material fails. The concentrated stress,  $\sigma_m$ , is given by  $\sigma_m = \sigma_0 \sqrt{a/\rho_t}$ , where  $\sigma_0$  is the nominal stress,  $a$  is the depth of the surface crack and  $\rho_t$  is the tip radius of the crack. Which is stronger when two identical samples with same crack depth but different crack tip sharpness were compared?

(b) (20) An alternative theory of the brittle fracture is by Griffith, which states the strength is given by  $\sigma_c = \sqrt{2E\gamma_s/\pi a}$ , where  $E$  is Young's modulus,  $\gamma_s$  is fracture surface energy. The fracture toughness is given by  $K_{Ic} \approx \sigma_c \sqrt{\pi a}$ .

If a glass had  $K_{Ic} = 0.74 \text{ MPa}\sqrt{\text{m}}$ , what would be the fracture stress (or strength) of the glass sample with surface crack  $a = 5 \text{ }\mu\text{m}$ ? What would be the strength if the surface crack depth  $a = 10 \text{ }\mu\text{m}$ ?

$$1\mu\text{m} = 10^{-3} \text{ mm} = 10^{-6} \text{ m}$$

3. (a) (10) Describe a strengthening method in a metal.

(b) (10) Explain why fcc metals are easier to deform plastically compared with bcc or hcp metals.

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1. Plastic deformation

(a) (20) A single crystal fcc was subjected to a tensile stress in [001] direction. It exhibited a slip on (111) plane in  $[0\bar{1}1]$  direction when the tensile stress was 34.3 MPa. Calculate the shear stress, which caused the slip. This is called the critical resolved shear stress.

$$\tau = \sigma \cos \phi \cos \lambda = 34.3 \text{ MPa} (1/\sqrt{3}) (1/\sqrt{2}) = 14.1 \text{ MPa.}$$

(b) (20) A zinc with hexagonal close packed structure is oriented with the normal to the basal plane making an angle of  $60^\circ$  with the tensile axis, and the three slip directions making angles of  $38^\circ$ ,  $45^\circ$  and  $84^\circ$  with the tensile axis. If the critical resolved shear stress for zinc is 53.2 MPa, what would be the minimum tensile stress needed to cause plastic deformation in this specimen. (Hint: In hcp structure, slip takes place on the basal plane and there are three equivalent slip directions on the plane.)

$$\tau = \sigma \cos \phi \cos \lambda.$$

Tensile stress,  $\sigma$ , which is needed to produce the critical resolved shear stress,  $\tau_c$ , will be calculated for three directions.

$$53.2 \text{ MPa} = \sigma \cos 60 \cos 38 = \sigma \times 0.5 \times 0.788 = 0.394 \sigma \rightarrow \sigma = 135 \text{ MPa}$$

$$53.2 \text{ MPa} = \sigma \cos 60 \cos 45 = \sigma \times 0.5 \times 0.707 = 0.3535 \sigma \rightarrow \sigma = 150.5 \text{ MPa}$$

$$53.2 \text{ MPa} = \sigma \cos 60 \cos 84 = \sigma \times 0.5 \times 0.105 = 0.0525 \sigma \rightarrow \sigma = 1,013 \text{ MPa}$$

Answer 135 MPa.

## 2. Brittle fracture

(a) (20) One theory of brittle fracture is based upon the idea of stress concentration. When the concentrated stress at the crack tip of a brittle material reached the theoretical strength, the material fails. The concentrated stress,  $\sigma_m$ , is given by  $\sigma_m = \sigma_0 \sqrt{a/\rho_t}$ , where  $\sigma_0$  is the nominal stress,  $a$  is the depth of the surface crack and  $\rho_t$  is the tip radius of the crack. Which is stronger when two identical samples with same crack depth but different crack tip sharpness were compared?

$$\text{The fracture strength } \sigma_c = \sigma_{th} \sqrt{\rho_t/a}$$

Therefore, the sample with the less sharper crack tip or larger  $\rho_t$  would be stronger.

(b) (20) An alternative theory of the brittle fracture is by Griffith, which states the strength is given by  $\sigma_c = \sqrt{2E\gamma/\pi a}$ , where  $E$  is Young's modulus,  $\gamma_s$  is fracture surface energy. The fracture toughness is given by  $K_{Ic} \approx \sigma_c \sqrt{\pi a}$ .

If a glass had  $K_{Ic} = 0.74 \text{ MPa}\sqrt{\text{m}}$ , what would be the fracture stress (or strength) of the glass sample with surface crack  $a = 5 \mu\text{m}$ ? What would be the strength if the surface crack depth  $a = 10 \mu\text{m}$ ?

$$1 \mu\text{m} = 10^{-3} \text{ mm} = 10^{-6} \text{ m}$$

$$\sigma_c \approx K_{Ic} / \sqrt{\pi a}$$

For the sample with crack  $a = 5 \mu\text{m}$ ,

$$\sigma_c \approx K_{Ic} / \sqrt{\pi a} = 0.74 / \sqrt{3.14 \times 5 \times 10^{-6}} = 0.74 / (3.96 \times 10^{-3}) = 187 \text{ MPa}$$

For the sample with crack  $a = 10 \mu\text{m}$ ,

$$\sigma_c \approx K_{Ic} / \sqrt{\pi a} = 0.74 / \sqrt{3.14 \times 10 \times 10^{-6}} = 0.74 / (5.60 \times 10^{-3}) = 132 \text{ MPa}$$

3. (a) (10) Describe a strengthening method of metal.

Strain hardening—simply strain a metal. It becomes stronger with its dislocation motion becoming harder to move due to the fact that dislocations with easier motion have already moved.

Solution hardening—dislocation motion becomes difficult due to strain field.

Small grain size—grain boundary blocks the dislocation motion. Smaller grains have more dislocation.

(b) (10) Explain why fcc metals are easier to deform plastically compared with bcc or hcp metals.

Numerous slip systems plus lower critical resolved shear stress.