

Hard Data In Defense of Logical Minds

“In Defense of Logical Minds”

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“The Logical Minds Manifesto”

and a book?

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Piagetian Views Dead

\mathcal{F} Humans naturally develop a context-free deductive reasoning scheme at the level of elementary first-order logic.

\mathcal{F}' If educated in logic as they are in arithmetic, humans develop a context-free deductive reasoning scheme at the level of elementary first-order logic — a scheme that will allow them to solve the logic problems studied in the psychology of reasoning literature, and significantly harder problems as well.

\mathcal{F} Killed by “The Lightning Bolt”

Suppose that I have a pack of cards each of which has a letter written on one side and a number written on the other side. Suppose in addition that I claim the following rule is true:

- If a card has a vowel on one side, then it has an even number on the other side.

Imagine that I now show you four cards from the pack:

$\boxed{\text{E}}$ $\boxed{\text{T}}$ $\boxed{4}$ $\boxed{7}$

Which card or cards should you turn over in order to decide whether the rule is true or false?

The Three Responses

1. Mental Logic (Rips)
2. Mental Models (Johnson-Laird)
3. Pragmatic Reasoning Schemas (Cheng & Holyoak)
 - All three reflect some reasoning of some people — that's all and so, so what?
 - Piaget was basically right
 - The psychology of deductive reasoning needs to be “destroyed” and rebuilt

Mental Logic

Rule *R*:

$$\frac{\neg(P \rightarrow Q)}{P \wedge \neg Q}$$

ML Logically untrained people, when reasoning deductively, do so by employing a set of formal rules of inference containing all those in standard FOL save for *R*.

Refutation of Mental Logic

- (1) If ML is true, then (all) logically untrained people are unable to reason deductively in order to solve X = an AR problem like those on the LSAT, except the problem involves “the missing rule” R .
- (2) Some logically untrained people are able to reason deductively in order to solve X .

Therefore:

- (3) ML is not true.

Mental Models — Works With Syllogisms

- All artists are beekeepers.
- All beekeepers are chemists.

Therefore:

- All artists are chemists.

artist = beekeeper = chemist

artist = beekeeper = chemist

artist = beekeeper = chemist

(beekeeper) = (chemist)

(beekeeper) = (chemist)

(chemist)

The Mental Models Doctrine

MM Logically untrained people, when reasoning deductively in an attempt to solve problem P , do so by constructing mental models in which the true premises in P are captured, but in which the false premises in P are not.

Refutation of Mental Models

(4) If MM is true, then logically untrained people do not (cannot?) solve problem $X =$ Dreadsbury Mansion Mystery via a formal proof.

(5) Some logically untrained people can solve X (as my “pine database” shows in the case of Dreadsbury).

Therefore:

(6) MM is false.

The Dreadsbury Mansion Mystery

Someone Who lives in Dreadsbury Mansion killed Aunt Agatha. Agatha, the butler, and Charles live in Dreadsbury Mansion, and are the only people who live therein. A killer always hates his victim, and is never richer than his victim. Charles hates no one that Aunt Agatha hates. Agatha hates everyone except the butler. The butler hates everyone not richer than Aunt Agatha. The butler hates everyone Agatha hates. No one hates everyone. Agatha is not the butler.

Now, given the above clues, there is a bit of a disagreement between three (incompetent?) Norwegian detectives: Inspector Bjorn is sure that Charles didn't do it. Is he right? Inspector Reidar is sure that it was a suicide. Is he right? Inspector Olaf is sure that the butler, despite conventional wisdom, is innocent. Is he right?

And Can MM Be Used to Solve This One?

- Is it true that there exists something which is such that if it's a sneeber then everything is a sneeber?
- Alternatively: $\vdash \exists x(Sx \rightarrow \forall ySx)$?

Pragmatic Reasoning Schemas

P1 If the actions is to be taken, then the precondition must be satisfied.

P2 If the actions is not taken, then the precondition need not be satisfied.

P3 If the precondition is sat, then the action may be taken.

P4 If the precondition is not satisfied, then the action must not be taken.

So compare:

- If a card has an vowel on one side, then it must have an even number on the other.
- If a person is drinking beer, then they must have a license indicating they are over 18 yrs of age.

Fatal Problems Infecting Pragmatic Reasoning Schemas

- By definition only applies to problems that have conditionals from standard FOL that can map to P1-P4.
 - so impotent in face of Dreadsbury and “sneeber” problem
- Once again: many subjects solve problems by *using* standard FOL.

Cheng and Holyoak's Paper

- “Wait a minute. Maybe Piaget isn't too popular these days, but is it literally the case that logic instructors strive to teach what psychologists of reasoning say cannot be taught?”
- C&H's instruction didn't include three teaching techniques we believe are crucial:
 1. disproofs;
 2. diagrammatic inference rules; and
 3. rigorous and general-purpose procedures for formalizing natural language logic problems in first-order logic so that they can then be solved by automated theorem provers

Structure of Latest Study

- pre-test/post-test design carried out during Bringsjord's *Introduction to Logic*.
- Hypothesized that by teaching logic using these three key techniques, direct evidence for the Piagetian pair could be obtained.
- In order to use these three techniques, we used both Barwise and Etchemendy's Hyperproof system (which allows for both disproofs and diagrammatic inference) and the first-order theorem-prover OTTER (in which even robust natural language logic problems can be expressed).

Lots of Interesting Data – But Reductio Perhaps Most Interesting

- 2% correct pre-test (0% w/o some training)
- 80% correct (correlate) post-test
- students (and math teachers) haven't a clue about reductio
- what about psychologists of reasoning?

Sample Problem From Pre-Test (no reductio)

We will use lower-case Roman letters a, b, c, \dots to represent propositions. Let the symbol ' \neg ' stand for 'it is not the case that.' Let the symbol ' \vee ' stand for 'or.' Let the symbol ' \rightarrow ' stand for if-then, so that $p \rightarrow q$ means 'if p then q .'

Now, given the statements

$$\neg a \vee \neg b$$

$$b$$

$$c \rightarrow a$$

which one of the following statements must also be true? (Check the correct answer.)

c

$\neg b$

$\neg c$

h

a

none of the above

- **Have you seen this problem before?** Yes No
(circle one)

- **Justification:**

Sample Problem Post-Test (reductio)

Once again, we will use lower-case Roman letters a, b, c, \dots to represent propositions. Let the symbol ' \neg ' stand for 'it is not the case that.' Let the symbol ' \vee ' stand for 'or.' Let the symbol ' \rightarrow ' stand for if-then, so that $p \rightarrow q$ means 'if p then q .'

Given the statements

$$\neg\neg c$$

$$c \rightarrow a$$

$$\neg a \vee b$$

$$b \rightarrow d$$

$$\neg(d \vee e)$$

which one of the following statements must also be true? (Check the correct answer.)

$\neg c$

e

h

$\neg a$

all of the above

- **Have you seen this problem before?** Yes No
(circle one)

- **Justification:**

Two “Cognitive Illusions,” The First Clever, The Second a Bad Mistake (Johnson-Laird)

1. If there is a king in the hand, then there is an ace, or else if there isn't a king in the hand, then there is an ace.
2. Only one of the following two assertions is true.
 - Albert is here or Betty is here, or both.
 - Charlie is here or Betty is here, or both.

This assertion is definitely true.

- Albert isn't here and Charlie isn't here.

“These premises yield the illusion that Betty is here.”
(p. 431, *Minds & Machines* exchange, 7.3 1997)

Johnson-Laird's Challenge to the "Other" APA

If one of the following assertions is true then so is the other:

1. There is a king in the hand if and only if there is an ace in the hand.
2. There is a king in the hand.

Which is more likely to be in the hand, if either: the king or the ace?

Solution

You could obviously prove $K \& A$ in the cases where

(1)	(2)
T	T
T	F
F	T

so everything must boil down to the F F case. It's easy here to do a proof that can get A (constructive dilemma and reductio), but $\neg K$ instead of K. Ergo, the answer must be A.

The Planet Wearth

On Wearth, all human children without significant cognitive deficiencies are educated so as to routinely exploit formal logic. Starting in the 1st grade, students learn the propositional calculus — alongside arithmetic of the sort their counterparts learn on Earth. At the start of the 3rd grade, children on Wearth crack into quantifier logic, and are given problem after problem that yields to FOL. For every conventional math word problem children receive on Earth, children on Wearth get five word problems to be solved by formal logic. At the sixth grade, Wearth children are taught n -order extensional logic, and the simplest of intensional logics, and are asked to provide formal proofs in these systems alongside the usual “plug and chug” calculation seen on Earth for elementary algebra, geometry, and calculus.

Wearth will produce many cognizers that exceed Piaget’s standards; such cognizers will not be rare at all. Again, that *our* psychologists need to search hard for such people is just a coincidence, a function of the way we teach our young.