Linear Algebra, MATH–410 Gregor Kovačič

Homework #8 Due: Wednesday, October 27, 1999

1. Compute the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}.$$

2. Compute the eigenvalues and eigenvectors of the matrix

$$B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 2 \end{pmatrix}.$$

3. (a) Compute the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 4/5 & 2/5 \\ 1/5 & 3/5 \end{pmatrix}.$$

(b) Find A^n for $n = 1, 2, \ldots$. Does A^n tend to a limit?

4. Let V be the linear span of the functions 1, $\cos x$, $\sin x$. Let the operator T on V be given by the rule $Ty(x) = y(x + \pi/4)$. Find the eigenvalues and eigenvectors of T in V.

5. Compute the eigenvalues and eigenvectors of the symmetric matrix

$$C = \begin{pmatrix} 2 & 2 & 1 \\ 2 & 2 & -1 \\ 1 & -1 & -1 \end{pmatrix}.$$

Verify that the eigenvectors you have found are orthogonal, and find an orthonormal basis of eigenvectors of the matrix C.

6. Extra Credit: Find the eigenvalues and eigenvectors of the operator A on $\mathbb{R}^3$ given by $A\mathbf{x} = |\mathbf{a}|^2\,\mathbf{x} - (\mathbf{a}\cdot\mathbf{x})\,\mathbf{a}$, where $\mathbf{a}$ is a given constant vector. How do you know without any calculations that A must have an orthonormal eigenbasis?