

Why lattice supersymmetry?

- To answer this you might ask:
 - ◆ Q. Why supersymmetry?
 - ◆ A. Because it is a candidate solution to the hierarchy problem.
- That is, we want to explain why the Higgs is so light.

Our communal anthem...

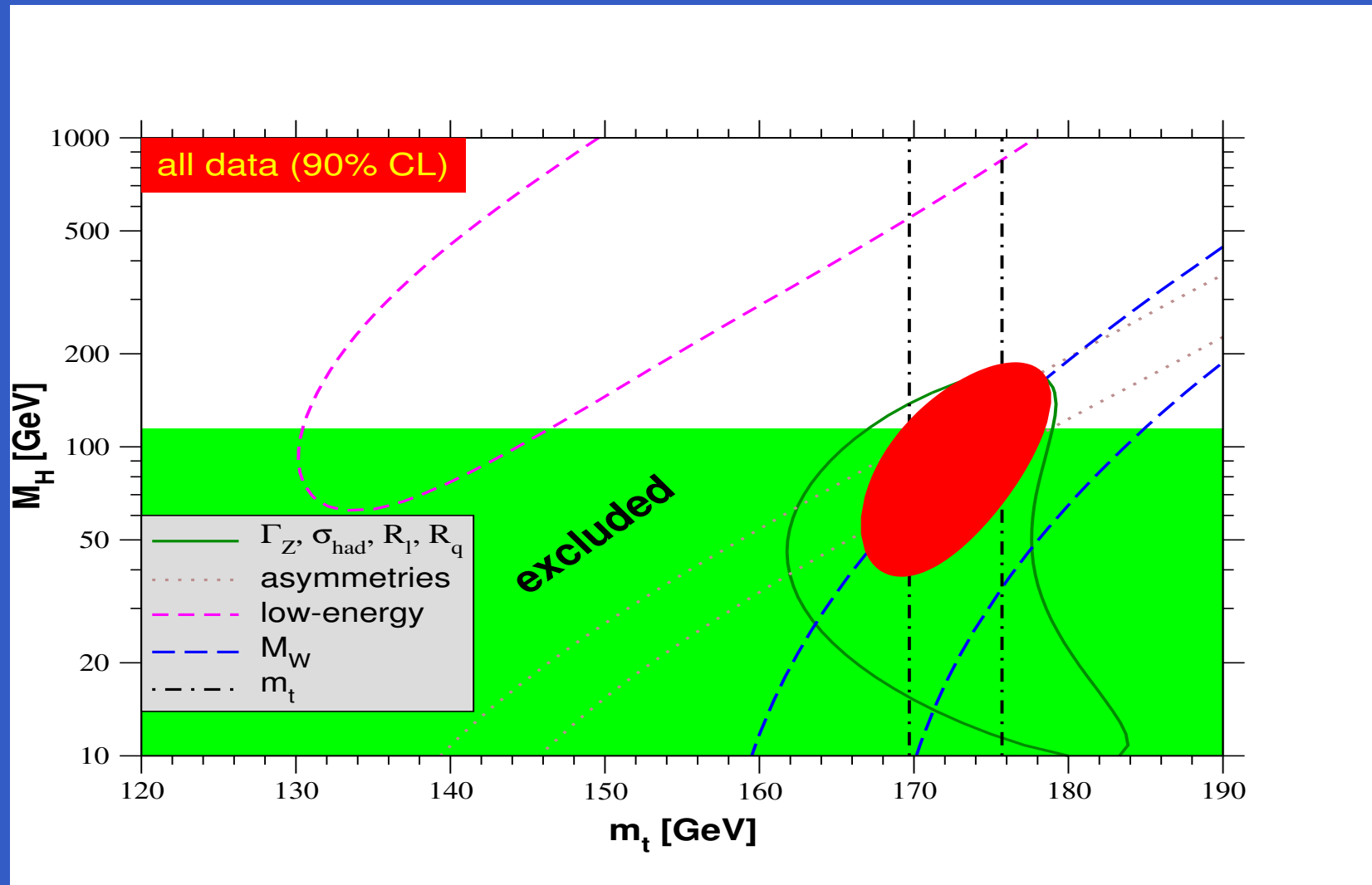
- The classical Higgs mass in the Standard Model is naturally of order:

$$m_H^2 \sim m_Z^2.$$

- The quantum corrections shift this mass by:

$$\Delta m_H^2 \sim m_{\text{cutoff}}^2 \lesssim m_P^2.$$

Experimental fit constraints



Particle Data Group, 2006

Keeping Δm_H^2 small

- Typically, we add new particles and symmetries such that:

$$\Delta m_H^2 \lesssim m_H^2.$$

- For the cancellations to work:

$$\text{mass}(\text{new particles}) \sim m_Z.$$

- Result:

We expect to see particles
beyond the Standard Model
at energies near m_Z .

The LHC is almost here!

- We will be looking for these beyond-the-Standard Model particles at the Large Hadron Collider, which turns on at the end of the year.
- It will be an exciting period in physics.

Studying the proposals

- Beyond the Standard Model \approx theories that solve the hierarchy problem.
- Roughly speaking, three types of extensions to the Standard Model:

Classes of solutions

- **Cancellations:** supersymmetry (topic of this talk).
- **Fluffiness:** Composite Higgs, transparent to high frequencies.
 - ◆ Old: Technicolor, ...
 - ◆ New: Warped extra dims. (via AdS/CFT), ...
- **Hierarchy is a fake:** Large extra dimensions.

Supersymmetry

- It is a symmetry that transforms
bosons \leftrightarrow **fermions**.
- If the symmetry is exact, the masses of particles show a **Bose-Fermi degeneracy**:

$$m_B = m_F.$$

Superpartners

- Supersymmetry predicts **superpartners**:

electron (fermion) \Rightarrow
scalar electron (boson) = “selectron”.

- Recall, however, that supersymmetry predicts:

$$m_B = m_F.$$

- Because we have not observed superpartners, supersymmetry must be spontaneously broken. E.g.,

$$m_{\text{selectron}} \gtrsim m_Z.$$

Embracing the nonperturbative

- Models for SUSY-breaking involve **nonperturbative dynamics** of supersymmetric gauge theories.
- Higgs at weak coupling \rightarrow **calculable models**.
- Other possibilities no-can-do: strongly interacting \rightarrow **noncalculable models**.

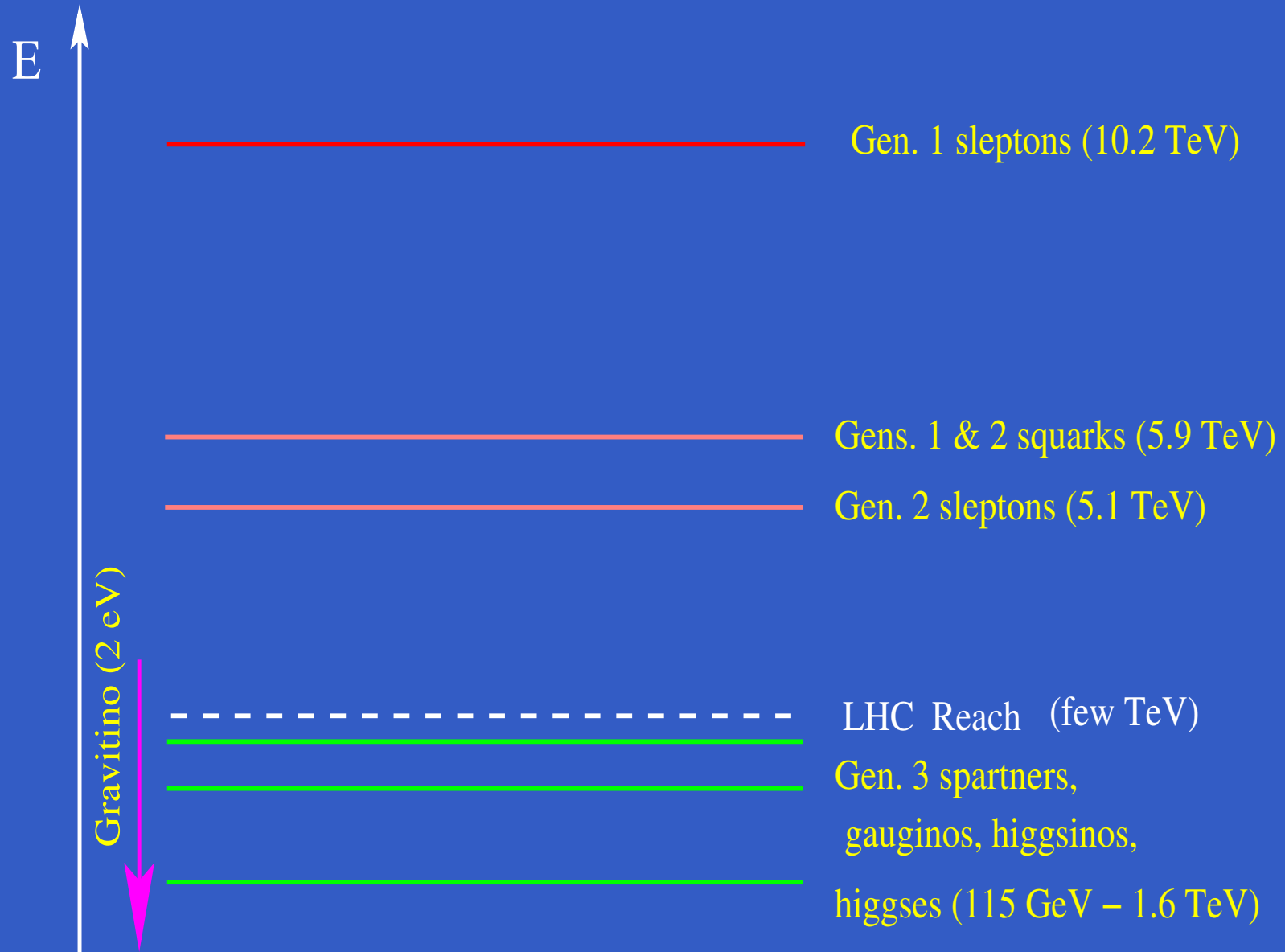
Gravity dual

- One approach:
Use a weakly coupled gravity dual.
- In work with T. Gherghetta, M. Gavelle (Minnesota), we are exploring string-inspired generalizations of:
warped MSSM in the bulk.
- Susy-breaking from deformed metric, inspired by Type IIB solutions. [Borokhov, Gubser 02; Kuperstein, Sonnenschein 03]

Dual description

- Partially composite MSSM.
- Composite scale 100 TeV.
- SUSY-breaking in the 100 TeV strong dynamics.
- Elementary MSSM states experience gauge mediation.

Observable spectrum



Blast from the past

- The susy-breaking is similar to what occurs in
 - ◆ “more minimal SSM”
[Cohen, Kaplan, Nelson 96]
 - ◆ “single sector models”
[Arkani-Hamed, Luty, Terning 97-98]where much of the MSSM is composite.
- The new tool:
5d calc of composite spectrum.

LHC predictions

- Like gauge mediation, but lower rates.
- NLSP decays

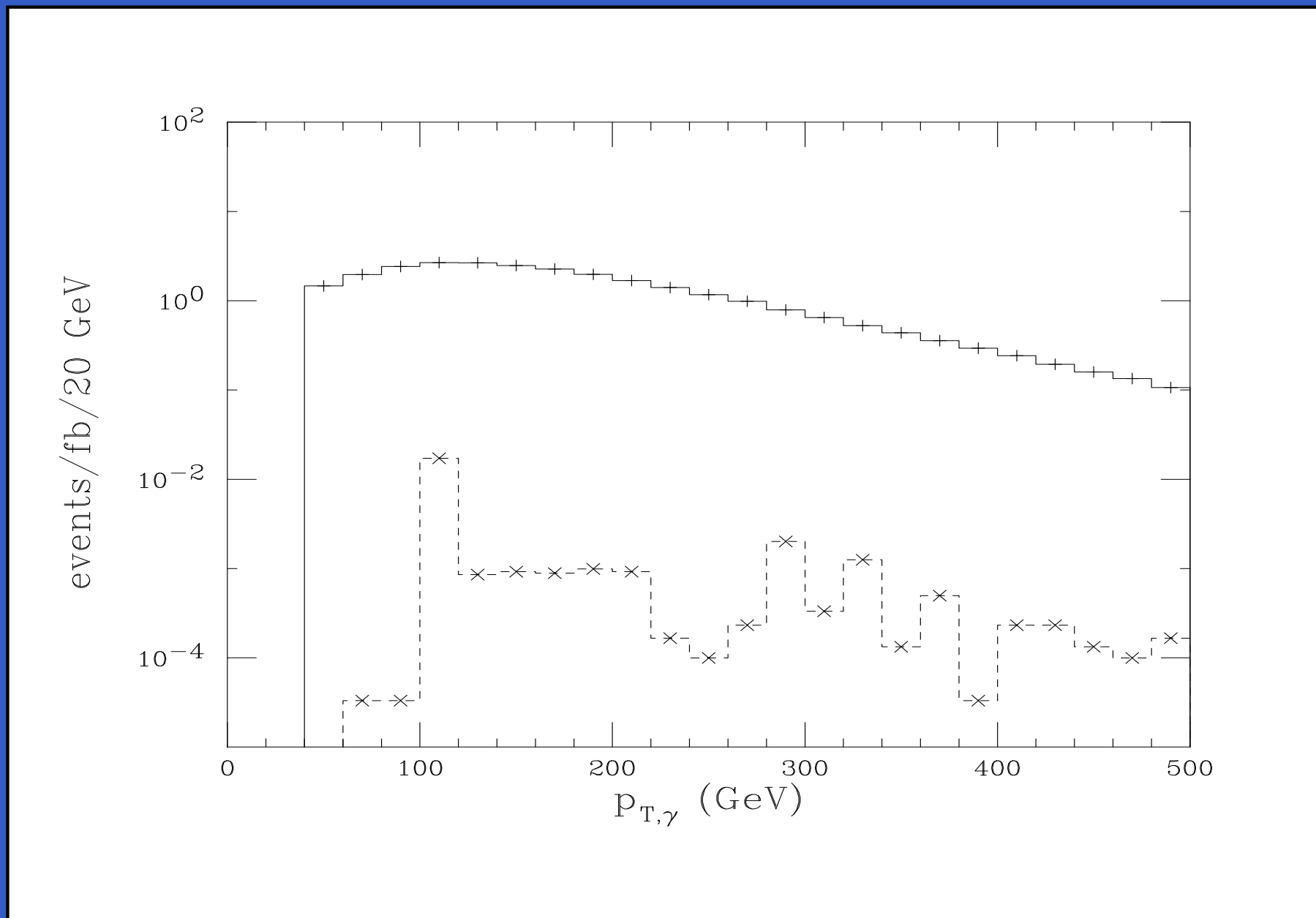
$$\begin{array}{ccc} \tilde{\chi}_1^0 & \rightarrow & \gamma + \tilde{G} \\ 288 \text{ GeV} & & 2 \text{ eV} \end{array}$$

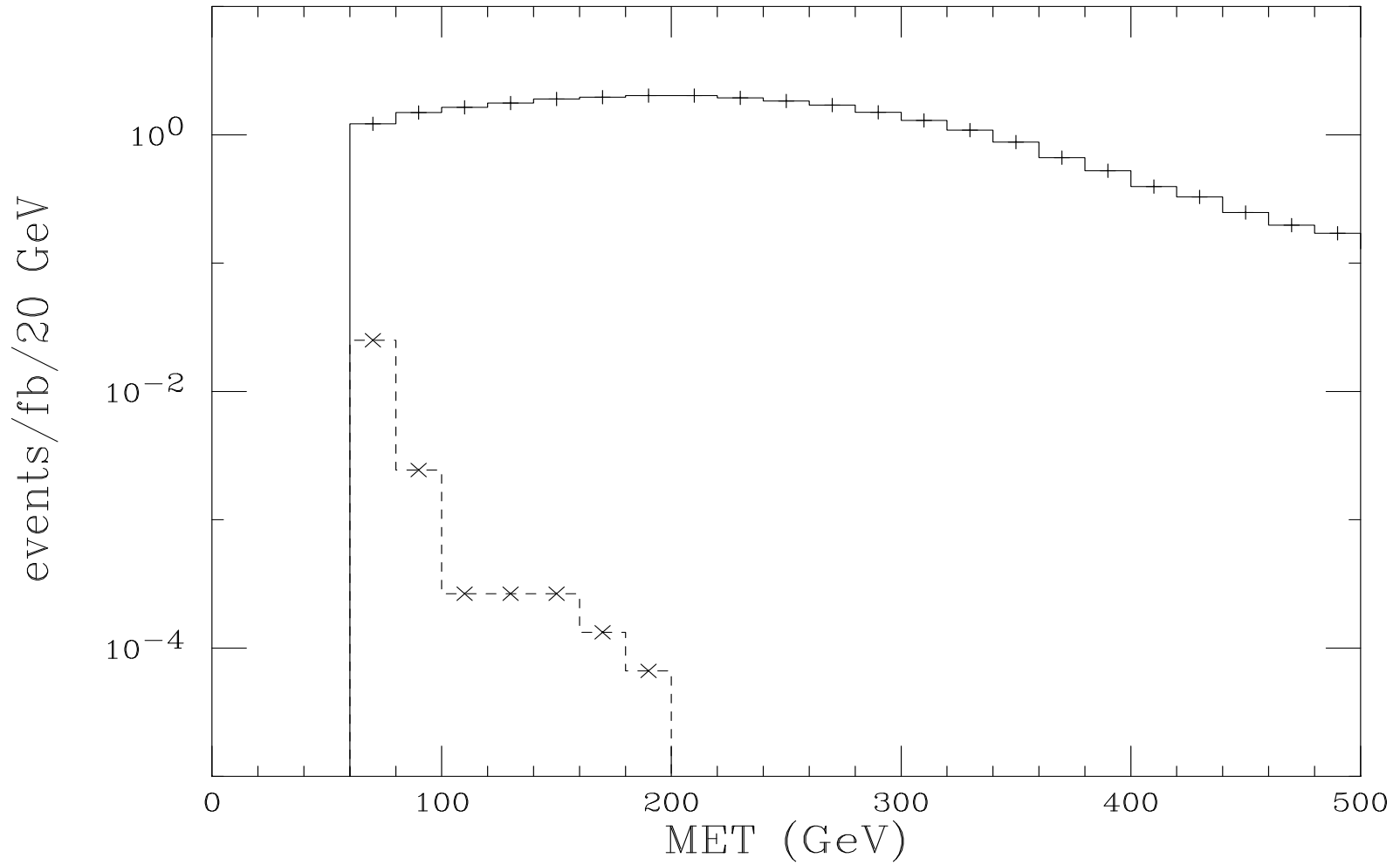
give strong signal:

$$\begin{array}{ccc} p\bar{p} & \rightarrow & \gamma\gamma + \cancel{E}_T + X \\ & & \text{hard} \quad \text{lots} \end{array}$$

pT w/cuts

[SUSY = solid, SM($\gamma\gamma$) = dashed]





Summary w/cuts

$$p_{T,\gamma} \geq 40 \text{ GeV}, \quad \cancel{E}_T \geq 60 \text{ GeV}$$

Integrated Luminosity	SUSY	SM 2γ	SM 2γ + fakes
1 fb^{-1}	27.6	0.0285	$\lesssim 0.1$
10 fb^{-1}	276	0.285	$\lesssim 1$

EWSB Strong Dynamics and New Lattices

- In many SUSY and non-SUSY models, we want to have **additional nonperturbative tools**.
- In particular, strong dynamics in **chiral gauge theories**.
- Lattice would be nice, but
- Chiral gauge theory on the lattice is an **unsolved problem**.
- \exists a new proposal for lattice chiral gauge theory.
[Bhattacharya, Martin, Poppitz 06]

Splitting left and right

- In recent work with Erich Poppitz, I studied this proposal.
 - ◆ Sectors: **Light (R)** + **Mirror (L)**.
 - ◆ In absence of interaction w/ **unitary Higgs**: exactly massless, no doublers.
 - ◆ Ginsparg-Wilson chiral symmetry.
 - ◆ GW chiral coupling Higgs-Mirror.
 - ◆ Strong Yukawa-Higgs to lift mirrors without breaking gauge symmetry.

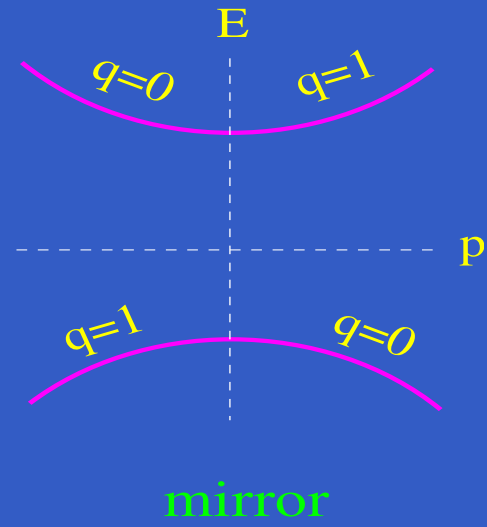
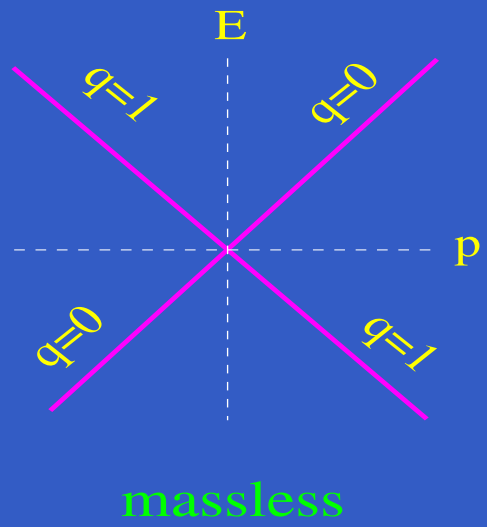
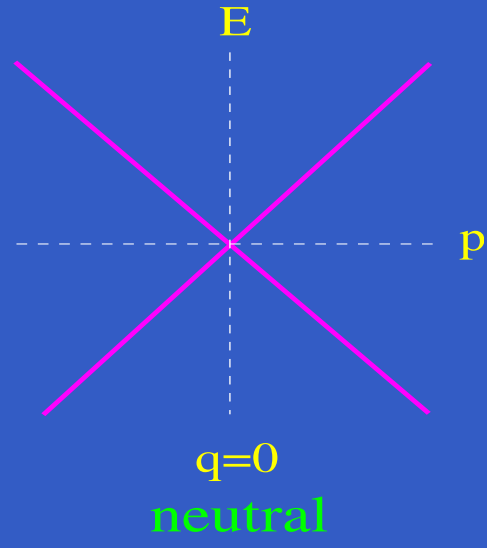
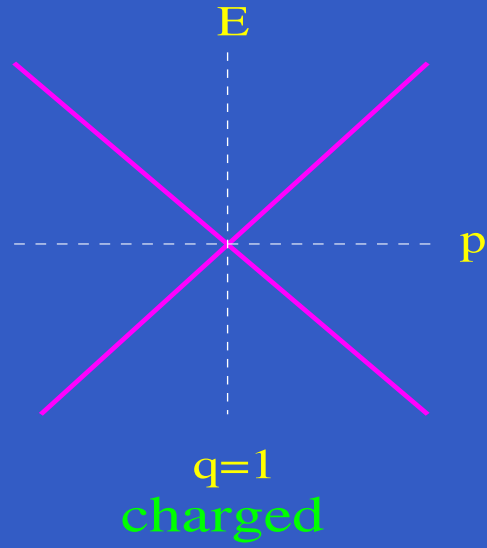
Splitting left and right

- A **dispersion relation** tells you how the energy of a state depends on momentum. E.g.:

$$E(p) = \pm \sqrt{p^2 + m^2}$$

- For $m = 0$, there is a linear dispersion: $E \sim p$.
- For $m \neq 0$, the dispersion is hyperbolic:
 $E^2 - p^2 = \text{const.}$
- We want:
 - ◆ Light \rightarrow linear,
 - ◆ Mirror \rightarrow hyperbolic.

Effect of bosons on fermion modes



The mirror is massive

- We have confirmed this picture with Monte Carlo lattice simulations.
- The mirror fermions get masses $\mathcal{O}(a^{-1}) \rightarrow \infty$, as had been hoped for.

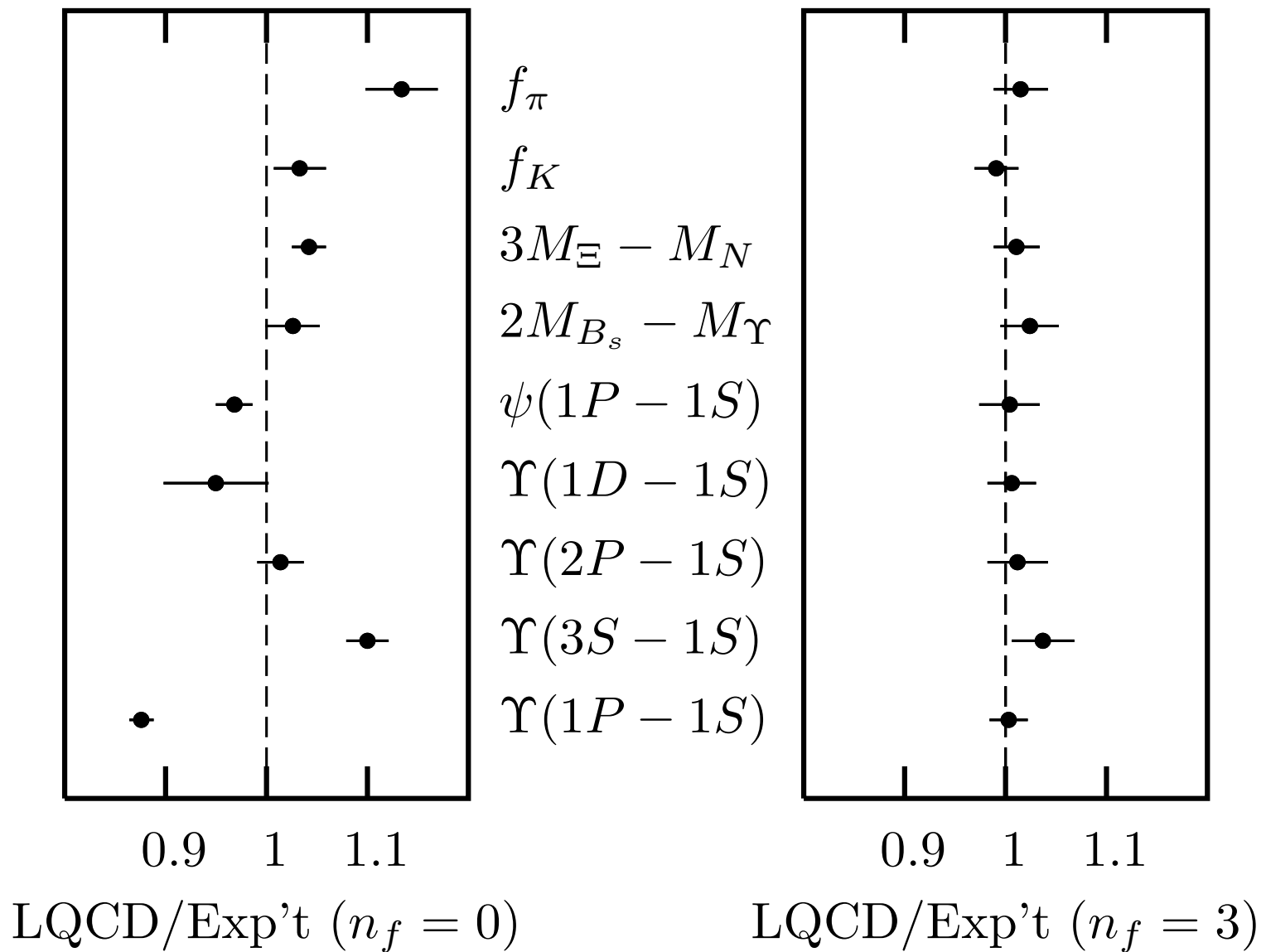
Outlook

- Our simulations are encouraging, and can be extended to $(1+3)$ -dimensions.
- Important questions remain to be studied in this proposal; in progress.

Lattice supersymmetry

- Since strong coupling is hard,
we want as many tools as possible.
- One method for the study of strongly coupled gauge theories:
formulate on lattice, simulate.
- Successful in the case of the strong interaction in the Standard Model (QCD):

A recent success in lattice QCD



Davies et al., 2004

Lattice supersymmetry

- However, lattice supersymmetry has problems.
- Lattice breaks supersymmetry: discretization errors.
- Divergences in quantum field theories \Rightarrow errors can be dangerously amplified:

$$\epsilon \times \infty = \infty.$$

Lattice supersymmetry

- For a few years now, I have studied ways that exact lattice symmetries might be used to overcome this problem.
- Collaborators: Poppitz, Koniuk, Yavin.
- Many encouraging results.
- I will describe one example in detail and
- Mention others briefly.

Lattice susy is problematic

- Super-Poincaré contains generator of continuous translations:

$$\{Q_{\alpha}^A, \bar{Q}_{\dot{\alpha}}^B\} = 2\delta^{AB}\sigma_{\alpha\dot{\alpha}}^{\mu}P_{\mu}$$

- Only the discrete group of translations

$$T_n = \exp(iaP_{\mu}n_{\mu}), \quad n_{\mu} \in \mathbf{Z}$$

survive on the lattice.

- What symmetry generator should the Q's close on?

Lattice susy is problematic

- Answer seems to be: replace

$$P_\mu = i\partial_\mu \rightarrow i\Delta_\mu$$

in the algebra, where Δ_μ is a finite difference operator that approximates differentiation.

- But Δ_μ does not satisfy Leibnitz rule, and this leads to difficulties getting an invariant action [Dondi, Nicolai 77].

Lattice susy is problematic

- Form of variation:

$$\begin{aligned}\epsilon Q S &= \epsilon \sum_x (\Delta f_x g_x + f_x \Delta g_x) \\ &= \epsilon \sum_x \Delta(fg)_x + \epsilon a^p \mathcal{O}_S\end{aligned}$$

- With periodic BCs:

$$\sum_x \Delta(fg)_x = 0$$

- Thus:

$$Q S = a^p \mathcal{O}_S$$

Lattice susy is problematic

- In correlation functions:

$$\langle Q\mathcal{O} \rangle = a^p \langle \mathcal{O}_S \mathcal{O} \rangle$$

- UV degree: $\langle \mathcal{O}_S \mathcal{O} \rangle \sim a^{-p+\delta}$.

- Depending on the operator

$\delta < 0$ ∞ counterterm,

$\delta = 0$ finite/log counterterm,

$\delta > 0$ no counterterm.

Counterterm approach

- Suppose we introduce the necessary counterterms to the lattice action.
- These have their own variation:

$$QS_{CT} = a^q \mathcal{O}_{S_{CT}}$$

- They are tuned to restore the supersymmetric Ward identities in the continuum limit ($a \rightarrow 0$):

$$a^p \langle \mathcal{O}_S \mathcal{O} \rangle + a^q \langle \mathcal{O}_{S_{CT}} \mathcal{O} \rangle \sim a^r$$

- Generically, at least a few counterterms are required: **no symmetry to prevent.**

Counterterm approach

- In SUSY-QM, we were able to determine the counterterm (only one required) exactly.
[JG, Koniuk, Poppitz, Yavin 05]
- The 1-loop counterterm was everything, as demonstrated by nonperturbative calculation.
- Analytic & Monte Carlo, independently.

Counterterm approach

- Another super-renormalizable theory where perturbative counterterm has been computed exactly: 3d $\mathcal{N} = 2$ SYM.
[Elliott, Moore 05]
- Nonperturbative:
 - ◆ IR no problem (see arguments of E&M).
 - ◆ Nonperturbative UV?
I'll come back to that at the end!

Q-exact approach

- However, we would like to have methods that are free of counterterm adjustment.
- This is true for Q-exact ($S = QX, Q^2 = 0$) SUSY-QM. Analytic proof: [JG, Poppitz 03]
- Consistent w/ MC findings [Catterall, Gregory 00].
- I next present a (1+1)-dim'l Q-exact lattice field theory where counterterms are absent.

(2,2) 2d Wess-Zumino model

- Dim'l reduction of 4d $\mathcal{N} = 1$ Wess-Zumino model with cubic superpotential.
- A.k.a. $\mathcal{N} = 2$ Landau-Ginsburg.

A very nice toy model, b/c:

- Much is known from continuum.
- The field theory at its critical point is “solved:”
 - ◆ Primary operators known.
 - ◆ Conformal dimensions known.
 - ◆ Thus, correlation functions determined.
- Continuum theory perturbatively super-renormalizable.
(Reasonable to expect this of lattice theory too.)
- The extended supersymmetry $\Rightarrow Q^2 = 0$ subalgebra.

More on how nice 2dWZ is

- $Q^2 = 0$ allows Q-exact lattice formulation:

$$S = QX \Rightarrow QS = 0. \quad (*)$$

- Here Q involves finite-difference operators and X consists of lattice fields.
- Shown [JG, Poppitz 04] to enjoy:
 - ◆ Lattice theory perturbatively super-renormalizable.[†]
 - ◆ Correct quantum continuum limit to all loop orders. [The property $(*)$ is essential.]

[†] Lattice power-counting can differ from continuum; cf. unsolved problem for staggered fermions (~ 25 yrs.) until recently [JG 06].

Open question

- Thus, in perturbation theory no counterterms are needed!
- What about nonperturbative?
- Some Ward identities OK.
[Catterall, Karamov 01]
- Let's look at what happens to $\mathcal{O}(a)$ -violated R-symmetries.
[JG 05]

Continuum action

- In imaginary time $S =$

$$\int d^2 z \left[-4\bar{\phi}\partial_z\partial_{\bar{z}}\phi - 2i\bar{\psi}_-\partial_{\bar{z}}\psi_- + 2i\psi_+\partial_z\bar{\psi}_+ \right. \\ \left. -\bar{F}F + W'(\phi)F + \bar{W}'(\bar{\phi})\bar{F} \right. \\ \left. -W''(\phi)\psi_+\psi_- - \bar{W}''(\bar{\phi})\bar{\psi}_-\bar{\psi}_+ \right]$$

- As usual:

$$W' = \partial W / \partial \phi, \quad W'' = \partial^2 W / \partial \phi^2.$$

- Choose cubic superpotential, with mass term:

$$W(\phi) = \frac{m}{2}\phi^2 + \frac{g}{3!}\phi^3$$

Symmetric choice

- Better to make linear shift $\phi = \sigma - (m/g)$, obtaining:

$$W(\sigma) = -\lambda\sigma + \frac{g}{3!}\sigma^3, \quad \lambda = \frac{m^2}{2g}$$

- Integrating out aux. field, scalar potential:

$$V = \left| \lambda - \frac{1}{2}g\sigma^2 \right|^2$$

- Yukawa interaction:

$$-g\sigma\psi_+\psi_-$$

Looking at

$$V = \left| \lambda - \frac{1}{2}g\sigma^2 \right|^2, \quad -g\sigma\psi_+\psi_-$$

we see a $Z(2)_R$ symmetry

$$\sigma \rightarrow -\sigma, \quad \bar{\sigma} \rightarrow -\bar{\sigma}, \quad \psi_{\pm} \rightarrow \pm\psi_{\pm}, \quad \bar{\psi}_{\pm} \rightarrow \pm\bar{\psi}_{\pm}$$

that relates the classical vacua:

$$\sigma_{\pm} = \pm\sqrt{2\lambda/g} = \pm m/g$$

Ensembles, subensembles and $Z(2)_R$

- Denote the real and imaginary parts of σ_m as $\sigma_{R,m}$ and $\sigma_{I,m}$ respectively.
- Suppose we have a ensemble of configurations Γ .
- We partition this ensemble as $\Gamma = \Gamma_+ + \Gamma_- + \Gamma_0$ where:
 - ◆ $\sigma_{R,m} > 0$ on the subensemble Γ_+ ,
 - ◆ $\sigma_{R,m} < 0$ on the subensemble Γ_- , and
 - ◆ $\sigma_{R,m} = 0$ on the subensemble Γ_0 .
- Note that Γ_0 is a set of measure zero in any expectation value over Γ , since $\sigma_{R,m}$ is a continuous variable.

Ensembles, subensembles and $Z(2)_R$

- Let:
 - ◆ $\langle \sigma_R \rangle_+$ be the average of $\sigma_{R,m}$ over the subensemble Γ_+ and
 - ◆ $\langle \sigma_R \rangle_-$ be the average for the subensemble Γ_- .
- The order parameter $\langle |\sigma_R| \rangle$ will denote the average of $|\sigma_{R,m}|$ over the full ensemble Γ .

Ensembles, subensembles and $Z(2)_R$

- Suppose $Z(2)_R$ is a good symmetry in the IR of the lattice theory.
- Then we expect

$$\langle \sigma_R \rangle_+ = -\langle \sigma_R \rangle_- = \langle |\sigma_R| \rangle \equiv v$$

- Now look at the Monte Carlo evidence:

Evidence of $Z(2)_R$

N	$\langle \sigma_R \rangle_+$	$\langle \sigma_R \rangle_-$	$\langle \sigma_R \rangle$
4	3.505(52)	-3.518(52)	3.512(14)
8	3.337(49)	-3.353(48)	3.3456(87)
16	3.342(60)	-3.349(41)	3.3471(45)

Table 1: Comparison of subensemble order parameters to that of the full ensemble, for $(m, g) = (0.10, 0.03)$.

Sourcing

- We introduce a constant external field h

$$\Delta V(h) = -h \sum_m (\sigma_m + \bar{\sigma}_m) = -2h \sum_m \sigma_{R,m}$$

- This allows us to explore the effective potential and to study the extent to which it is symmetric w.r.t. $\sigma \rightarrow -\sigma$.

- As usual, we obtain the effective potential from the Legendre transformation of the generating function

$$w(h) = \ln Z(h),$$

- Where $Z(h)$ is the partition function that is obtained when $\Delta V(h)$ is added to the lattice action.
- $Z_2(R)$ symmetry of the effective potential is equivalent to

$$w(-h) = w(h).$$

- Note also that

$$\langle \sigma \rangle_h = \partial w(h) / \partial h,$$

where $\langle \sigma \rangle_h$ is the expectation value of σ in the background h .

- It follows that in the case of $Z_2(R)$ symmetry we have the prediction

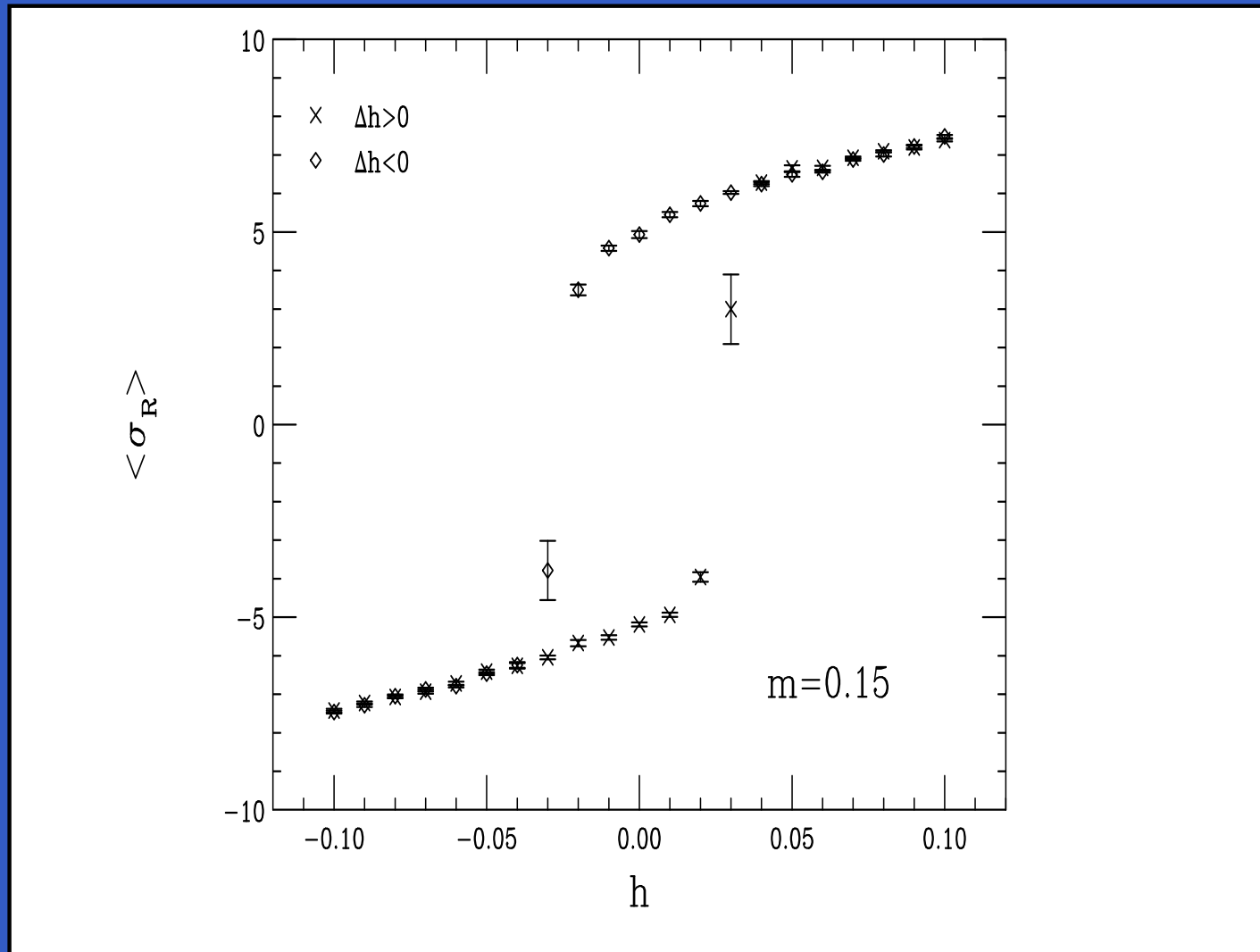
$$\langle \sigma \rangle_{-h} = -\langle \sigma \rangle_h$$

Evidence of $Z(2)_R$

$ h $	$\langle \sigma_R \rangle (h < 0)$	$\langle \sigma_R \rangle (h > 0)$
0.10	-8.427(19)	8.405(17)
0.01	-6.663(61)	6.7231(45)
0.005	-5.395(56)	5.428(53)
0.003	-3.776(79)	3.897(79)
0.001	-1.408(95)	1.520(98)
0.0005	-0.667(99)	0.694(25)
0.0001	-0.235(96)	0.145(96)
0.0	-0.038(97)	—

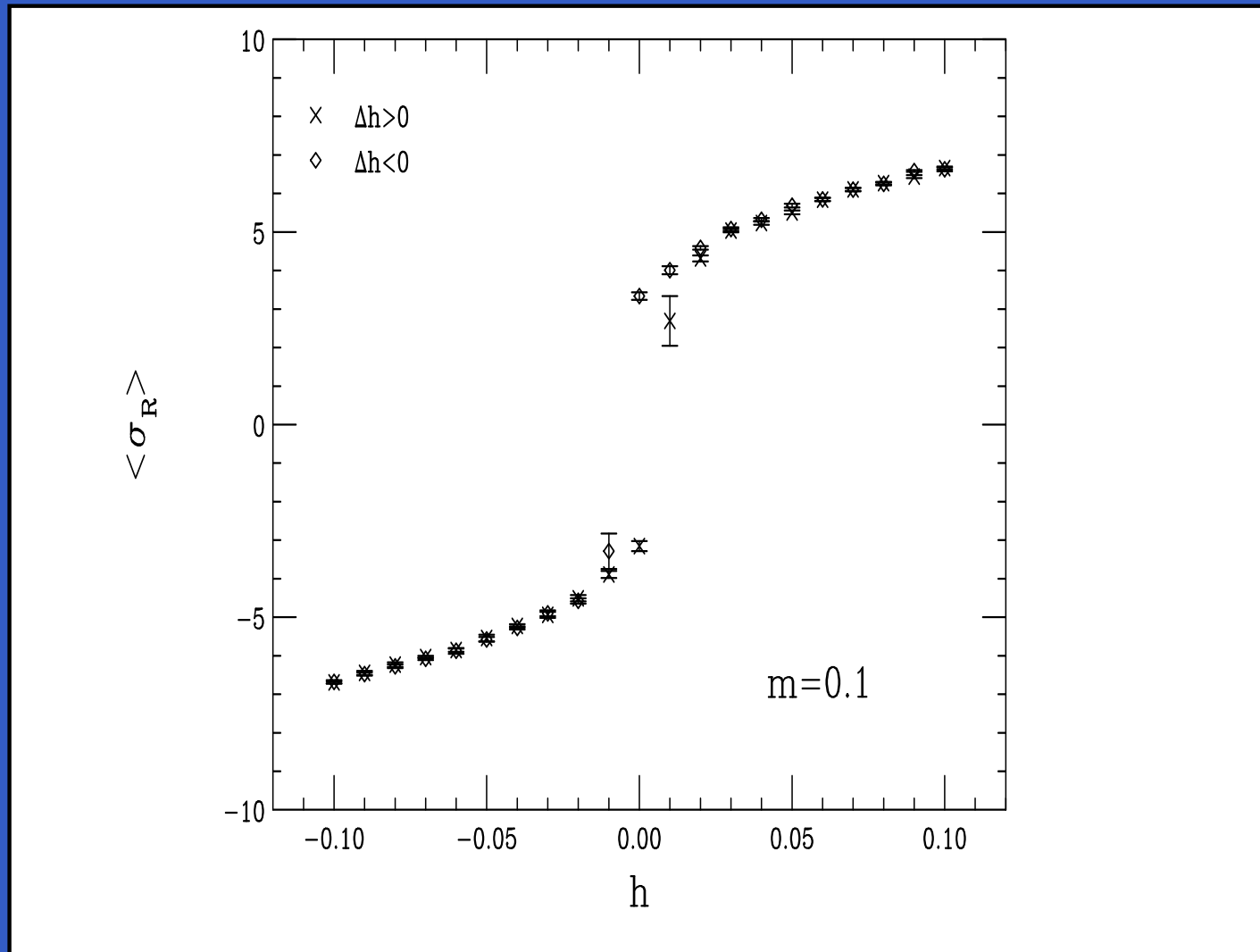
Table 2: $\langle \sigma_R \rangle_h$ as a function of the source h . Note that $\langle \sigma_I \rangle_h = 0$ for h real.

Order of transition



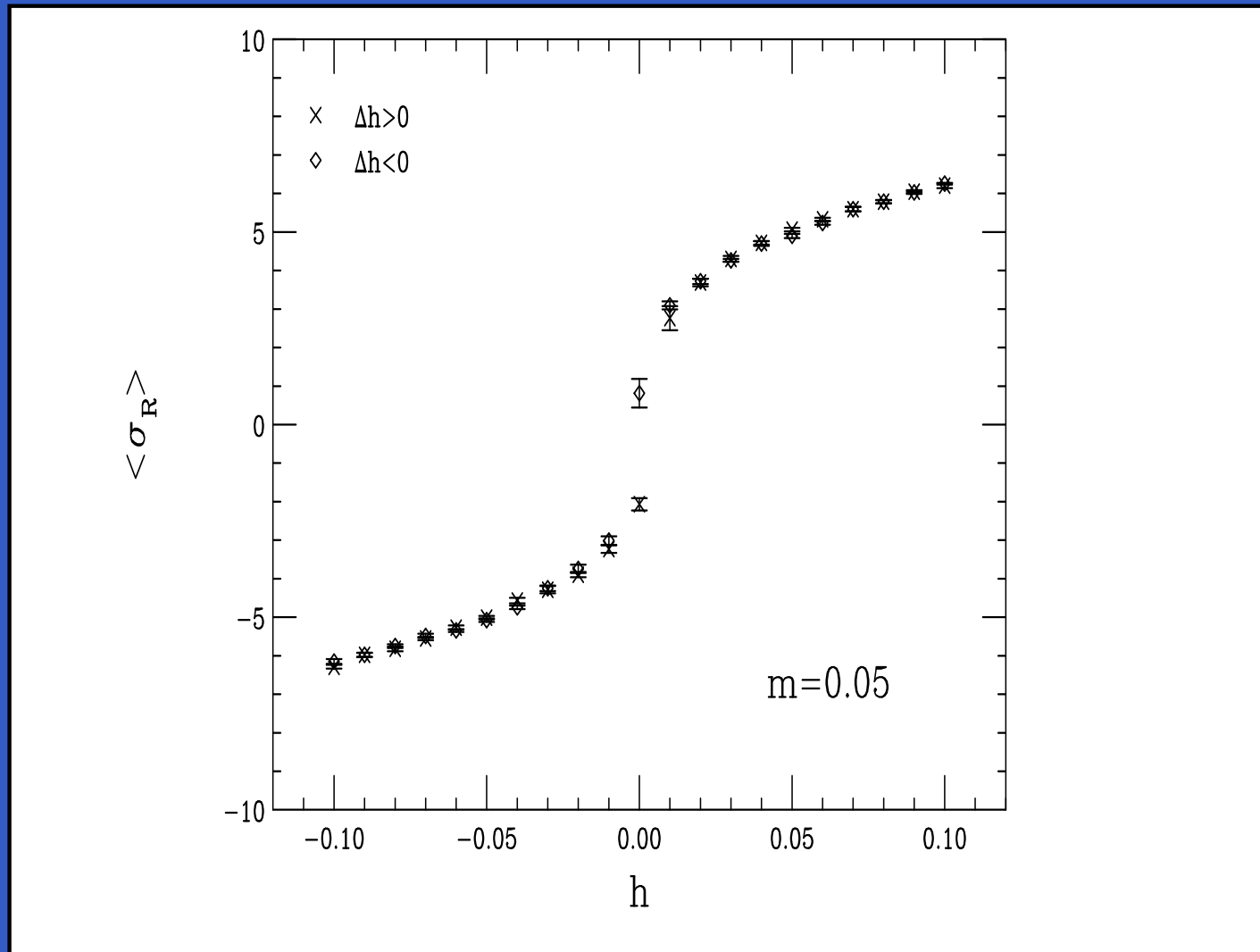
Hysteresis cycles, $\langle \sigma_R \rangle$ versus h , various m .
20 config. avg.'s.

Order of transition



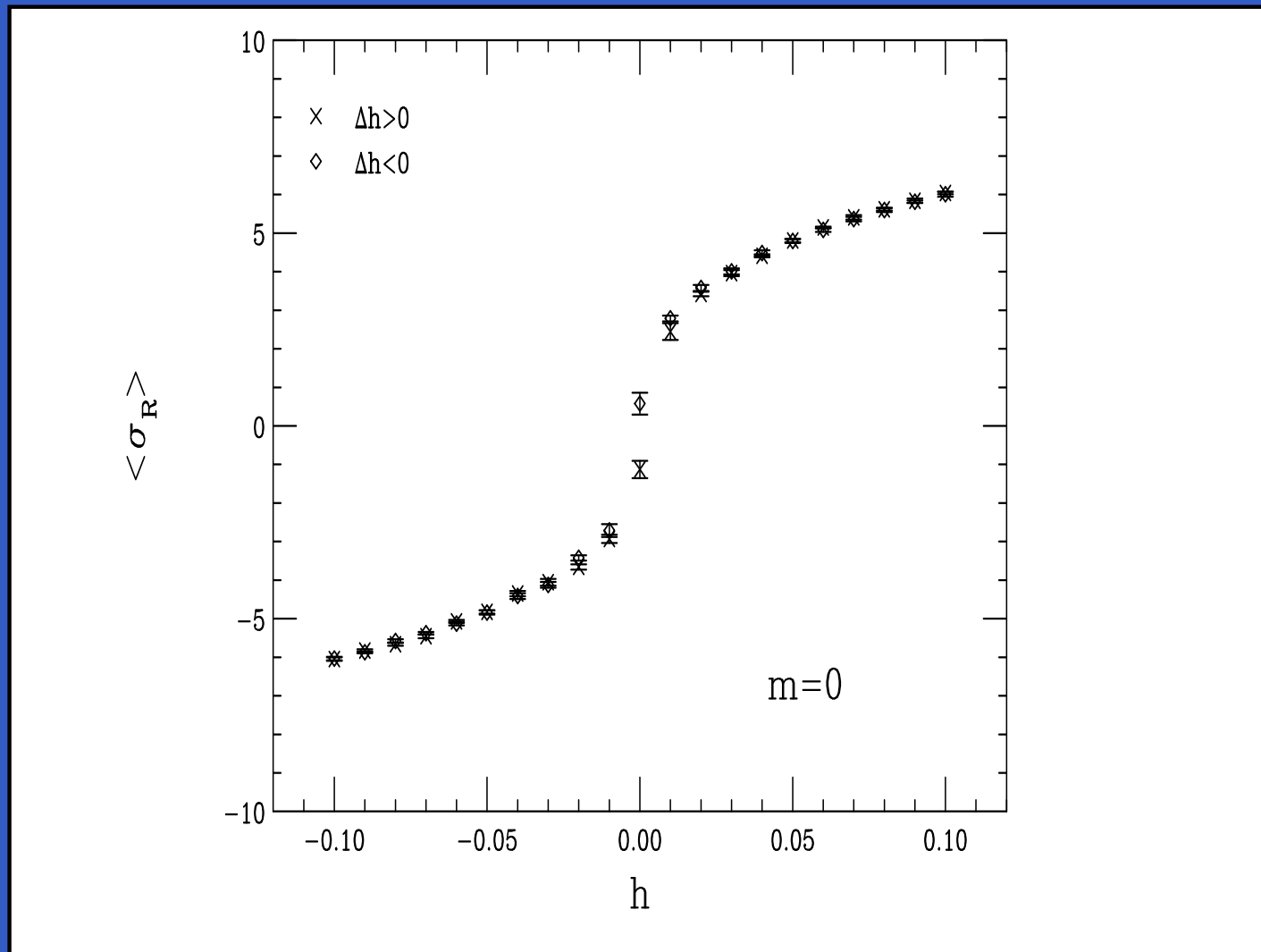
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$U(1)_R$ at the critical point

- In the continuum, at $m = 0$ (equiv. $\lambda = m^2/2g = 0$), the scalar potential is $V \propto |\sigma^2|^2$.

- Then symmetry enhancement $Z(2)_R \rightarrow U(1)_R$:

$$\sigma \rightarrow e^{2i\alpha/3} \sigma, \quad \psi_{\pm} \rightarrow e^{-i\alpha/3} \psi_{\pm}, \quad F \rightarrow e^{-4i\alpha/3} F$$

$U(1)_R$ at the critical point

- This chiral symmetry leads to multiplicative renormalization of the fermion mass.
- By supersymmetry, there is multiplicative renormalization of the scalar mass as well.
- Implies critical line is just:

$$m = 0, \quad g = \text{anything}$$

- The hysteresis curves indicate this is also the critical line in the lattice theory, even though $U(1)_R$ is explicitly broken at $\mathcal{O}(a)$.

Sourcing again

- We study recovery of $U(1)_R$ in the lattice by this technique:

$$\Delta V(h) = - \sum_m (\bar{h}\sigma_m + h\bar{\sigma}_m)$$

where h is site-independent complex source field.

- $U(1)_R$ symmetry of the effective potential is equivalent to $w(e^{i\theta}h) = w(h)$.

Sourcing again

- Note also that $\langle \sigma \rangle_h = \partial w(h) / \partial \bar{h}$.
- In the case of $U(1)_R$ symmetry we have the (strong) prediction

$$\langle \sigma \rangle_{e^{i\theta} h} = e^{i\theta} \langle \sigma \rangle_h$$

Sourcing again

- Equivalently,

(1)

$$\arg \langle \sigma \rangle_h = \arg h,$$

(2)

$$|\langle \sigma \rangle_h| = \text{const.}, \quad \text{fixed } |h|$$

is the prediction of $U(1)_R$ symmetry.

Evidence of $U(1)_R$

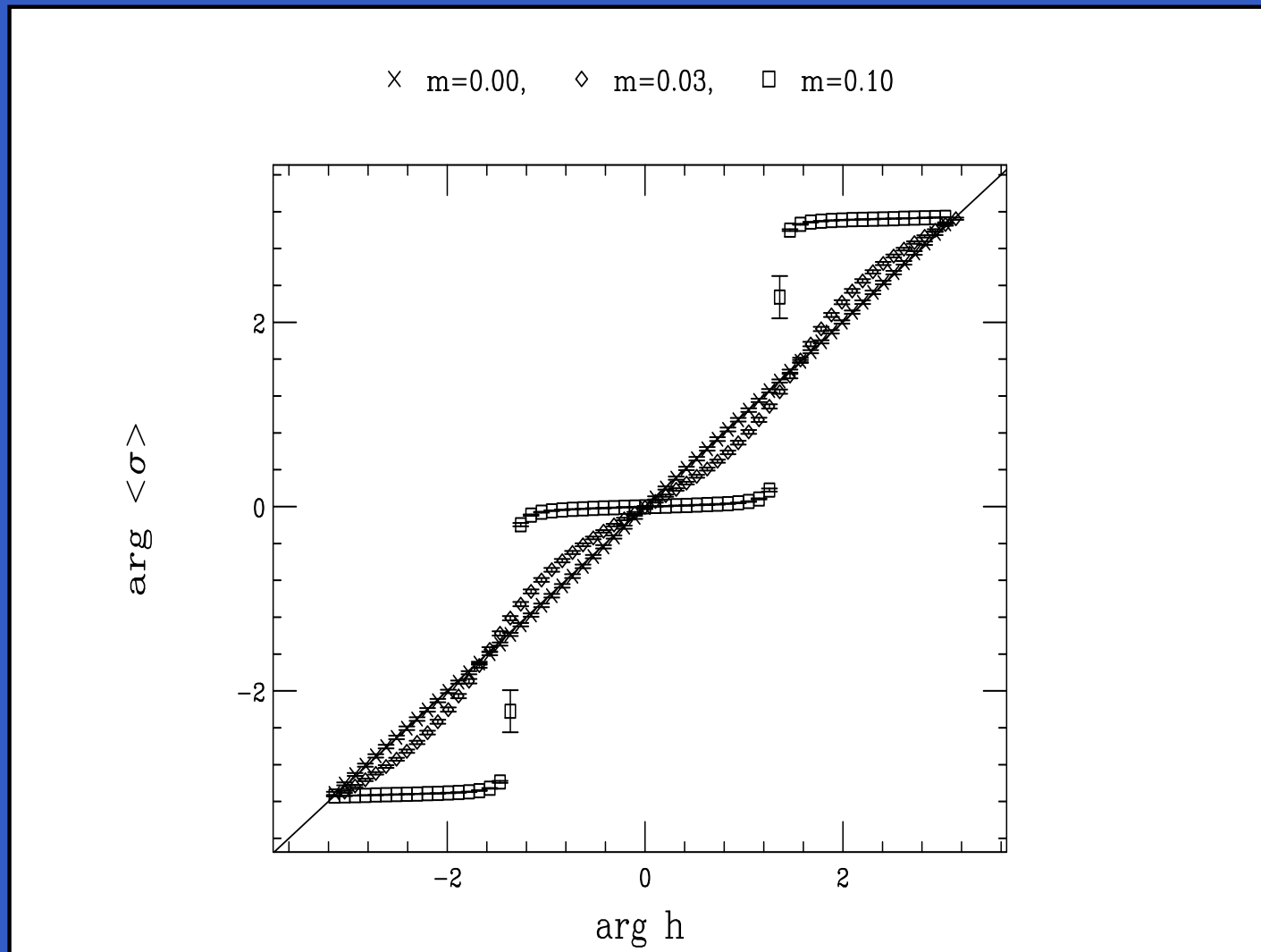


Figure 1: Comparison to the prediction $\arg \langle \sigma \rangle_h = \arg h$.

Evidence of $U(1)_R$

- The second part of the conjecture was studied through the quantity

$$R(|\langle \sigma \rangle_h|) = \frac{|\langle \sigma \rangle_h| - \overline{|\langle \sigma \rangle|}}{\overline{|\langle \sigma \rangle|}}, \quad \overline{|\langle \sigma \rangle|} = \frac{1}{n} \sum_{j=1}^n |\langle \sigma \rangle_{h_j}|$$

- Here, $h_j = |h| \exp(2\pi i j/n)$ corresponds to the values of h that were used in the data set.
- Thus, R measures the relative shift of $|\langle \sigma \rangle_h|$ away from the mean, where the mean is taken w.r.t. $\arg h$.

Evidence of $U(1)_R$

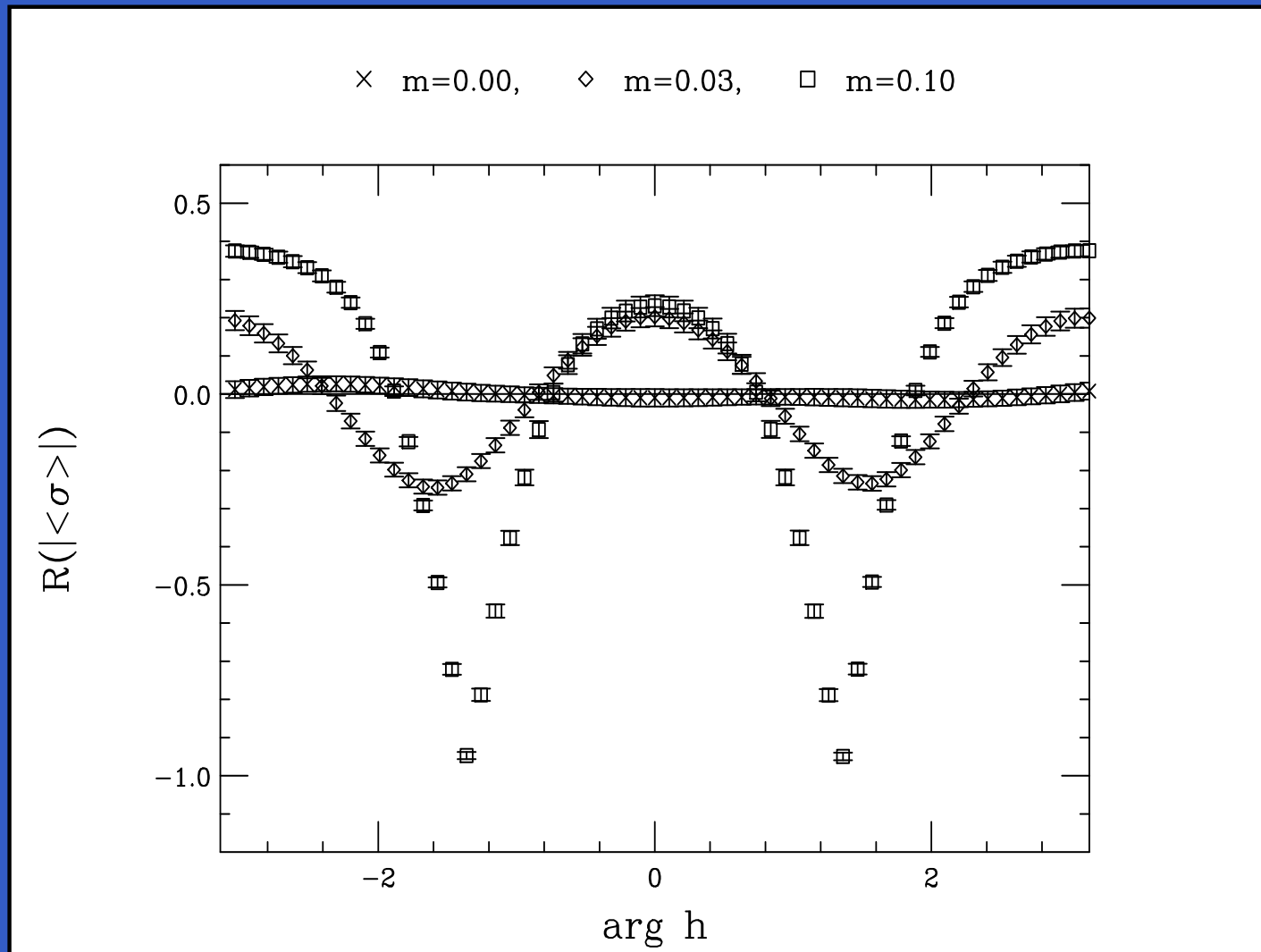


Figure 2: Comparison to the prediction that $|\langle \sigma \rangle_h|$ should be independent of $\arg h$.

Conclusions

- It is interesting that good perturbative continuum limit and good nonperturbative continuum limit coincide for 2dWZ.
- Seen also in susy-QM models:
 - ◆ Q-exact susy-QM [Catterall, Gregory 00; JG, Poppitz 05].
 - ◆ Naive susy-QM once perturbative counterterm was introduced [JG, Koniuk, Poppitz, Yavin 05].

Conclusions

- Leads to:

CONJECTURE If the quantum continuum limit is correct (supersymmetric) in perturbation theory, then it will also be correct nonperturbatively.

Conclusions

- It would be interesting to study in other models, such as 3d SYM, where perturbative counterterms were determined by Elliott & Moore.
- It would also be interesting to study perturbative continuum limit of the 2d SYM model of Catterall, which has good behavior in Monte Carlo simulation.

Conclusions

- There is an argument:
 - ◆ The violation of supersymmetry is short distance.
 - ◆ Suppose there are **no important** nonperturbative effects at short distance, due to weak coupling (above: ag is the dimensionless coupling): $\sim \exp[-C/(ag)]$.
 - ◆ Thus, once the perturbative counterterms are included, the correct continuum limit should be obtained.
- Note that it is essential that the continuum theory is asymptotically free.