Final Exam

Introduction to Logic, Fall 98

February 19, 1999

Instructions

Put all your answers in this test booklet. You may use scrap paper of course, but your answers must be given on the appropriate pages provided in this booklet. In particular, this means that your solution to the Hyperproof problem (Part I) must be given on the sheet(s) provided.

Student Info

- $\Rightarrow$ Name:
- $\Rightarrow$ Social Security #:

Part I

The Hyperproof problem is Proof 9.49, directly from the Hyperproof book and disk. Notice that there are three goals, hence the three figures shown below. (If you are unsure about any of the visual information in these hard-copy diagrams, ask the proctor, who can refer to a pristine original.) An acceptable answer to this question must be a Hyperproof proof, specified line by line, with the appropriate icon to indicate a connection to visual information; in other words, provide exactly the sort of answer you were directed to provide on the Midterm. If you need to present visual information, draw a grid or grids populated in the customary way with appropriate symbols, e.g., □ for cubes. Make sure you show which line in the proof connects with which grid. **Please write your proof and diagrams in this booklet, on the pages provided**. You may of course use scrap paper — but your final answer should be written on the pages provided.

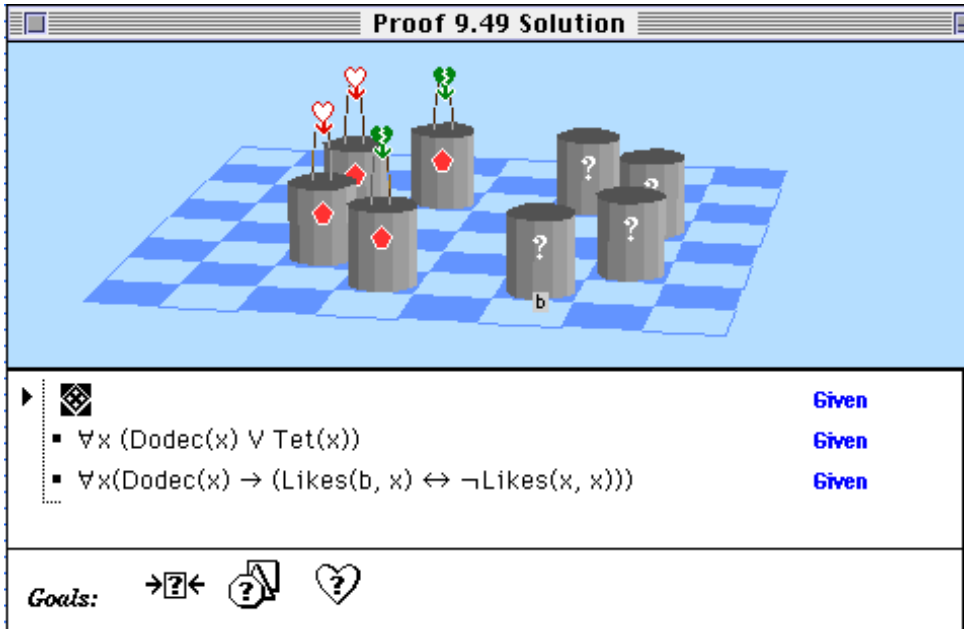

Figure 1: Proof 9.49 – Goal 1.

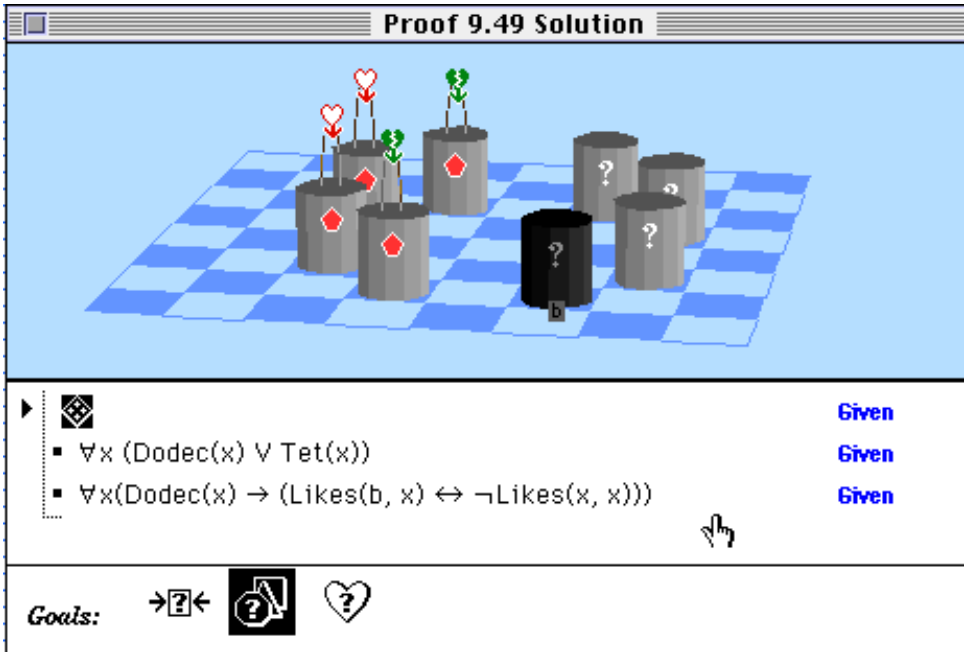

Figure 2: Proof 9.49 – Goal 2.

Space for Answer to Part I

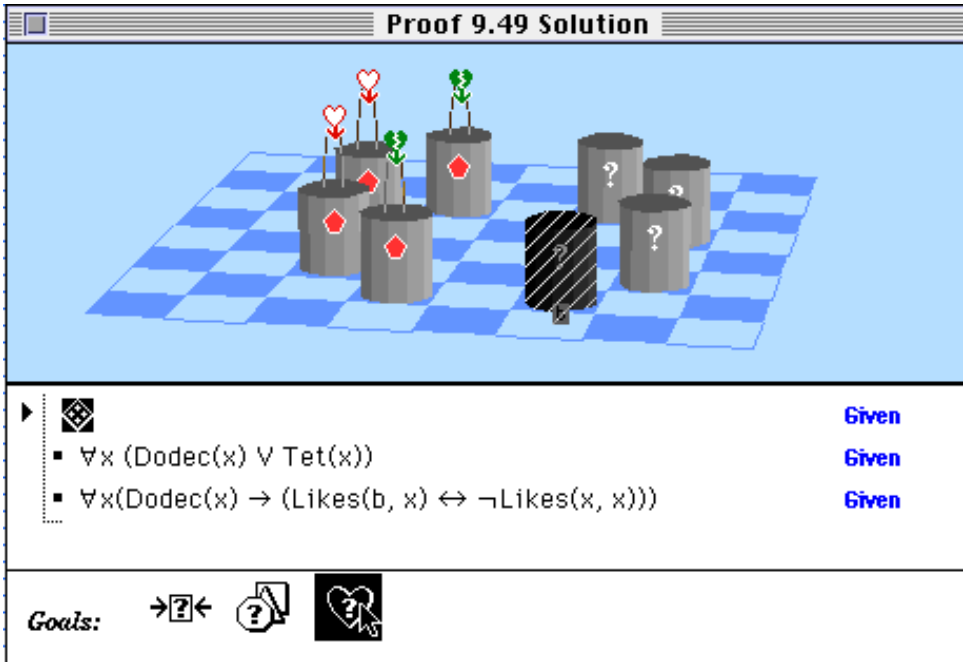

Figure 3: Proof 9.49 – Goal 1.

Space for Answer to Part I

Space for Answer to Part I

Part II

Note: The problems in this part generally get more challenging the later they fall. If you are unable to make progress toward solving a particular problem after sustained thought, move on to the next one, but be aware of the fact that the later problems are quite challenging.

1

Suppose that you are doing an assignment for a biology expedition. You learn before starting on this expedition that insects can be one of two kinds, a spade fly or a bevel wasp, and that insect color is either black or green. Your task is to study insects in order to find out if a certain rule is false. The rule is:

- If an insect is a spade fly, then it is black.

You see an insect that is green. Which of the following would be true about the insect if it violates the rule?

a The insect is a spade fly.

b The insect is a bevel wasp.

c The type of the insect does not matter.

- **Answer**:
- **Justification**:

2

Suppose that there are four possible kinds of objects:

- a smart tetrahedron
- a stupid tetrahedron
- a smart cube
- a stupid cube

Suppose as well that I have written down on a hidden piece of paper one of the mental attributes (smart/stupid), and one of the shapes (tetrahedron/cube). Now read the following rule carefully:

- An object is a LOKE if and only if it has neither the mental attribute I have written down, nor the shape I have written down..

I will tell you that the stupid tetrahedron is a LOKE. Which of the other objects, if any, is a LOKE?

- **Answer**:
- **Justification**:

3

We will use lower-case Roman letters $a, b, c, \ldots$ to represent propositions. Let the symbol '$\neg$' stand for 'it is not the case that.' Let the symbol '$\vee$' stand for 'or.' Let the symbol '$\rightarrow$' stand for 'if-then,' so that $p \rightarrow q$ means 'if p then q.' And let the symbol '$\leftrightarrow$' stand for 'if and only if,' so that $p \leftrightarrow q$ means 'p if and only if q.'

Now, given the statements

$\neg a \vee b$

$\neg\neg a$

$b \rightarrow \neg d$

$e \leftrightarrow d$

which one of the following statements must also be true? (Check the correct answer.)

☐ e

☐ $\neg b$

☐ $\neg e$

☐ h

☐ $\neg a$

☐ none of the above

- **Justification**:

4

Which one of the following statements is logically equivalent to the following statement: "If you are not for us, then you are against us." (Check the correct answer.)

☐ If you are not against us, then you are for us.

☐ If you are not for us, then you are not against us.

☐ If you are for us, then you are against us.

☐ If you are against us, then you are not for us.

- **Justification**:

5

There are only two sorts of people: knights, who always tell the truth, and knaves, who always lie. Suppose there are three individuals, A, B, and C, each of whom is either a knight or a knave. Also, suppose that two people are of the same type if they are both knights or both knaves.

A says: "B is a knight."
B says: "A and C are of the same type."
Question: Is C a knight or a knave?

- **Answer**:
- **Justification**:

6

Once again, we will use lower-case Roman letters $a, b, c, \ldots$ to represent propositions. Let the symbol '$\neg$' stand for 'it is not the case that.' Let the symbol '$\vee$' stand for 'or.' Let the symbol '$\wedge$' stand for 'and.' Let the symbol '$\rightarrow$' stand for if-then, so that $p \rightarrow q$ means 'if p then q.' And let the symbol '$\leftrightarrow$' stand for 'if and only if,' so that $p \leftrightarrow q$ means 'p if and only if q.'

Given the statements

$\neg\neg a$

$a \rightarrow b$

$\neg b \vee c$

$c \leftrightarrow e$

$\neg(e \vee f)$

which one of the following statements must also be true? (Check the correct answer.)

☐ $\neg a$

☐ f

☐ h

☐ $\neg b$

☐ all of the above

- **Justification**:

7: The Mystery of the Missing Diamond

When Queen Elizabeth, readying herself for her kingdom's annual gala, reached into her safe for the largest diamond on Earth, she recoiled in horror, and screamed so loud that the King, who was shaving, gave himself a nasty little cut.

Within one hour Sherlock Hemlock has assembled four suspects: Count Miles, Prince Henry, Dr. Devin, and Palsgraff.

"Did you steal it, Miles?" Holmes asked.

"Most certainly not!" the Count replied. He also volunteered: "Palsgraff is the culprit, Mr. Holmes."

"What about you, Prince Henry?" Holmes asked. "Are you by any chance the thief?"

"One of us stole it," Henry replied, "but I assure you that it wasn't me!"

"And what about you?" continued the master detective to Dr. Devin. "What do you have to say about all of this? Did Miles and Henry both tell the truth?"

"Well, one of them did," replied Devin, who then feinted and remained unconscious for the remainder of the investigation.

It wasn't long before Holmes discovered that it was not the case that both Palsgraff stole the diamond and Devin spoke the truth. At that point the great detective was able to deduce the solution.

Who stole the diamond? Indicate your answer and, in the **Justification** space below, try to give Holmes' deduction.

- **Answer**:
- **Justification**: