Linear Algebra, MATH–410 Gregor Kovačič

Homework #10 Due: Wednesday, November 10, 1999

1. For the quadratic form

$$A(x,x) = 2x_1^2 + 3x_2^2 - x_3^2 + 4x_1x_2 + 5x_1x_3 - 6x_2x_3, \qquad x = (x_1, x_2, x_3)$$

find the symmetric matrix M such that $A(x,x) = (Mx, x)$.

HINT: Since $M_{ij} = M_{ji}$, the coefficient in front of x_ix_j, for $i \neq j$, equals $2M_{ij}$.

2. Consider the bilinear form $F(x,y)$ on $\mathbb{R}^2$ defined by $F(x,y) = 3x_1y_1 - 2x_1y_2 + 4x_2y_1 - x_2y_2$. Find the matrix A such that $F(x,y) = (x, Ay)$ in the usual basis of $\mathbb{R}^2$, and the matrix B for $F(x,y)$ in the basis $(1,1)$ and $(1,2)$.

3. Diagonalize the quadratic form $A(x,y) = 3x^2 - 12xy + 7y^2$ by completing the squares.

4. Diagonalize the quadratic form $A(x,x) = 2x_1^2 - 1/2x_2^2 - 2x_1x_2 - 4x_1x_3$ by completing the squares, and find the change of basis matrix and the new basis in which A is diagonalized.

HINT: The change of basis matrix is the inverse of the change of coordinates matrix.