

Lattices, Strong Dynamics  
and  
Warped Extra Dimensions  
at the Frontier

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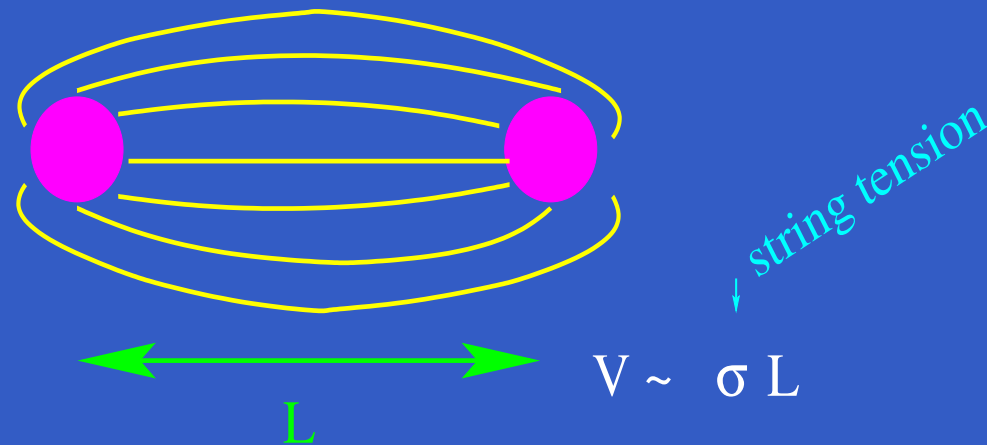
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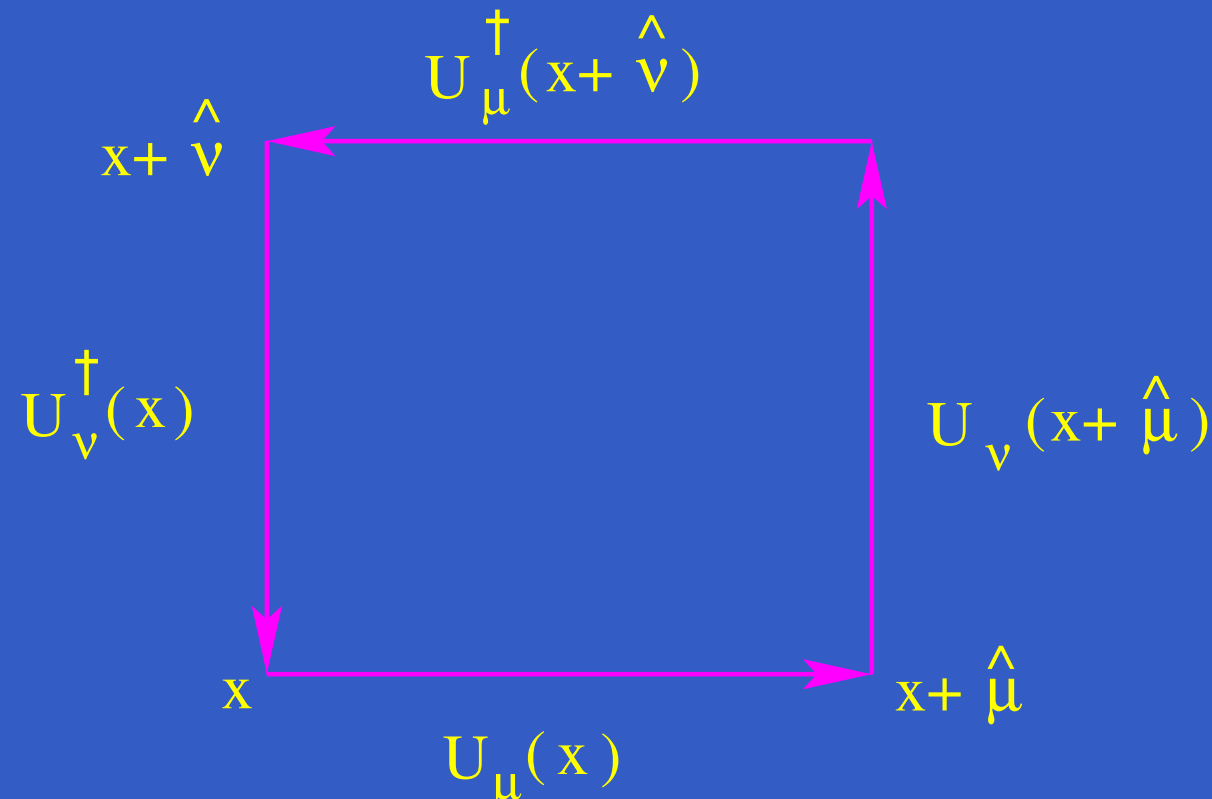
# A wonderful model

- 30 years of success for the **Standard Model**
  - = Electroweak (EW) +  
Quantum Chromodynamics (QCD)
- 70s to 90s, confirmed by experiment.
- Aspects of the theory still being worked out:
  - ◆ Mechanism of EW symmetry breaking.
  - ◆ Phase diagram of QCD ( $T$  vs.  $\mu$ ).
  - ◆ Difficult strong coupling problems ( $\Delta I = 1/2$ ,  $\epsilon'/\epsilon$ , ...).
  - ◆ Confinement mechanism (\$1 million q.).

# Confinement

- QCD = theory of “color charge” (quarks, gluons).
- All states we observe are color neutral.
- Short distance: evidence of colored constituents.
- Color is “confined” within color neutral objects: mesons, baryons, glueballs, exotics...





Link  $U_\mu(x)$  points in  $\hat{\mu}$  direction from site  $x$ .  
The  $\mu\nu$  plaquette:

$$U_{\mu\nu}(x) = U_\mu(x)U_\nu(x + \hat{\mu})U_\mu^\dagger(x + \hat{\nu})U_\nu^\dagger(x)$$

# Continuum limit

$$U_\mu(x) = \exp(iagA_\mu(x)), \quad A_\mu = \sum_{a=1}^{n^2-1} A_\mu^a T^a$$

$$\begin{aligned} U_{\mu\nu}(x) &= \exp[iag(A_\nu(x + \hat{\mu}) - A_\nu(x)) \\ &\quad - iag(A_\mu(x + \hat{\nu}) - A_\mu(x)) + \mathcal{O}(a^2)] \\ &= \exp[ia^2 g F_{\mu\nu}(x) + \mathcal{O}(a^4)] \end{aligned}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \sim (\vec{E}^a, \vec{B}^a).$$

Color electric  $\vec{E}^a$ , color magnetic  $\vec{B}^a$ .

# Continuum limit

- Continuum action obtained from:

$$S = \sum_{x, \mu\nu} \frac{2n}{g^2} [1 - (1/n) \mathbf{Re Tr} U_{\mu\nu}(x)]$$

$$= - \sum_{x, \mu\nu} \frac{a^4}{4} F_{\mu\nu}(x) F_{\mu\nu}(x) + \mathcal{O}(a^8)$$

$$\sum_x a^4 \rightarrow \int d^4x$$

# Gauge invariance

$$U_\mu(x) \rightarrow g^\dagger(x) U_\mu(x) g(x + \hat{\mu})$$

Since  $U^\dagger U = 1$ ,

$$g(x) = U_\mu(x), \quad g(x + \hat{\mu}) = 1 \quad \Rightarrow \quad U_\mu(x) \rightarrow 1.$$

- $N^4$  sites  $\rightarrow N^4$   $g$ 's.
- $\mu = 1, 2, 3, 4 \rightarrow 4N^4$   $U$ 's.
- Can only fix 1/4 of links to 1.

E.g.,  $U_4(x) \equiv 1 \rightarrow A_4(x) = 0$  (temporal gauge).

# Maximal gauges in $SU(2)$

Maximal abelian gauge:

$$\max \sum_{x,\mu} \text{Tr} [U_\mu(x) \sigma_3 U_\mu^\dagger(x) \sigma_3]$$

Direct maximal center gauge:

$$\max \sum_{x,\mu} \text{Tr} [U_\mu(x) U_\mu(x)]$$

# Direct maximal center gauge

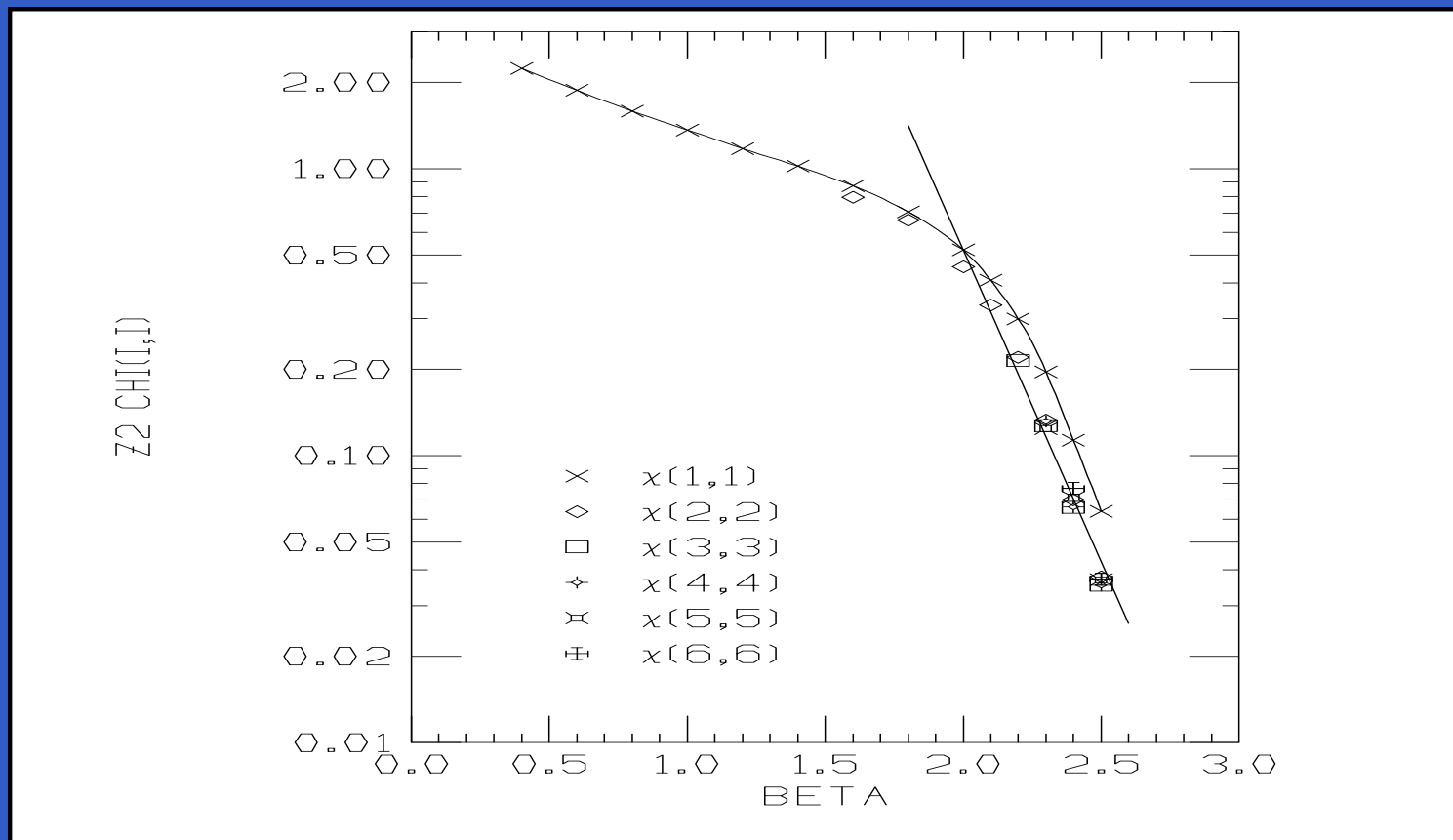
$$U_\mu(x) = \exp(iag\vec{A}_\mu(x) \cdot \vec{\sigma}/2)$$

$$\text{Tr } U_\mu^2(x) = 2 \cos(ag|\vec{A}_\mu(x)|)$$

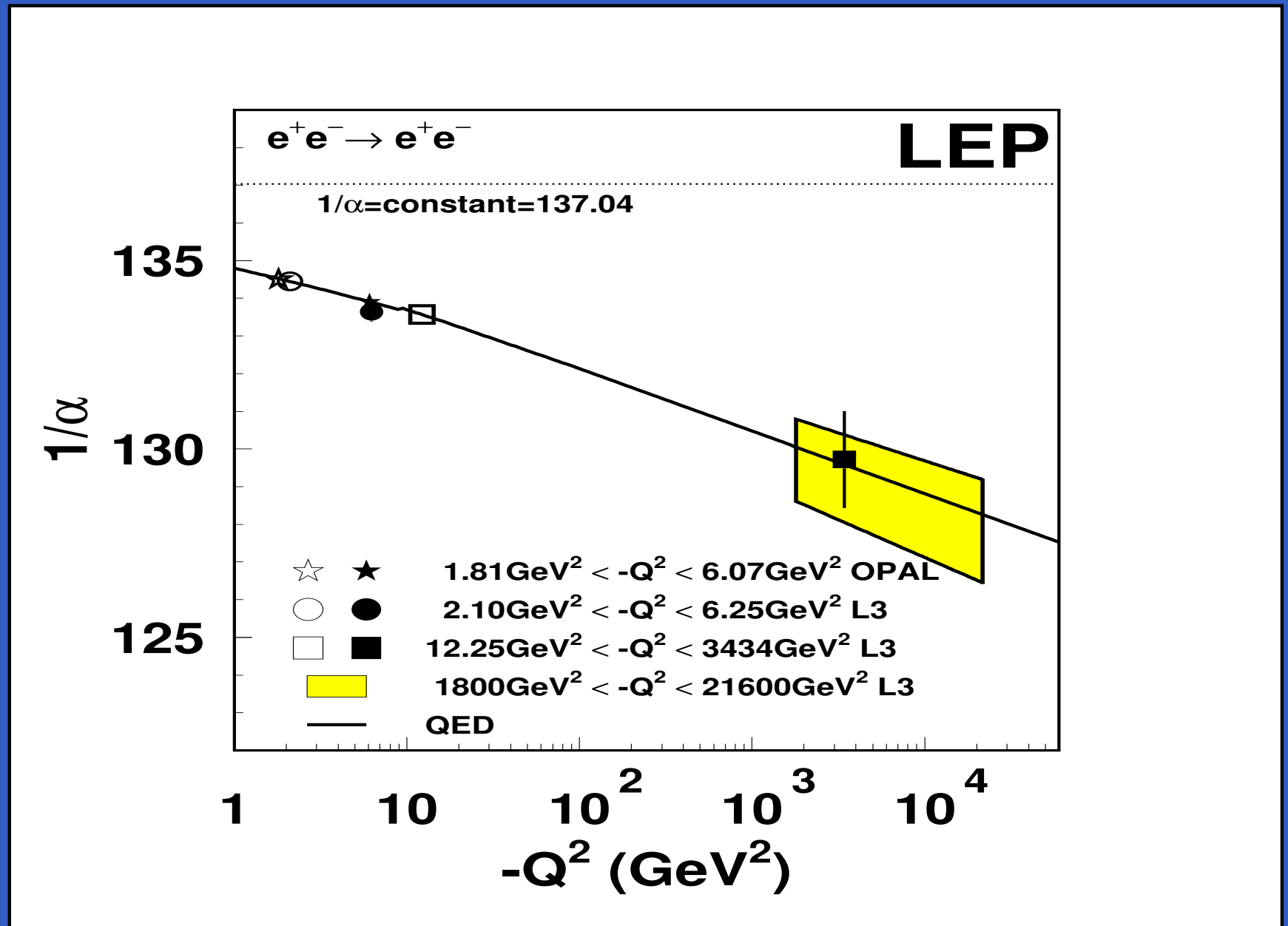
$$\max \Rightarrow ag|\vec{A}_\mu(x)| = 0 \pmod{2\pi}$$

$$U_\mu(x) = \underbrace{\cos\left(\frac{ag|\vec{A}_\mu(x)|}{2}\right)}_{\pm 1} + \frac{i}{2}\vec{\sigma} \cdot \hat{A}_\mu(x) \underbrace{\sin\left(\frac{ag|\vec{A}_\mu(x)|}{2}\right)}_0$$

- (1) Fix to maximal center gauge,  $U_\mu(x) \approx \pm 1$ .
- (2) Project links  $U_\mu(x) \equiv \pm 1$ , exactly.
- (3) Measure confinement observable: string tension. [Del Debbio, Faber, JG, Greensite, Olejnik 98]



# Effective electromagnetic coupling $\alpha$ [Mele, hep-ex/0610037]



# Strange predictions

- Some Standard Model interactions  $\rightarrow \infty$  at  $E$  finite:

(1) Weak hypercharge:

$$B = \cos \theta_W A - \sin \theta_W Z^0.$$

photon                  Z-boson

(2) Higgs self-coupling:

$$H + H \rightarrow H + H.$$

# Gravity turns strong too

- In theorists' units  $\hbar = c = 1$ , Newton's constant  $G$  has dimensions:

$$[G] = (\text{mass})^{-2}.$$

- The mass scale—**Planck scale**—is:

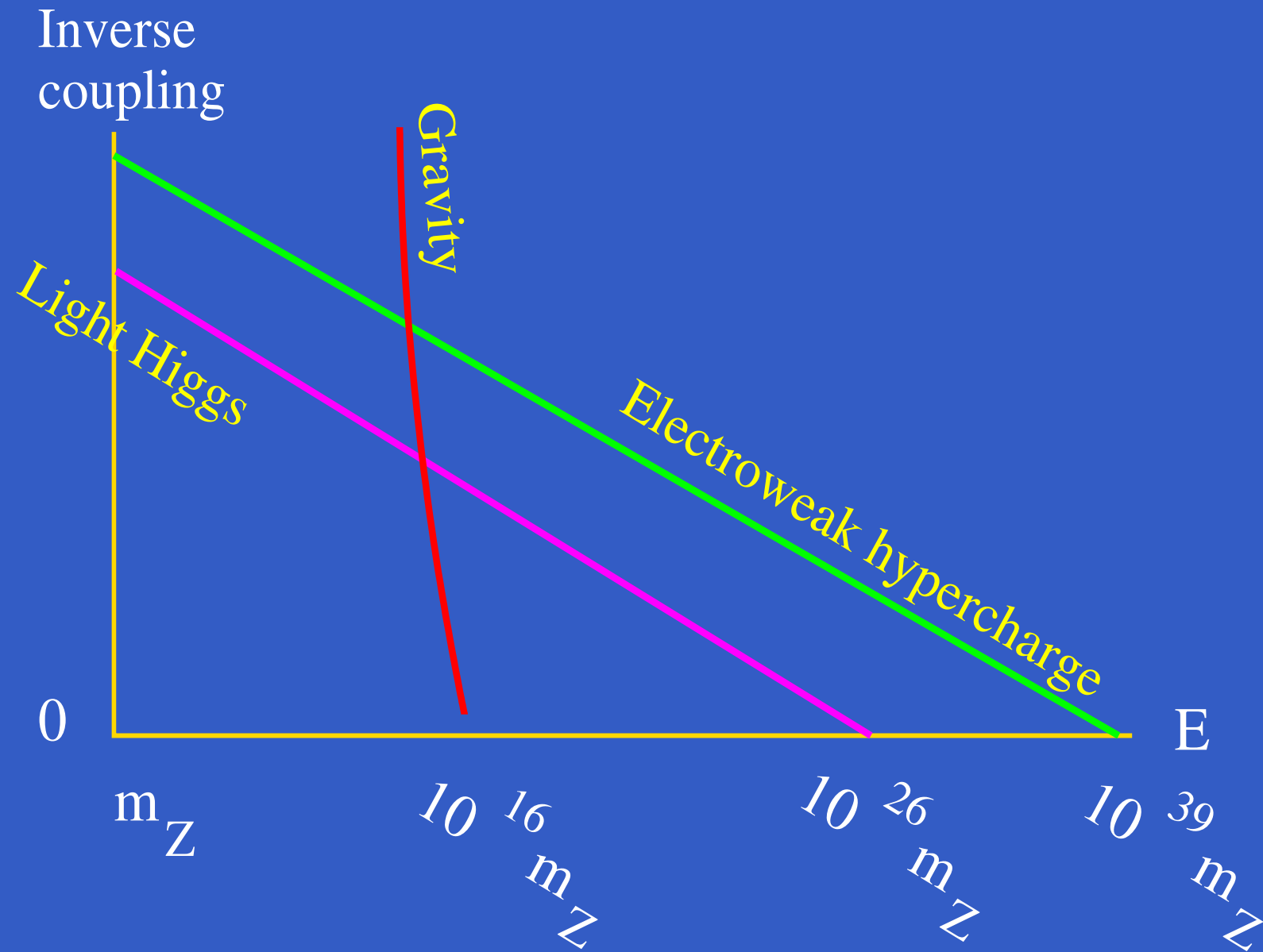
$$m_P = 1/\sqrt{8\pi G} \sim 10^{16} m_Z.$$

- Effective gravitational coupling

$$\lambda_{\text{grav.}} \sim E^2 G \sim (E/m_P)^2.$$

- Becomes  $\mathcal{O}(1)$ —**strong**—at  $E \sim m_P$ .

# Scales of mystery



# What does this mean?

- $\infty \rightarrow$  unphysical  $\rightarrow$  approximation breaking down:

## triviality problem

- Standard Model (SM) = “effective” or “cutoff” theory = finite range of validity:



$$m_{\text{cutoff}} \lesssim m_P = \text{unobtainable.}$$

# If cutoff so big, who cares?

- The classical Higgs mass in the Standard Model is:

$$m_H^2 \sim m_Z^2.$$

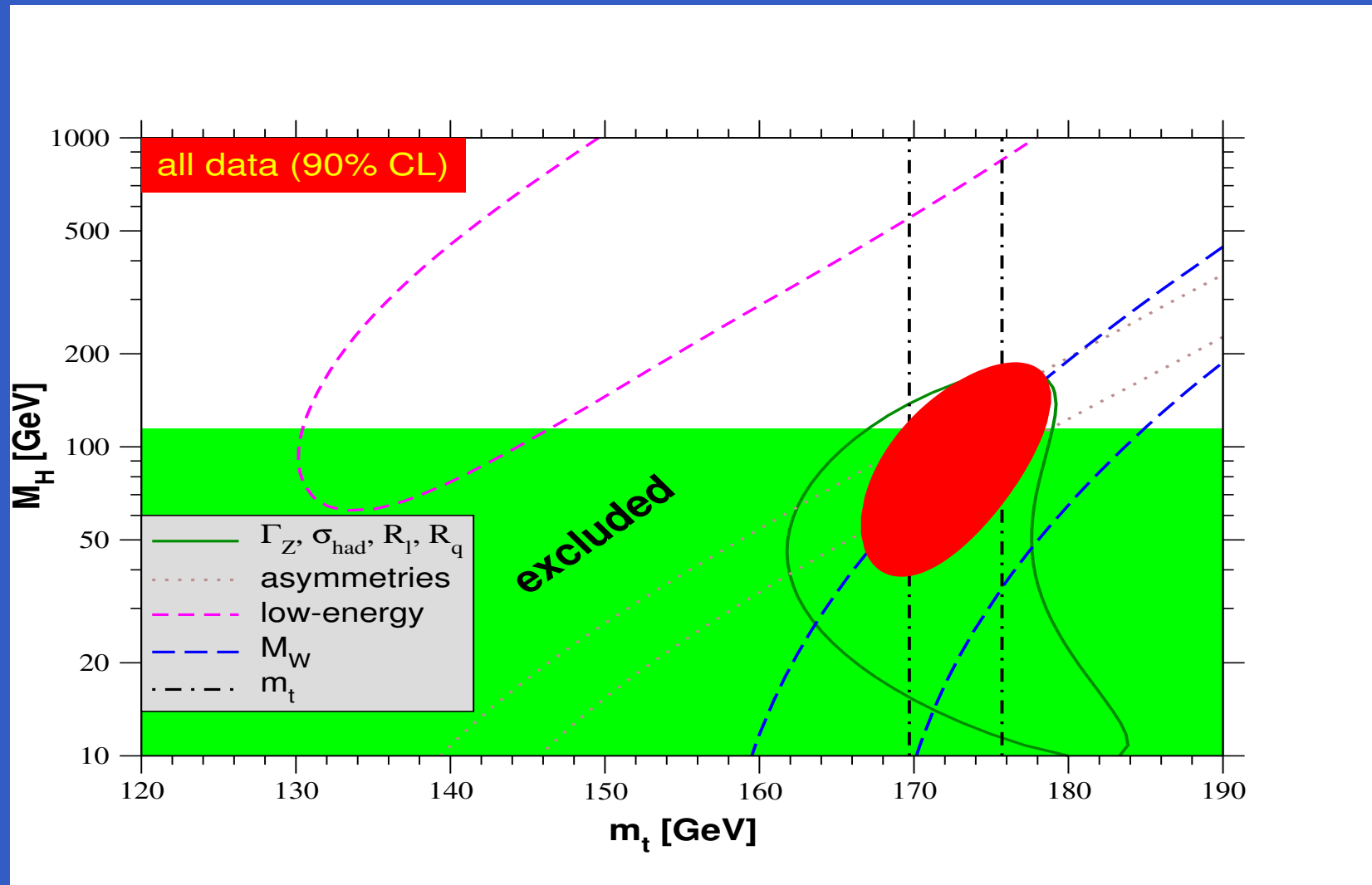
- The quantum corrections shift this mass by:

$$\Delta m_H^2 \sim m_{\text{cutoff}}^2.$$

- Physical mass = total:

$$m_H^2 + \Delta m_H^2 \sim m_{\text{cutoff}}^2 \lesssim m_P^2.$$

# Experimental fit constraints



Particle Data Group, 2006

# What gives?

- But the Standard Model quantum corrections predict a Higgs mass of the cutoff scale.
- What keeps the Higgs so light?
- This is the **gauge hierarchy problem**.

# Keeping $\Delta m_H^2$ small

- All solutions in a nutshell:

(1)  $m_{\text{cutoff}} \lesssim 1000 \text{ GeV} = 1 \text{ TeV}.$

$\Rightarrow$  Beyond the Standard Model  
at our fingertips!

- (2) New particles and symmetries such that:

$$\Delta m_H^2 \lesssim m_H^2,$$

in the Beyond the Standard Model theory.

# Around the corner

- For  $\Delta m_H^2 \lesssim m_H^2$  to work:

$$\text{mass(new particles)} \sim m_Z.$$

- We will be looking for these Beyond the Standard Model particles at the Large Hadron Collider.

# LHC is coming...



- 225 MB of data will be recorded every second!  
(20000 GB = 4400 DVD's per day, full luminosity)
- It will be an EXCITING period in physics.

# Solutions to the hierarchy problem

- **Cancellations:** supersymmetry (SUSY).
- **Fluffiness:** Composite Higgs, transparent to high frequencies.
  - ◆ Old: Technicolor, ...
  - ◆ New: Warped extra dims. (via AdS/CFT), ...
- **Hierarchy is a fake:** Large extra dimensions.

# Supersymmetry

- It is a symmetry that transforms  
**bosons**  $\leftrightarrow$  **fermions**.
- If the symmetry is exact, the masses of particles show a **Bose-Fermi degeneracy**:

$$m_B = m_F.$$

# Superpartners

- Supersymmetry predicts **superpartners**:

electron (fermion)  $\Rightarrow$   
scalar electron (boson) = “selectron”.

- Recall, however, that SUSY predicts:

$$m_B = m_F.$$

- Because we have not observed superpartners, SUSY must be spontaneously broken. E.g.,

$$m_{\text{selectron}} \gtrsim m_Z.$$

# Tackling the nonperturbative

- Models for SUSY-breaking involve **nonperturbative dynamics** of SUSY gauge theories.
- Higgs effect at high scale  
→ frozen at weak coupling  
→ **calculable models.**
- Other scenarios:  
strongly interacting  
→ **“noncalculable” models.**

# Gravity dual

- One approach:  
Find weakly coupled gravity dual.
- AdS/CFT correspondence:
  - ◆ Yang-Mills dynamics (=“CFT”)
  - ◆ takes on a gravitational meaning (=“AdS”).
- In recent work w/ T. Gherghetta (Minnesota), M. Gavelle (Lausanne):
  - ◆ MSSM in bulk.
  - ◆ Deformed  $\text{AdS}_5$ .
- Susy-breaking from deformed metric, inspired by Type IIB supergravity solutions.  
[Borokhov, Gubser 02; Kuperstein, Sonnenschein 03]

- Slice of  $\text{AdS}_5$ :

$$ds_5^2 = A^2(z) (-dt^2 + d\vec{x}^2 + dz^2),$$

$$A^2(z) = \frac{1}{z^2}.$$

- 1+3 dim's:  $t, \vec{x}$ .
- 5th dim.:  $1 \leq z \leq z_{\text{IR}}$ .
- Warp factor:  $A^2(z)$  (units of AdS curvature).
- UV brane:  $z_{\text{UV}} = 1$ .
- IR brane:  $z_{\text{IR}} = m_P / (0.1 \text{ to } 10 \text{ TeV})$ .

# Deforming AdS<sub>5</sub>

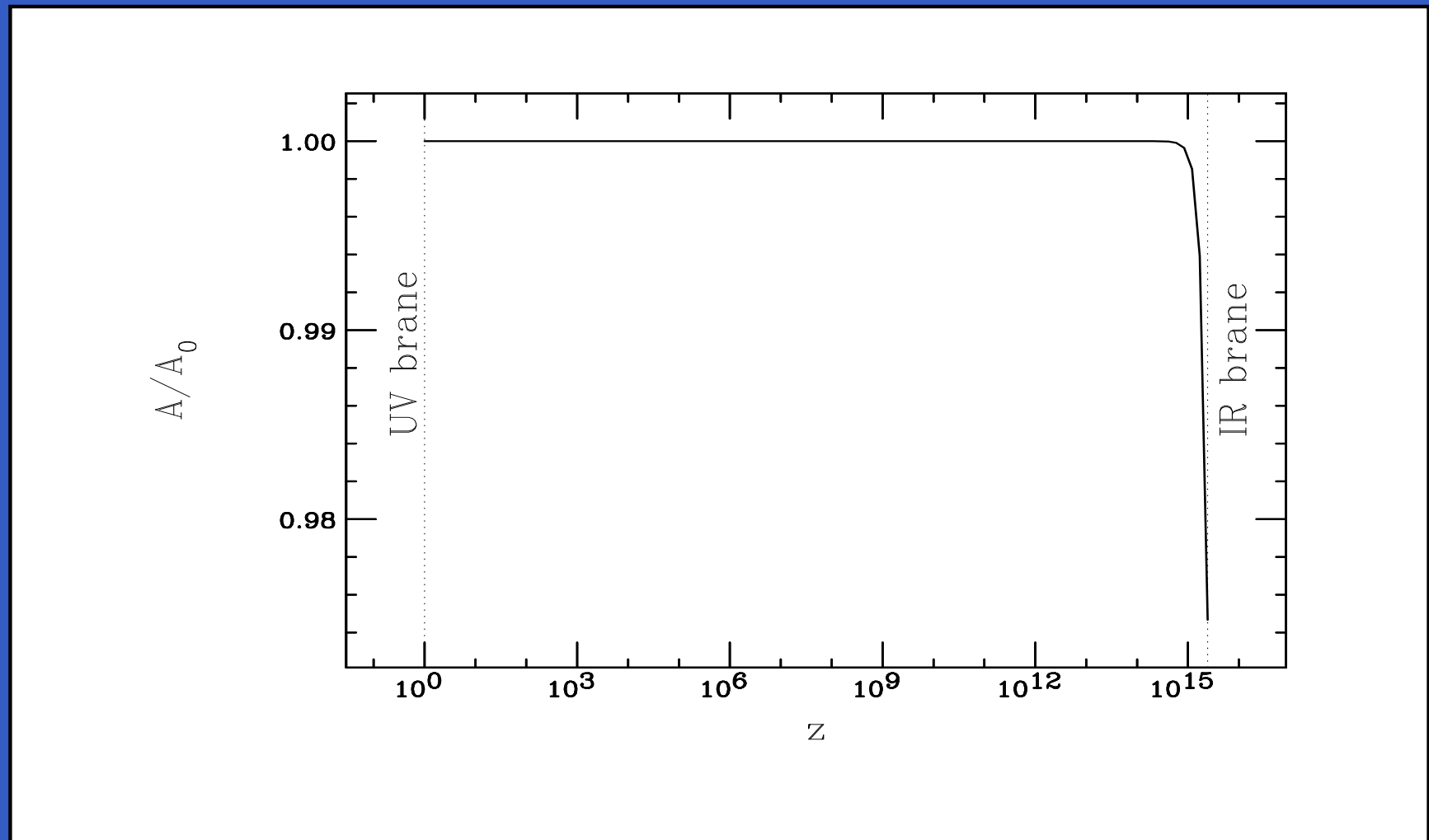
- Deform AdS according to the dim'l reduc. of Kup.-Sonn. soln.:

$$ds_5^2 = A^2(z) (-dt^2 + d\vec{x}^2 + dz^2),$$

$$A^2(z) = \frac{1}{z^2} \left( 1 - \epsilon \cdot (z/z_{\text{IR}})^4 \right).$$

- Domain:  $1 \leq z \leq z_{\text{IR}}$ .
- Dial 1:  $\epsilon \sim 0.1$ . [ $\epsilon = 0.05$  in following.]
- Dial 2:  $z_{\text{IR}} = m_P / (1-100 \text{ TeV})$ .
- SUSY lim.:  $\epsilon \rightarrow 0$ .
- Interpretation: **DSB in the  $SU(N_c)$ .**

# A few per cent violence in the IR: Will it matter?



- \* Ratio deformed/undeformed warp factor. \*
- \* Unchanged except very near IR brane. \*

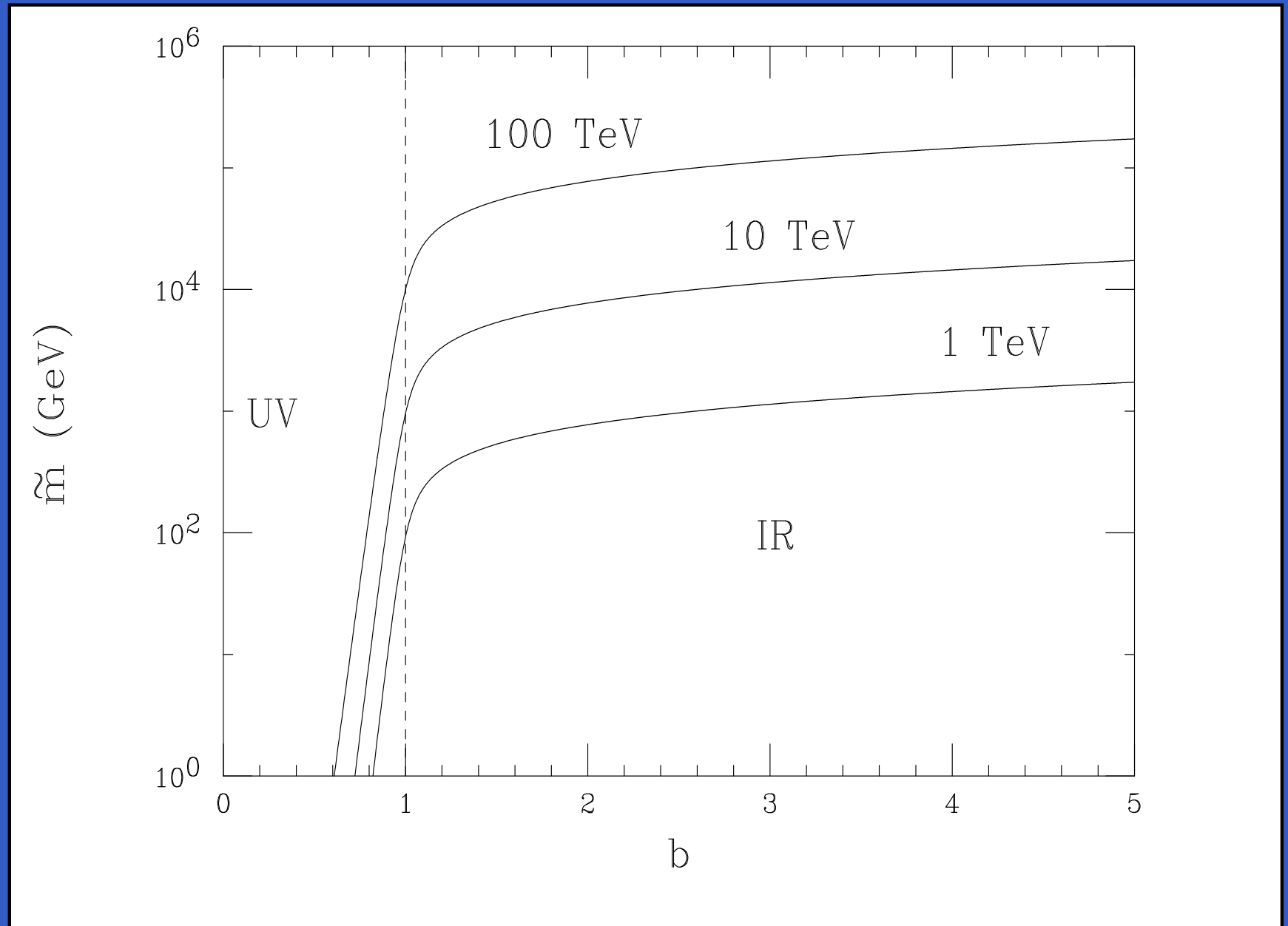
# Scalars

- Solutions parameterized by boundary mass  $b$ .
- LO profile  $\sim z^{b-1}$ , unchanged.
- Zero modes lifted:

$$\tilde{m} \approx \sqrt{\epsilon(b-1)(b+10)} z_{\text{IR}}^{-1} \quad (b > 1)$$

$$\tilde{m} \approx \sqrt{\epsilon(1-b)(b+10)} (z_{\text{IR}})^{b-2} \quad (0 < b < 1)$$

# Masses of quasi-zero mode scalars



# Fermions

- Exact zeromodes persist.

$$0 = (\gamma^\alpha \partial_\alpha + \gamma_5 \partial_z + cA) \hat{\psi}, \quad \hat{\psi} \equiv A^2 \psi,$$

$$\hat{\psi} = \chi(x) A^2(z) f(z), \quad (\pm \partial_z + cA)(A^2 f) = 0$$

$$f_\pm = N_\pm A^{-2}(z) \exp\left(\mp c \int_1^z dz' A(z')\right).$$

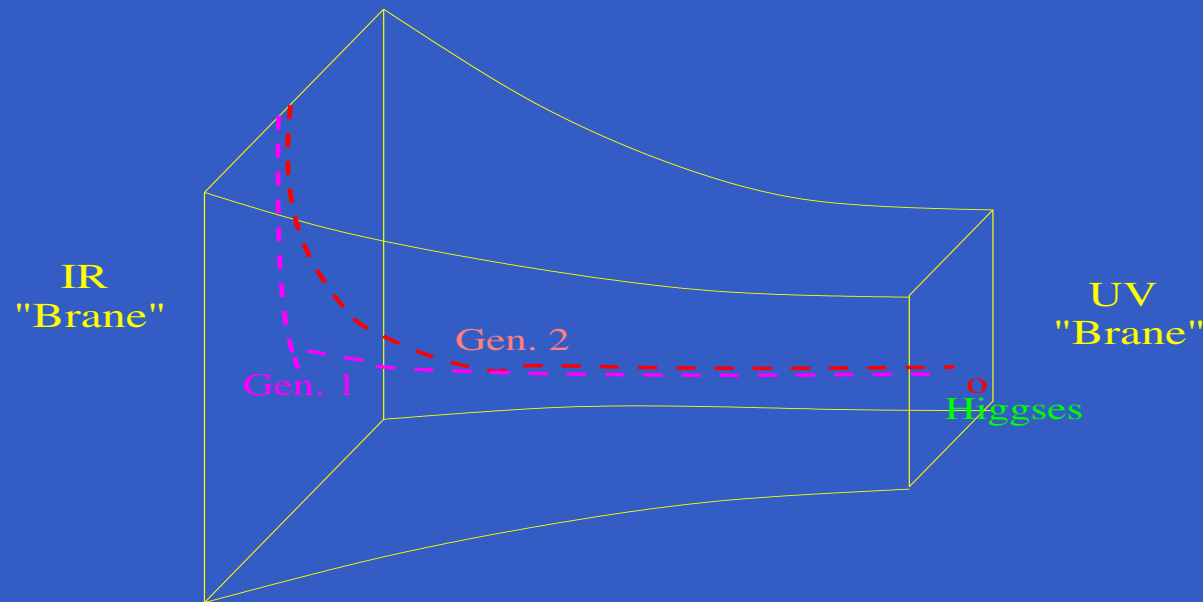
- Profile virtually unchanged:

$$f_\pm \approx N_\pm z^{\frac{1}{2} \mp c_\pm} \left[ 1 + \frac{\epsilon z^4}{4z_{\text{IR}}^4} \left( 1 - \frac{1}{2} c_\pm \right) \right].$$

# Generational hierarchies

- Higgses at UV brane.
- Use **profiles**  $\sim z^{\frac{1}{2}-c_L}$ ,  $z^{\frac{1}{2}+c_R}$  for ferm. hier.

$$m_{4d} = \frac{\langle H \rangle}{2} Y_{5d} \sqrt{\frac{1-2c_L}{z_{\text{IR}}^{1-2c_L} - 1}} \sqrt{\frac{1+2c_R}{z_{\text{IR}}^{1+2c_R} - 1}}.$$



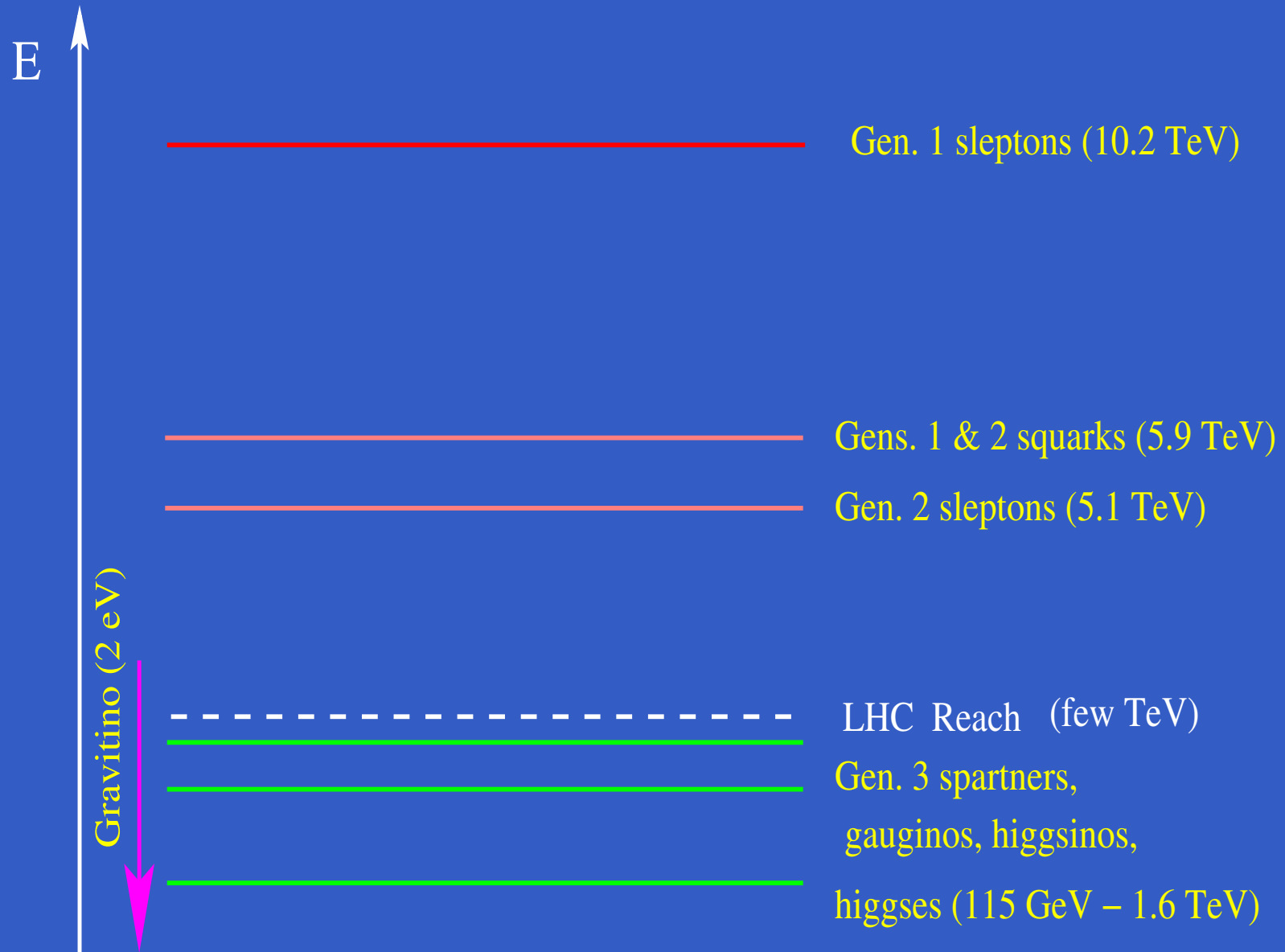
# Scalar-fermion correlation

- SUSY relates scalar/fermion masses:

$$b = \frac{3}{2} - c_L.$$

- Scalar/fermion profiles approx. same.
- $\Rightarrow$  scalar partners of light fermions IR localized ( $b > 1$ ), [HEAVY]
- partners of heavy fermions UV localized ( $b < 1$ ). [LIGHT]
- Gauginos, stop, sbottom, stau: radiative mass.

# Example spectrum



# Dual interpretation

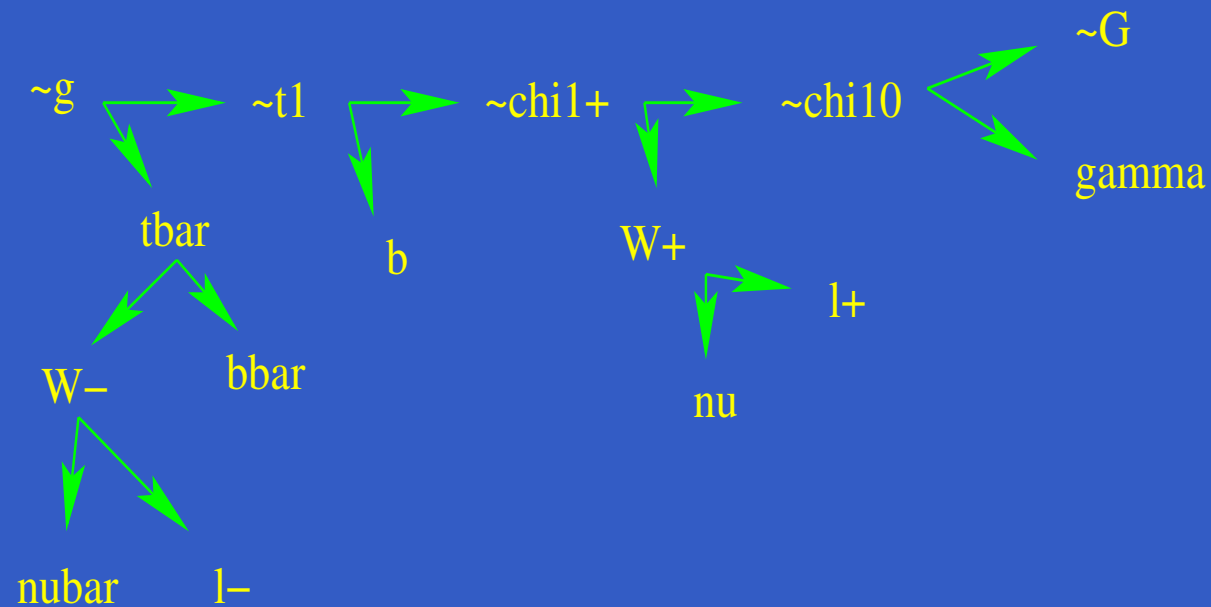
- IR-localized: composites of new strong interaction.
- UV-localized: elementary.
- Composite fermions massless: SM chiral gauge symmetry.
- SUSY-breaking similar to:
  - ◆ More minimal SSM. [Cohen, Kaplan, Nelson 96]
  - ◆ Single-sector models. [Arkani-Hamed, Luty, Terning 97-98]

# Example spectrum (5d numerical + 2-loop Softsusy [Allanach])

$\tilde{e}_L, \tilde{e}_R, \tilde{\nu}_{eL}$	10160, 10150, 10160 GeV
$\tilde{\mu}_L, \tilde{\mu}_R, \tilde{\nu}_{\mu L}$	5145, 5130, 5145 GeV
$\tilde{d}_L, \tilde{d}_R, \tilde{u}_L, \tilde{u}_R$	5905, 5885, 5970, 5890 GeV
$\tilde{s}_L, \tilde{s}_R, \tilde{c}_L, \tilde{c}_R$	5905, 5885, 5970, 5890 GeV
$\tilde{g}$	<b>1615</b> GeV
$\tilde{b}_1, \tilde{b}_2, \tilde{t}_1, \tilde{t}_2$	1354, 1369, 1253, 1369 GeV
$\tilde{\tau}_1, \tilde{\tau}_2, \tilde{\nu}_{\tau L}$	511, 630, 633 GeV
$\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$	478, 593 GeV
$\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$	<b>288</b> , 480, 511, 598 GeV
$h^0, A^0, H^0, H^\pm$	<b>115</b> , 646, 646, 651 GeV
$\tilde{G}$	<b>2.35</b> eV

# LHC predictions

- Typical cascade decay from  $gg \rightarrow \tilde{g}\tilde{g}$ :



- NLSP decays

$$\begin{array}{ccc} \tilde{\chi}_1^0 & \rightarrow & \gamma + \tilde{G} \\ 288 \text{ GeV} & & 2 \text{ eV} \end{array}$$

give strong signal:

$$\begin{array}{ccc} p\bar{p} & \rightarrow & \gamma\gamma + \cancel{E}_T + X \\ & & \text{hard} \quad \text{lots} \end{array}$$

## $\gamma\gamma + \cancel{E}_T$ in our model

- At Tevatron energy 1.96 TeV, PYTHIA says:  
 $\sigma \times BR = 2.5 \times 10^{-5}$  pb  
 $\sim 0$  events at DØ+CDF w/  $2 \text{ fb}^{-1}$ .
- No constraint.

## $\gamma\gamma + \cancel{E}_T$ in our model

- At LHC energy 14 TeV, PYTHIA says:  
 $\sigma \times BR = 3.5 \times 10^{-2}$  pb.
- 1000 times the rate of Tevatron!
- Obtain  $\sim 35$  events w/  $1 \text{ fb}^{-1}$  ( $\sim 1$ st year).
- Backgrounds larger at LHC:  
Will we see the signal?

# Backgrounds

- Real:

$$pp \rightarrow \{gg, q\bar{q}\} \rightarrow \gamma\gamma,$$

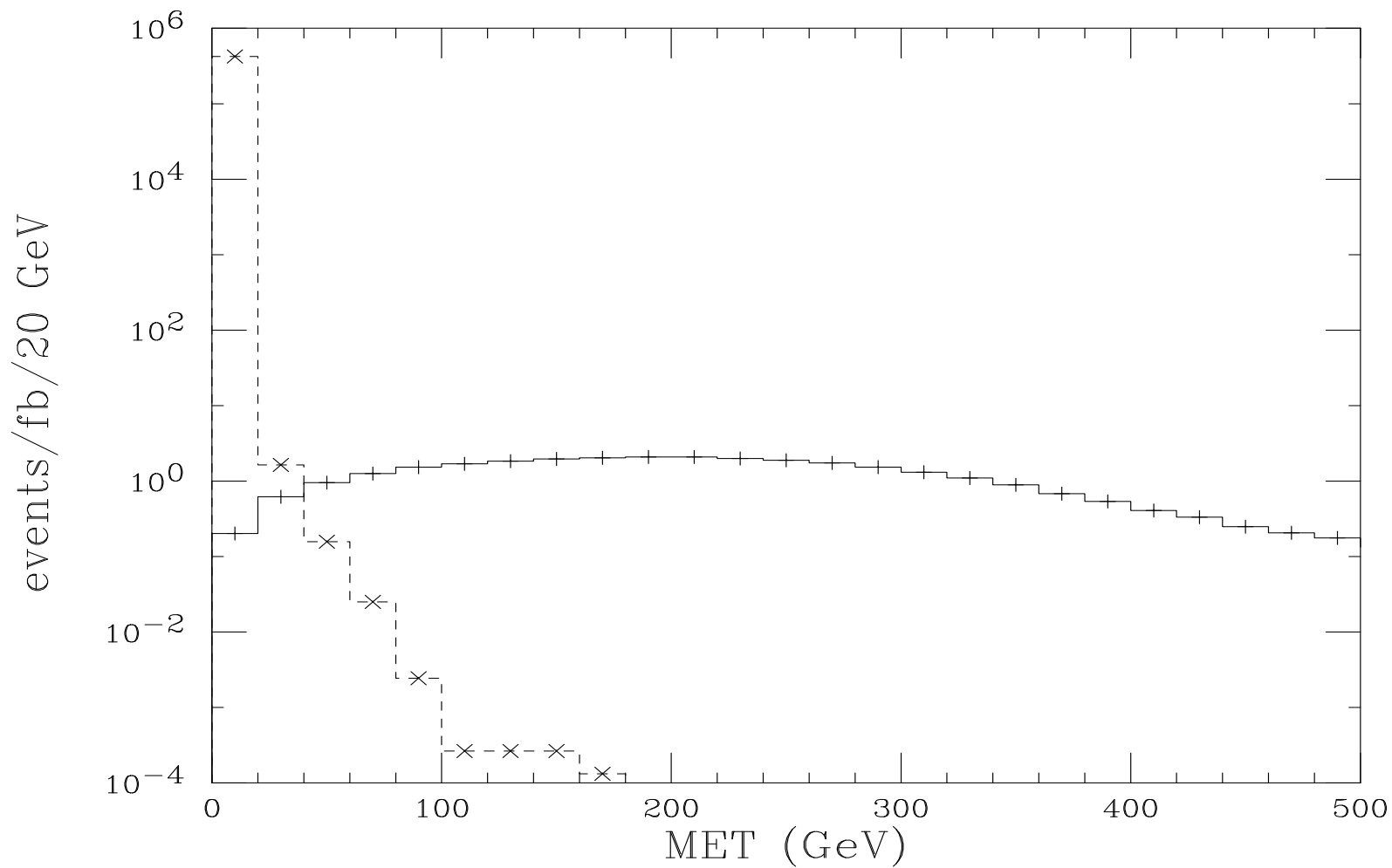
- Fake: (mainly  $j \sim \pi^0$ ):

$$pp \rightarrow \gamma j_{\text{fake}}, \quad pp \rightarrow j_{\text{fake}} j_{\text{fake}}.$$

channel	$\gamma\gamma$	$\gamma j$	$j j$
cross section	$0.15 \mu\text{b}$	$0.12 \text{ mb}$	$55 \text{ mb}$

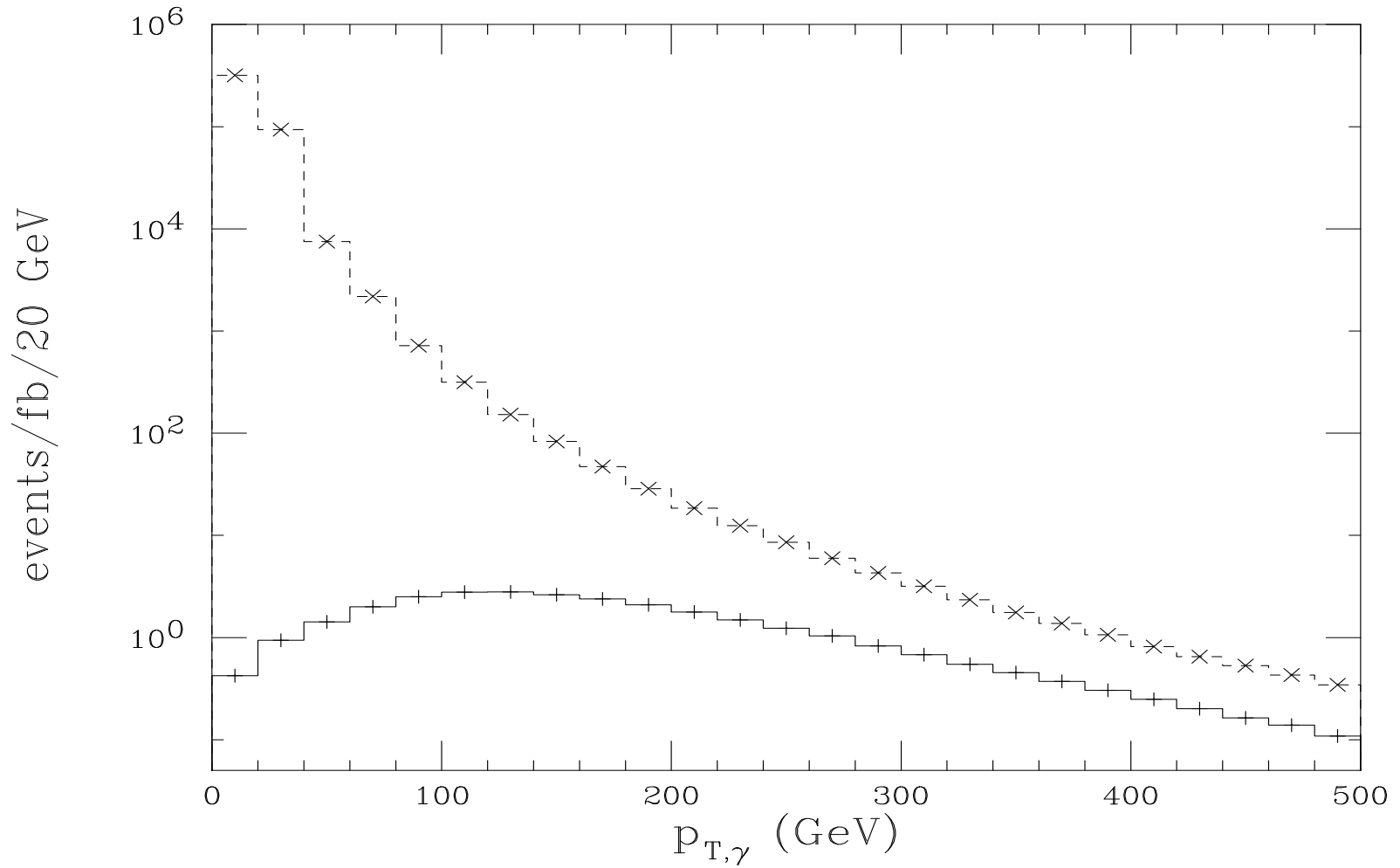
- Comparable since

$$j_{\text{fake}}/j \sim 10^{-3}$$



pT

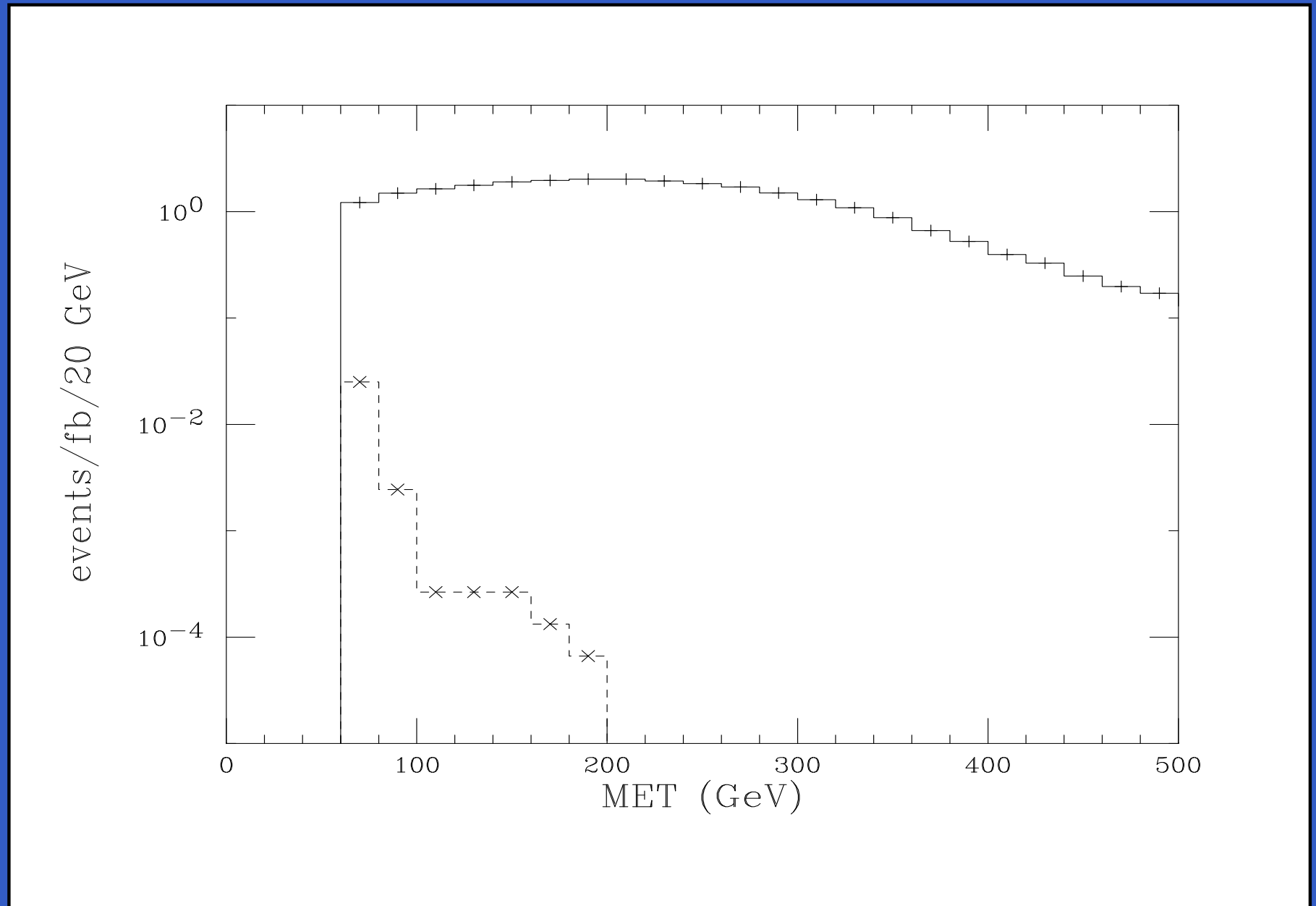
[SUSY = solid, SM( $\gamma\gamma$ ) = dashed]



- Distributions suggest:

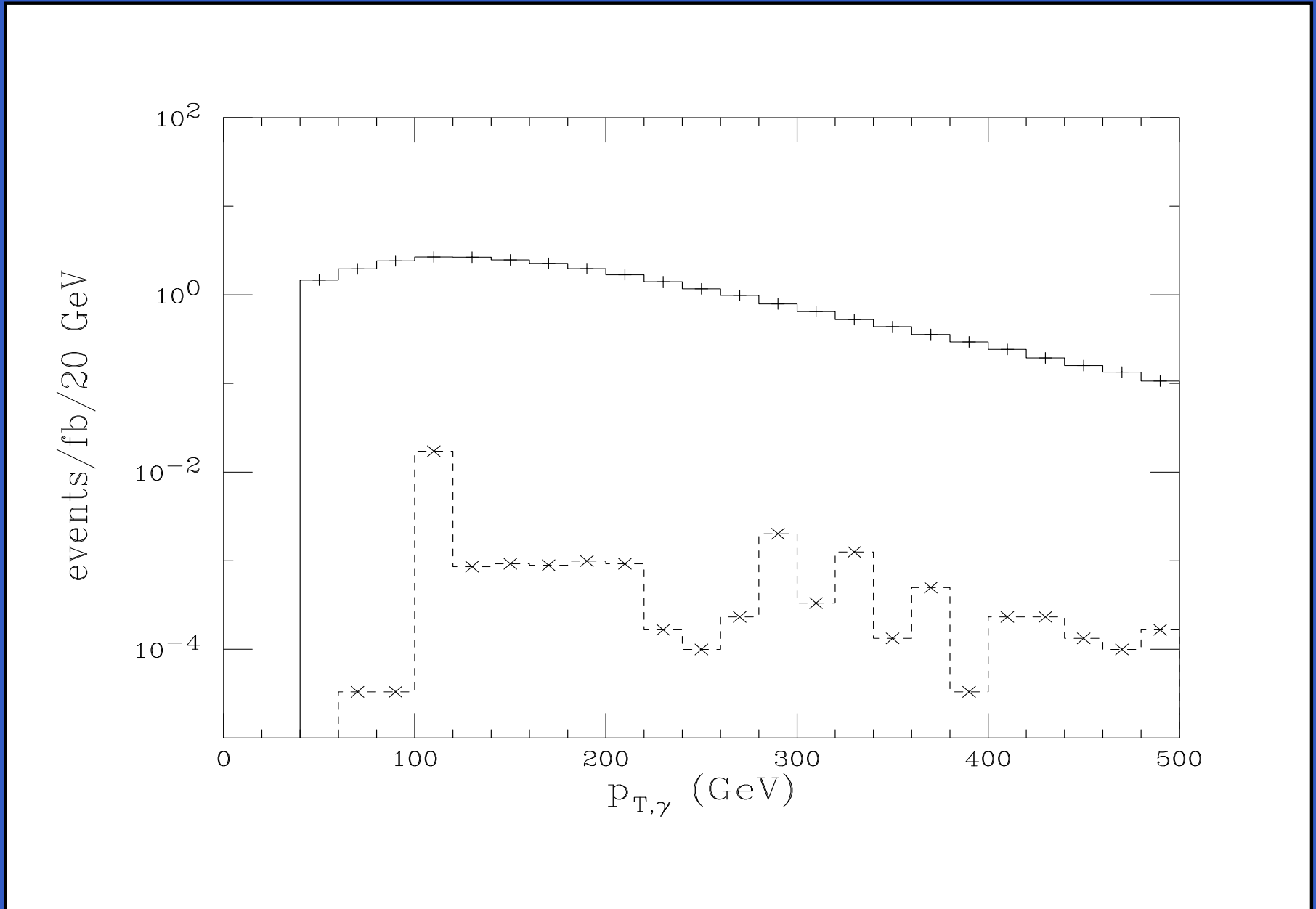
$$p_{T,\gamma} \geq 40 \text{ GeV}, \quad \cancel{E}_T \geq 60 \text{ GeV}$$

- Don't lose much signal.
- Background eliminated.



pT w/cuts

[SUSY = solid, SM( $\gamma\gamma$ ) = dashed]



# Summary w/cuts

Integrated Luminosity	SUSY	SM $2\gamma$	SM $2\gamma$ + fakes
$1 \text{ fb}^{-1}$	27.6	0.0285	$\lesssim 0.1$
$10 \text{ fb}^{-1}$	276	0.285	$\lesssim 1$

# Summary

- Far fewer parameters than new particles: predictive.  
(2-loop evaluation with publicly avail. code.)
- Gravity dual for **single-sector** models of spontaneous SUSY-breaking.
- Our simulations imply  $2\gamma + \cancel{E}_T$  is a very clean signal for the model at LHC.
- On edge of B-physics FCNC's and LFV's  
⇒ experiments, keep searching!  
(Upcoming experiments will test.)

# Conclusions

- Interesting questions remain to be answered in the Standard model:
  - ◆ EW symmetry breaking
  - ◆ strong effects in QCD [ $Z_3$  vortex confinement?]
- Problems of the Standard Model (triviality, hierarchy problem, etc.) imply “new physics” somewhere above 100 GeV.
- Example given: new strong dynamics and SUSY-breaking modeled with 5d warped extra dimension.
- Encouraging recent progress in strong SUSY & chiral gauge theory on lattice. [4d simulations needed]