Linear Algebra, MATH–410 Gregor Kovačič

Homework #5 Due: Wednesday, October 6, 1999

1. Show that if A is a linear transformation such that $A^2 - A + I = 0$, then A is invertible (i.e., A^{-1} exists). What is A^{-1}?

2. Let

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

be the matrix of a linear operator in the usual basis. Find the matrix B of this operator in the basis $(1,0,0)$, $(1,1,0)$, $(1,1,1)$.

3. Let

$$A = \begin{pmatrix} 1 & 3 & 1 & 4 \\ 2 & 3 & 4 & 5 \end{pmatrix}$$

be the matrix of the linear mapping $F : \mathbb{R}^4 \to \mathbb{R}^2$ in the usual bases of $\mathbb{R}^4$ and $\mathbb{R}^2$. Find the matrix B of the mapping F in the bases $(1,2,3,4)$, $(1,2,4,7)$, $(0,1,1,1)$, $(0,1,1,2)$ and $(1,3)$, $(2,5)$.

4. Consider the linear transformation of $\mathbb{R}^3$ given by $A\mathbf{x} = (\mathbf{a}\cdot\mathbf{x})\,\mathbf{a} + |\mathbf{a}|^2\,\mathbf{x}$. Is A symmetric? Is it positive, that is, is $\mathbf{x}\cdot A\mathbf{x} > 0$ for all vectors $\mathbf{x}$?

5. A real matrix is orthogonal if $A^T A = AA^T = I$, where ^{T} denotes the transpose. Let the first two columns of the orthogonal matrix A be the vectors $(1/9, 4/9, 8/9)$ and $(8/9, -4/9, 1/9)$. What vectors qualify for the last column?