Linear Algebra, MATH–410 Gregor Kovačič

Homework #11 Due: Wednesday, November 17, 1999

1. Diagonalize the quadratic form

$$A(x,x) = x_1^2 + 3x_2^2 + 5x_3^2 + 4x_1x_2 + 2x_1x_3 + 8x_2x_3, \qquad x = (x_1, x_2, x_3)$$

by simultaneous row and column transformations. Also, find the new basis in which the form A is diagonal.

2. Same as Problem 1, but for the quadratic form

$$B(x,x) = x_1^2 + 3x_2^2 + 9x_3^2 + 4x_1x_2 - 8x_1x_3 - 12x_2x_3.$$

3. By using simultaneous row and column transformations to diagonalize the quadratic form

$$C(x,x) = x_1^2 + 2x_2^2 + 7x_3^2 - 4x_1x_3 - 4x_2x_3,$$

show that C is positive definite.

4. Which of the three procedures is the most efficient for determining whether a symmetric matrix is positive definite

   1. Finding its eigenvalues,
   2. Computing all its principal minors (i.e., all its subedeterminants that begin in the upper left corner),
   3. Diagonalizing the corresponding quadratic form by simultaneous row and column operations?

Explain your answer.

5. Consider the matrices

$$A = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 6 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \\ 3 & 1 & 5 \end{pmatrix}.$$

Show that A is positive definite. Solve the generalized eigenvalue problem $(B - \lambda A)\xi = 0$, where λ denotes a generalized eigenvalue and ξ a generalized eigenvector of the matrices A and B.