

Advances and applications of lattice supersymmetry

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Introduction

- Several motivations exist for efforts to formulate supersymmetric field theories on a lattice.
- It is difficult to formulate these theories in a way that avoids fine-tuning of counterterms (besides any bare/physical tuning that already occurs in the continuum).
- Nevertheless, there have been many promising formulations developed of late.
- But to what use should/would we put them?
- Look to motivations.

Definition

- Say what is meant by

$$Z[S] = \int \exp(-S).$$

- At a formal level we know

$$Z[S; a] = \int_{\mathcal{D}(a)} [d\mu(\phi; a)] \exp(-S[\phi; a])$$

and $a \rightarrow 0$ limiting sequence for ingredients.

Definition

- Due to universality, there is no unique definition of $Z[S; a]$.
- But one wants a classification based on the answer to:
 - ◆ Perturbative defn.:
no CT / local CTs / anomaly
 - ◆ Nonperturb. defn.:
no CT / local CTs / anomaly
- Favorite answer: no CT.

No CT examples

- Q-exact susy-QM:

$$S = QX, \quad Q^2 = 0.$$

- ◆ Monte-Carlo evidence (spectrum degeneracy, Ward identities) [Catterall, Gregory 00].
- ◆ All-order perturbative proof [JG, Poppitz 04].
- ◆ Nonperturbative proof by transfer matrix calculation [JG, Poppitz 04].
- ◆ Note: Q-exact \rightarrow cancellations \rightarrow no CT.

No CT examples

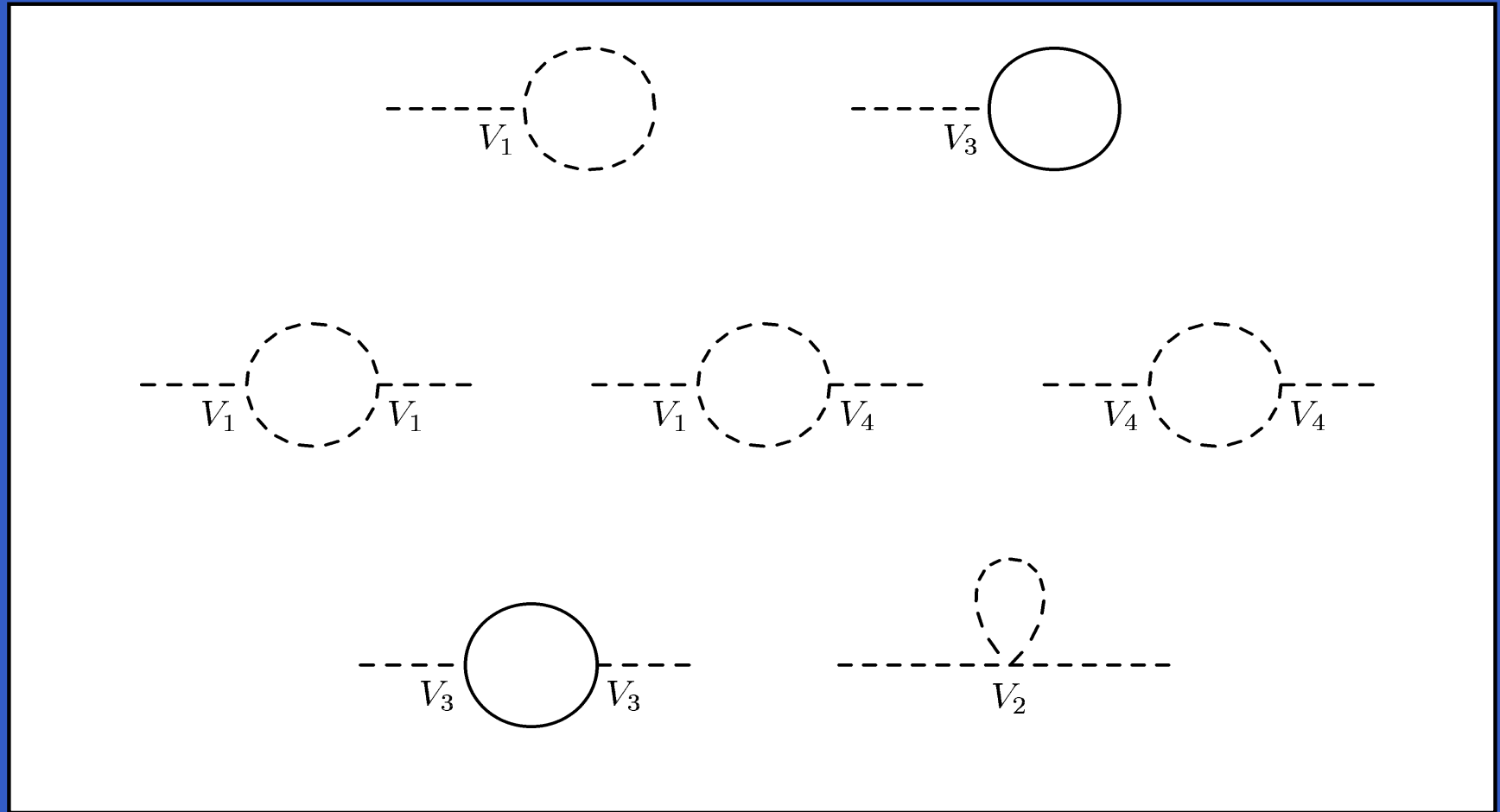
N	m_B	m_F
16	10.60(3)	10.64(5)
32	12.80(2)	12.91(4)
64	14.47(2)	14.52(2)
128	15.63(4)	15.63(4)
256	16.19(3)	16.28(4)

Spectrum degeneracy in Q-exact SQM. Taken from [Catterall, Gregory 00]. $L = Na = \text{fixed}$.

No CT examples

- Q-exact 2d Wess-Zumino (a.k.a. $\mathcal{N} = 2$ Landau-Ginsburg) [Elitzur, Schwimmer 83] [Sakai, Sakamoto 83]:
 - ◆ All orders perturbative proof [JG, Poppitz 04].
 - ◆ Similar to Q, Q^\dagger -preserving spatial [Elitzur, Schwimmer 83].
 - ◆ Note: Q-exact \rightarrow cancellations \rightarrow no CT.
 - ◆ Various Monte Carlo evidence: (spectrum degeneracy, Ward identities, R-symmetry) [Beccaria, Curci, D'Ambrosio 98] [Catterall, Karamov 01-02] [JG 05]

Cancellations in Q-exact 2d WZ



$D_{UV} \geq 0$ diagrams

$\mathcal{N} = 1$ 4d SYM w/ chiral fermions

- From [Curci, Veneziano 86] we know that $\mathcal{N} = 1$ 4d SYM with Ginsparg-Wilson fermions require no CTs.
- Overlap-Dirac was proposed [Narayanan, Neuberger 95] and sketched [Maru, Nishimura 97].
- But LO simulation studies, such as glueball spectra, have yet to be attempted.

$\mathcal{N} = 1$ 4d SYM w/ chiral fermions

- IR effective theories have been proposed by continuum theorists [Veneziano-Yankielowicz and extensions by Sannino et al., Gabadaze et al., Louis et al.]; it would be nice to test them!
- Domain-wall-Dirac fermions were proposed [Nishimura 97] [Neuberger 98] [Kaplan, Schmaltz 99] and briefly studied [Fleming, Kogut, Vranas 00].
- It deserves another push!

No CT examples

- Deconstruction models (quiver lattice, orbifold matrix model), various extended SYM in 2d, 3d and 4d [Cohen, Kaplan, Katz, Unsal 03].
 - ◆ The 2d examples definitely have no fine-tuning in perturbation theory.
 - ◆ For the $d > 2$ examples, it is less clear what really happens.
 - ◆ If $O(d)$ is recovered, an interesting susy theory is obtained.
- Q-exact compact (2,2) SYM in 2d [Sugino 03-06]. No fine-tuning in perturbation theory.

- Extended SYM in 2d & 4d [D'Adda, Kanamori, Kawamoto, Nagata 05].
- Modified Leibnitz rule for susy variation:

$$s_A[\Phi(x_1)P(x_2, \dots)] = s_A[\Phi(x_1)]P(x_2, \dots) + (-)^{F(\Phi)}\Phi(x_1 + a_A)s_A[P_2(x_2, \dots)].$$

- Action with 4 equivalent forms:

$$\begin{aligned} S &= \sum_x \text{Tr} \, s\tilde{s}s_1s_2\Psi_{x,x} = - \sum_x \text{Tr} \, \tilde{s}s s_1s_2\Psi_{x,x} \\ &= \sum_x \text{Tr} \, s_1s_2s\tilde{s}\Psi_{x,x} = - \sum_x \text{Tr} \, s_2s_1s\tilde{s}\Psi_{x,x}. \end{aligned}$$

- Nilpotency:

$$s^2 = \tilde{s}^2 = s_1^2 = s_2^2 = 0.$$

Thus action invariant under noncommutative (2,2) susy.

- Renormalization needs more study. Does noncommutativity matter?

Other no CT examples/claims

- Twisted (Q-exact) geometrical (2,2) SYM in 2d [Catterall 04-06]. MC data seems to indicate no need for CTs.
- 4d WZ with GW fermions [Bonini, Feo 04-05]. Nonlinear, perturbative definition of Q .
- Twisted (Q-exact) nonlinear σ model, (2,2), 2d [Catterall, Ghadab 03] [JG, Poppitz 04]. MC data seems to indicate no need for CTs [Catterall, Ghadab 06].

Nonholomorphic woes

- Continuum susy tricks usually partly fail to determine IR eff. theory.

- Schematically:

$$\int d^4\theta K(\Phi, \bar{\Phi}) + \left[\int d^2\theta W(\Phi) + \text{h.c.} \right]$$

- W **not renormalized** in perturbation theory.
- W sometimes **completely determined**, once symmetries accounted for.
- Generically none of these nice features hold for K .

Phenomenology

- Lack of control over nonholomorphic data is distressing.
- So-called **supersymmetry-breaking soft-terms** largely determine superpartner spectra and couplings for the **MSSM**.
- Depend on **Kähler potential** K .

Calculation

- Lattice Monte Carlo simulations would, as a first step, give us a handle on vevs ϕ_0 of scalars, and the spectrum of light states.
- This constrains:

$$V_\phi, \quad V_{\phi\phi}, \quad V_{\phi\bar{\phi}}$$

evaluated at ϕ_0 .

Calculation

- Both the Kähler potential K and superpotential W play a role in the scalar potential:

$$V = K^{k\bar{\ell}} W_k \bar{W}_{\bar{\ell}},$$

where $K^{k\bar{\ell}}$ is the inverse of the Kähler metric

$$K_{k\bar{\ell}} = \partial^2 K / \partial \phi^k \partial \bar{\phi}^{\bar{\ell}}$$

and

$$W_k = \partial W / \partial \phi_k, \quad \bar{W}_{\bar{\ell}} = (W_{\ell})^*.$$

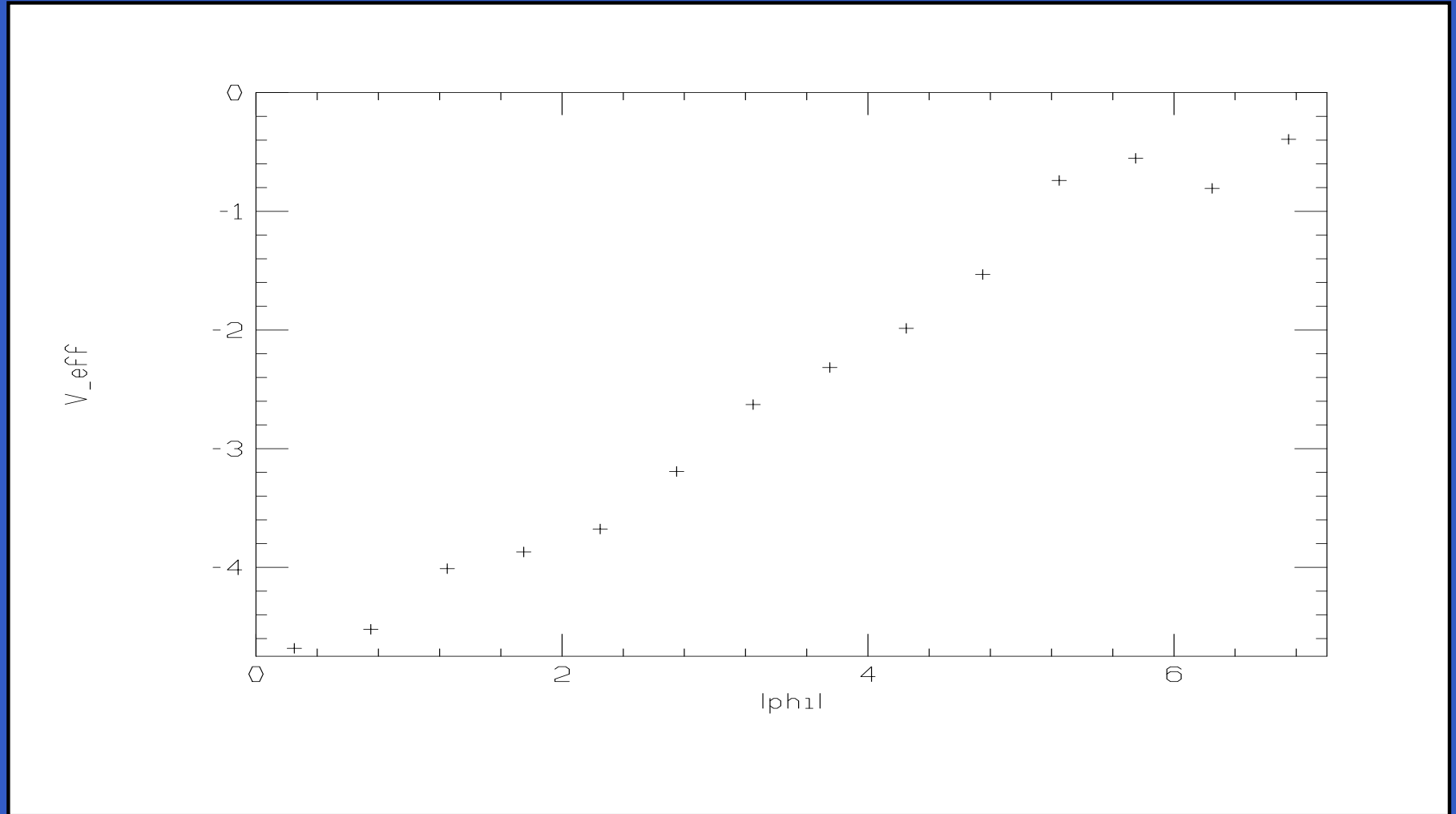
Extracting effective K

- Hypothesize effective Kähler potential K .
- Use known effective superpotential W .
- Match **microscopic lattice** and **IR effective lattice** data, to fit “phenomenological” constants in K .

Calculation

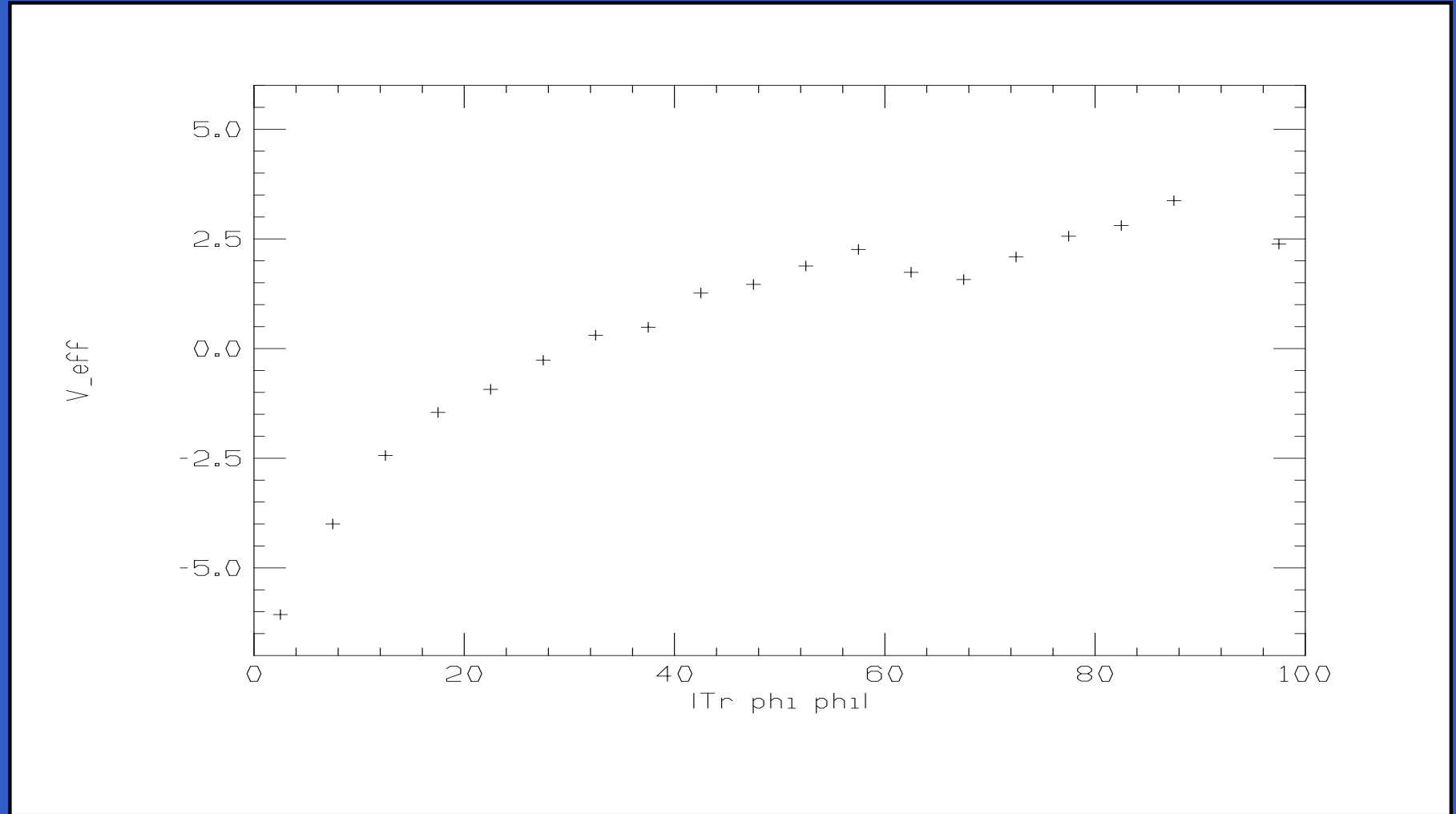
- These proposals illustrate how lattice simulations have the potential to teach us something about nonperturbative renormalization of nonholomorphic quantities.
- Work in progress (w/ Catterall): Determine (2,2) 2d twisted $N_{L\sigma M}$ IR effective theory that can reproduce the following (microscopic) constrained effective potentials (of (2,2) 2d $SU(2)$ SYM)...

Constrained effective potential



Potential for $|\phi|$ in $(2,2)$ $SU(2)$ SYM, using Catterall's construction and simulation code.

Constrained effective potential



Potential for $|\text{Tr } \phi^2|$ in $(2,2)$ $SU(2)$ SYM, using Catterall's construction and simulation code.

Susy breaking

- Third motivation: improve our understanding of **dynamical supersymmetry breaking**.
- Strong susy dynamics often invoked in models of soft susy breaking for **MSSM**.
- Any improvement of our understanding would be helpful.

3d $\mathcal{N} = 2$ SYM (perhaps feasible!)

- A simple theory where ground state susy is not yet fully understood.
- 3d reduction of 4d $\mathcal{N} = 1$ SYM.
- Unanswered questions arise in [Affleck, Harvey, Witten 82].
- Instanton-generated potential for modulus field ϕ (nonpert. renormalization), but **uncertainty** for small ϕ .
- Only known susy vac. is at $\phi \rightarrow \infty$.
- Is there a susy vacuum near origin?

3d $\mathcal{N} = 2$ SYM

- Use parity-preserving overlap-Dirac formulation, similar to 3d $\mathcal{N} = 1$ of [Maru, Nishimura 97]: **no CT fine-tuning?**
- Simple content (1 gluon, 1 adj. Maj. fermion, 1 adj. real scalar) \Rightarrow **efficient.**
- Addition of matter leads to very interesting vac. dynamics, even for the $U(1)$ gauge theory [Aharony, Hanany, Intriligator, Seiberg, Strassler 97] [de Boer, Hori, Oz 97]

Quantum gravity

- Evolving understanding of relationship between SYM and string/M-theory.
- Nonperturbative formulation of string/M-theory in **general backgrounds** is still lacking.
- But there are recent successes for special backgrounds:

Nonperturbative string theory

- D-branes [Polchinski 95].
- M(atr ix) theory [Banks, Fischler, Shenker, Susskind 96] [Ishibashi, Kawai, Kitazawa, Tsuchiya 96]
- AdS/CFT correspondence [Maldacena 97] [Gubser, Klebanov, Polyakov 98] [Witten 98]
- PP-wave limit [Metsaev 01] [Metsaev, Tseytlin 02] [Blau, Figueroa-O'Farrill, Hull, Papadopoulos 01] [Berenstein, Maldacena, Nastase 02]

Nonperturbative string theory

- Note that I include nontrivial semiclassical under what I call “nonperturbative.”
- It is of considerable interest to study these nonperturbative formulations in relation to SYM on the lattice.
- E.g., Matrix theory and AdS/CFT expressed in terms of **dimensionally reduced SYM**.

AdS/CFT correspondence

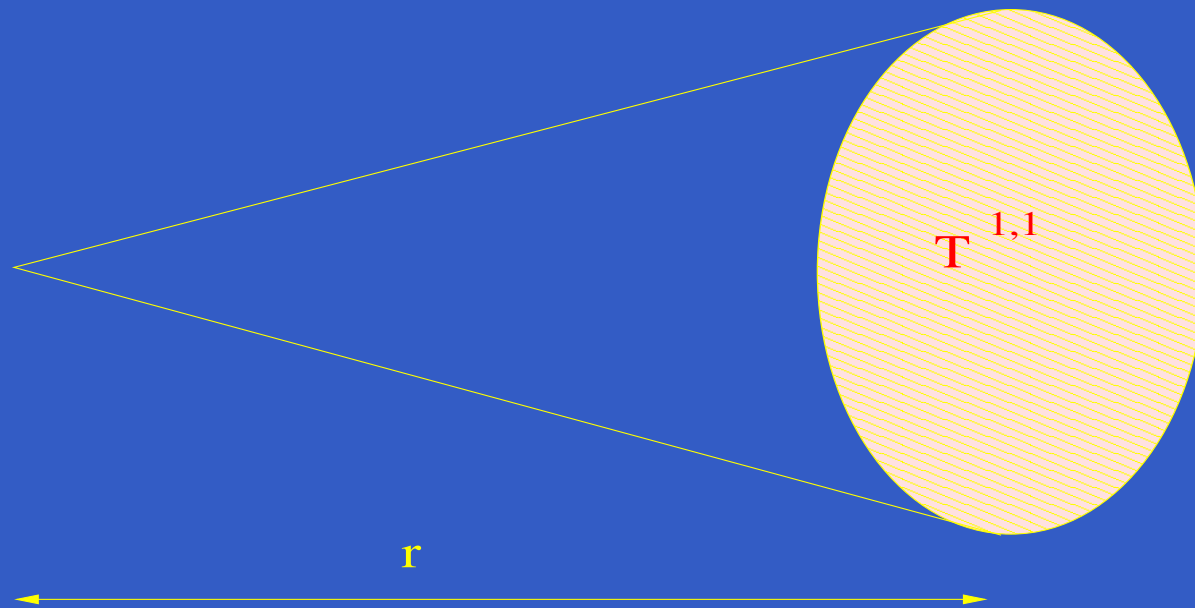
- The nontrivial SYM vacuum dynamics takes on a gravitational meaning.
- Everybody has heard about $AdS_5 \times S^5$.
- A far more interesting example is the **Klebanov-Strassler construction** [00].
- It is based on the Klebanov-Witten construction:

$$AdS_5 \times T^{1,1}.$$

- KW dual gauge theory: $\mathcal{N} = 1$ SCFT.

Conifold

- Type IIB theory is formulated on:
4d Minkowski \times conifold.
- The conifold has a cone-like geometry, with $T^{1,1}$ base:



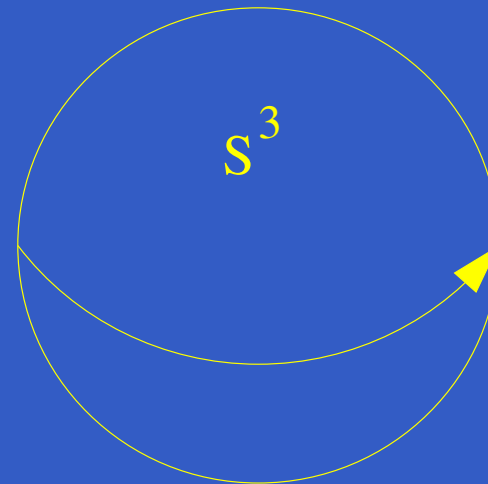
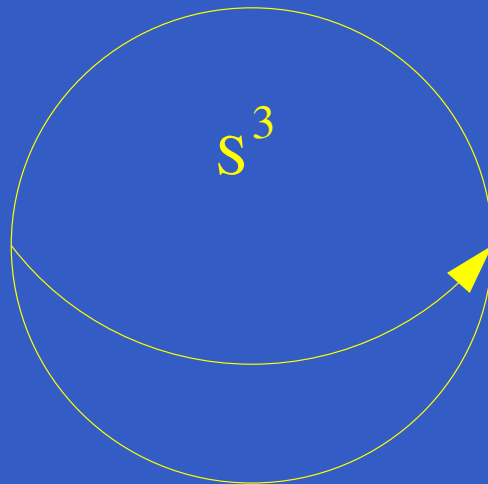
Conifold

- $T^{1,1}$ is a quotient manifold:

$$T^{1,1} = [SU(2) \times SU(2)]/U(1)$$

- The “1, 1” denotes the $U(1)$ quotient:

$$H = (\sigma_3 \otimes \mathbf{1}) + (\mathbf{1} \otimes \sigma_3).$$



Warping spacetime

- A stack of N_c D3 branes is placed at the tip, where the size of the base shrinks to zero.
- These branes are gravitating. They **backreact** on the geometry, warping it.
- Not too far from the branes, the geometry is $AdS_5 \times T^{1,1}$.

Killing spinors

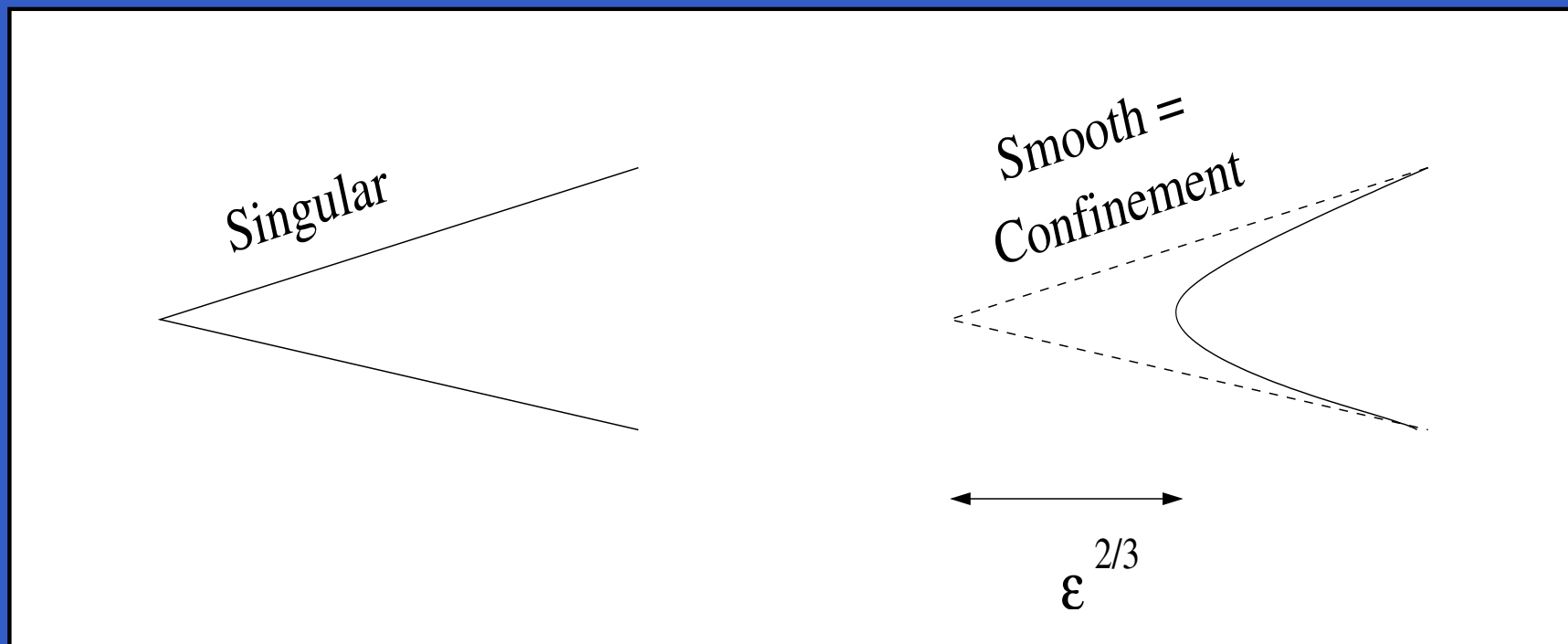
- One advantage: Type IIB SUGRA on $AdS_5 \times T^{1,1}$ only preserves **8 Killing spinors**, whereas
- Type IIB on $AdS_5 \times S^5$ preserves **32 Killing spinors**.
- In the dual gauge theory, we get $\mathcal{N} = 1$ SCFT rather than $\mathcal{N} = 4$ SCFT.

Singularity at the tip

- An $\mathcal{N} = 1$ SCFT is a much more promising start, being closer to the real world.
- There is a singularity at the tip of the conifold.
- In the dual gauge theory, this is reflected by the absence of an IR cutoff.

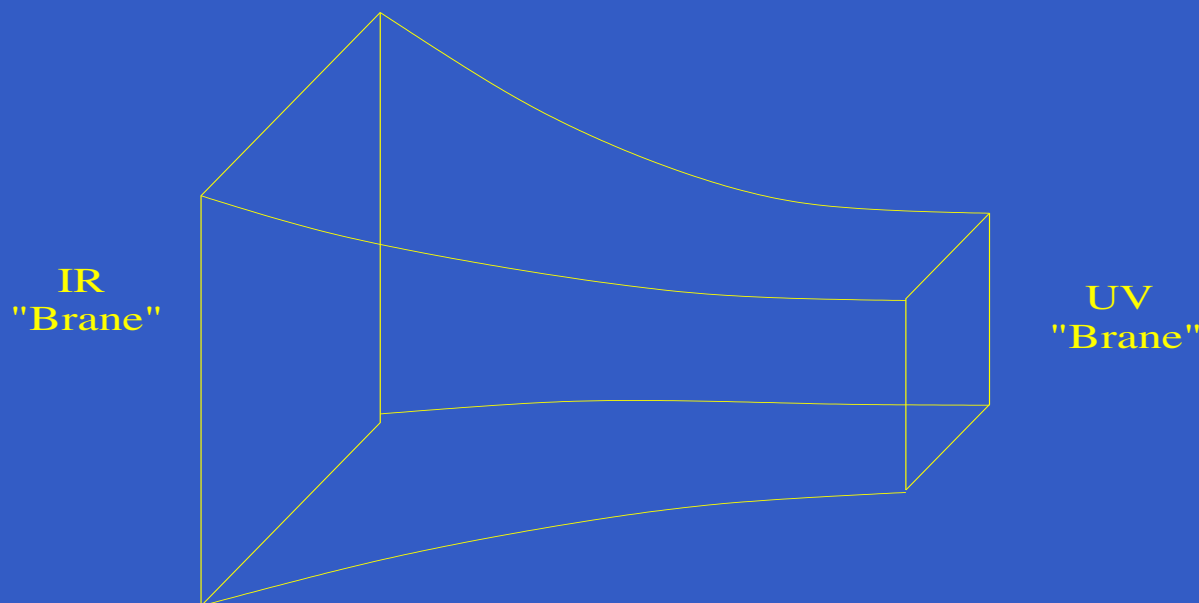
Singularity resolution

- Klebanov-Strassler resolve the singularity with the deformed conifold.
- They show that it is equivalent to **confinement** in the IR of the dual gauge theory.



Conformal symmetry breaking

- On the gravity side of the duality, this breaks half the Killing spinors.
- On the gauge theory side, this breaks the fermionic conformal charges, reducing to $\mathcal{N} = 1$ SUSY gauge theory.
- Now we are “very close” to the real world.
- I.e., semi-realistic susy AdS_5 models.



D7 probes = flavors

- Introduction of N_f D7 probe branes allows for a weakly coupled $U(N_f)$ gauge theory in the dual.

$$g_f^2 \sim (\text{volume})^{-1}.$$

- Embedding for D7s can generate bare masses for “quarks” of dual gauge theory.
- Low energy partons are really bound states, very much like in technicolor.
- Here, the $U(N_c)$ associated with the D3 branes is the technicolor group.

Noncompact \rightarrow compact CY

- Recently, studies of the low energy effective 4d and 5d theories have been conducted. [Sakai, Sonnenschein 03] [Kuperstein 04] [Levi, Ouyang 05] [Gherghetta, JG 06]
- In the latter work, we also imagined a regulator in the UV, following [Giddings, Kachru, Polchinski 02].
- This occurs by capping off the conifold with a compact Calabi-Yau manifold far from the tip.



Randall-Sundrum 1 interpretation

- The CY and the KS tip are, respectively, to be thought of as refinements of the UV and IR branes of, say, Randall-Sundrum 1.
- Potential—though challenging—interplay between **warped extra dimension** models, **AdS/CFT**, and **lattice SYM**.

- There are many more generalizations of AdS/CFT.
- In particular, AdS_3/CFT_2 may be accessible through lattice studies.
- The duality in the (4,4) 2d SQCD (D1-D5 system) is under study using a **deconstruction** lattice susy construction with matter [JG 06].

Deconstructed (4,4) SQCD

- The (4,4) lattice with matter is a generalization of the recent (2,2) SQCD constructions of Kaplan (see talk) and Endres [06].
- I now have a construction with 2 exact supercharges and only site/link/diagonal fields.

Conclusions

- Although it is challenging to write down supersymmetric lattice field theories that have a good quantum continuum limit, some **examples do exist**.
- A **wealth of exciting applications** await.
- At least for **lower dimensional examples**, certain nonperturbative features **can be studied** on the lattice with results that are of **broad** and of current **interest**.