

1. Consider the Carnot-cycle for the *ideal gas*. Show *explicitly*, that

$$\frac{Q_{low}}{Q_{high}} = \frac{T_{low}}{T_{high}},$$

i.e., the efficiency of this engine, using ideal gas as the working substance, is $\eta = 1 - \frac{T_{low}}{T_{high}}$. (This, of course, just provides an example for Carnot's principle, that is, all reversible engines operating between the same pair of temperatures have the above optimal efficiency.)

Hint: based on the equation of state, the expression for the internal energy, and the definition of work, find explicitly the heat transferred during the isothermal segments of the cycle, Q_{high} and Q_{low} .

2. Using the equation of state and the expression for the internal energy, find the entropy of the van der Waals gas, $S(E, V)$ (the number of particles N is fixed). (Hint: you may first try to obtain $S(T, V)$ and then using again the expression for the internal energy for the v.d.W. gas, obtain $S(E, V)$.)

3. Determine the relationship between T and V for the (quasistatic) *adiabatic* process for the v.d.W. gas. Then obtain the relationship between P and V . Sketch $P(V)$ for the above process and also sketch $P(V)$ for the isothermal process of the v.d.W. gas (on the same graph), starting at the same temperature and volume. Compare them and discuss your findings.

4. Prove the following important and general thermodynamic relationship:

$$\left(\frac{\partial E}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P.$$

Note that this relationship is *independent* of the specific substance. It explicitly shows that the equation of state $P(T, V)$ and the internal energy $E(T, V)$ are not independent of each other.

Hint: start with the First Law, $dE = TdS - PdV$, then use the appropriate Maxwell relationship to obtain the desired result.

5. Consider the van der Waal's equation of state $\left(P + a \frac{N^2}{V^2}\right)(V - bN) = NkT$. The gas

undergoes and *isothermal* expansion from volume V_1 to V_2 .

(a) Determine the total change in the Helmholtz free energy F .

(b) Calculate the total change in the internal energy E . *Pretend* that you do not know the explicit form of E given in class or in the text, i.e., you are only allowed to use the above equation of state.

Hint: write the internal energy as a function of T and V , i.e., $E(T, V)$. Then write the

differential $dE = \left(\frac{\partial E}{\partial T}\right)_V dT + \left(\frac{\partial E}{\partial V}\right)_T dV$ and employ the result of Problem 1. Now you

can easily integrate to obtain the total energy change.

6. The Joule process (or free expansion). Consider a gas, which *expands into a vacuum* from V_1 to V_2 while the system is *thermally isolated*.

(a) Show that the temperature change during this process is given by

$$\Delta T = T_2 - T_1 = - \int_{V_1}^{V_2} \frac{T(\partial P / \partial T)_V - P}{C_V} dV.$$

(b) Calculate the temperature change under the Joule process for the ideal gas.

(c) Calculate the temperature change under the Joule process for the van der Waal gas.