Linguistic Terms (Termset)

Label	Symbol	Universe of Discourse 1	2	3	4	5	6	7	8	9
Very Low	Ver_L	1	0.64	0.09	0.01	0	0	0	0	0
Low	L	1	0.80	0.30	0.10	0	0	0	0	0
More or less Low	Mor_L	1	0.89	0.55	0.32	0	0	0	0	0
Above Low	> L	0	0.20	0.70	0.90	1	1	1	1	1
Below Medium	< M	1	1.00	0.90	0.25	0	0	0	0	0
Above Low and Below Medium		0	0.20	0.70	0.25	0	0	0	0	0
Between Medium and H	L-M	0	0.29	1	0.36	0	0	0	0	0
Medium	M	0	0	0.10	0.75	1	0.75	0.10	0	0
Above Medium	>M	0	0	0	0	0	0.25	0.90	1	1
Below High	< H	1	1	1	1	1	0.90	0.70	0.20	0
Above Medium and Below High		0	0	0	0	0	0.25	0.70	0.20	0
Between Medium and H	M-H	0	0	0	0	0	0.36	1	0.29	0
More or less High	Mor_H	0	0	0	0	0	0.32	0.55	0.89	1
High	H	0	0	0	0	0	0.10	0.30	0.80	1
Very High	Ver_H	0	0	0	0	0	0.01	0.09	0.64	1

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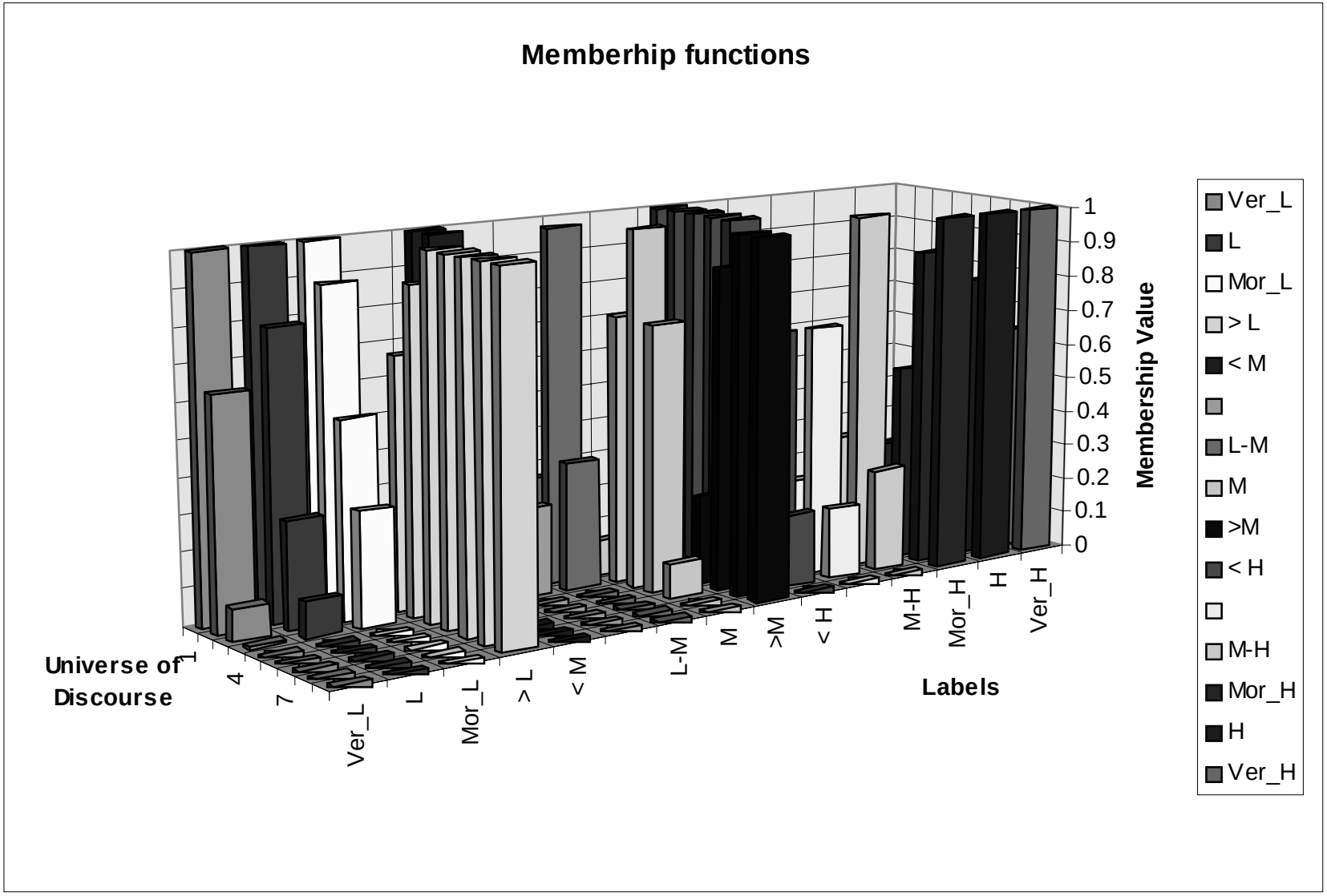
Termset Plot



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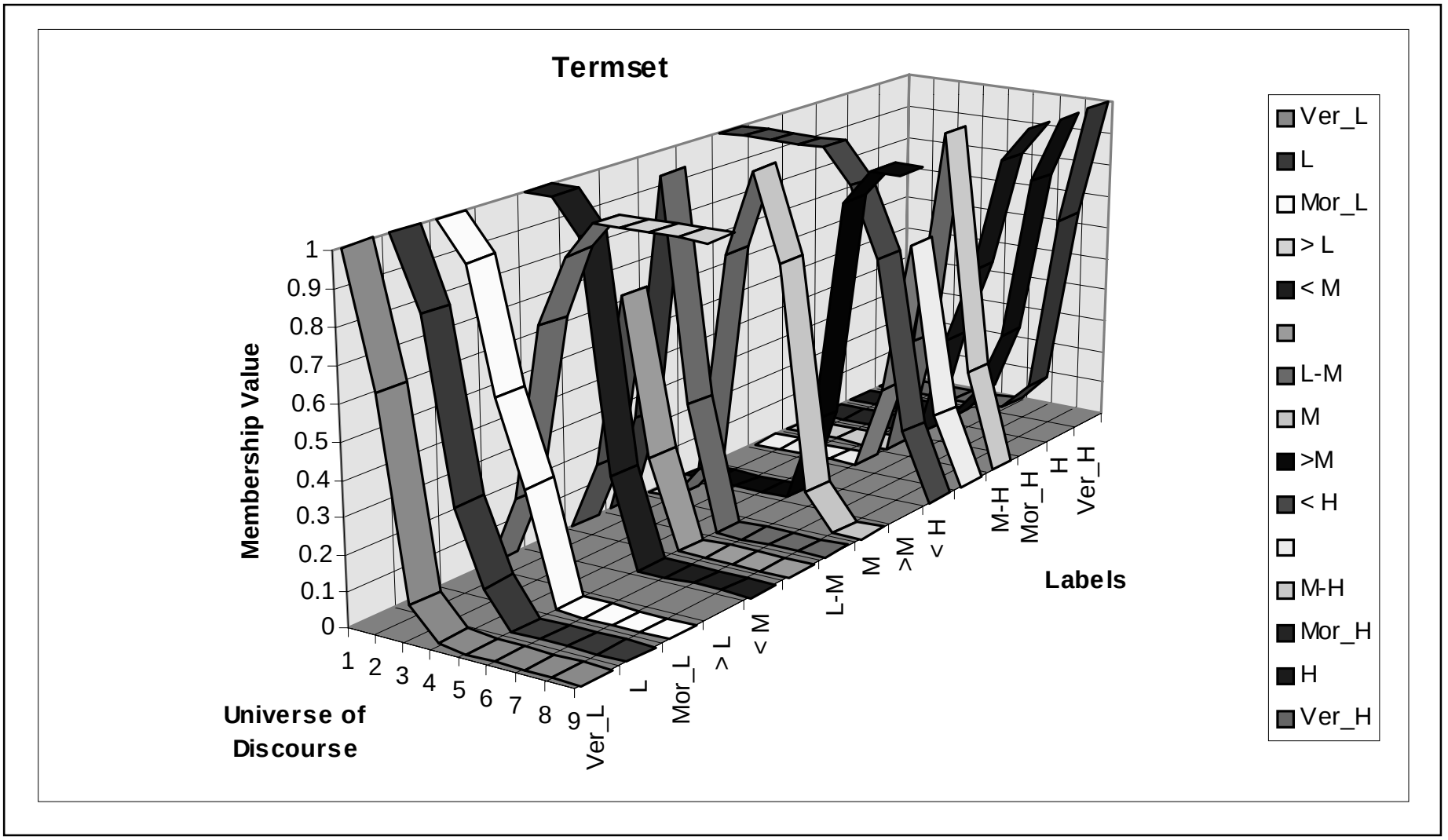
Termset Plot (Interpolated)



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Example of Fuzzy Relation R & Modus Ponens

		Universe of Discourse								
Label	**Symbol**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Very Low	Ver_L	1	0.64	0.09	0.01	0	0	0	0	0
Low	L	1	0.80	0.30	0.10	0	0	0	0	0
Between Low and Medium	L-M	1	0.80	0	0.10	0	0	0	0	0
Medium	M	0	0	0.10	0.75	1	0.75	0.10	0	0
Between Medium and High	M-H	0	0	0.1	0.75	1	0.75	0	0.00	0
High	H	0	0	0	0	0	0.10	0.30	0.80	1

$$R_i: [X \times Y] \rightarrow [0,1]$$

$$R = \bigcup_{i=1}^{5} R_i$$

R	X	Y
R_1	L	L
R_1	L-M	M
R_3	M	H
R_4	M-H	M
R_5	H	Ver_L

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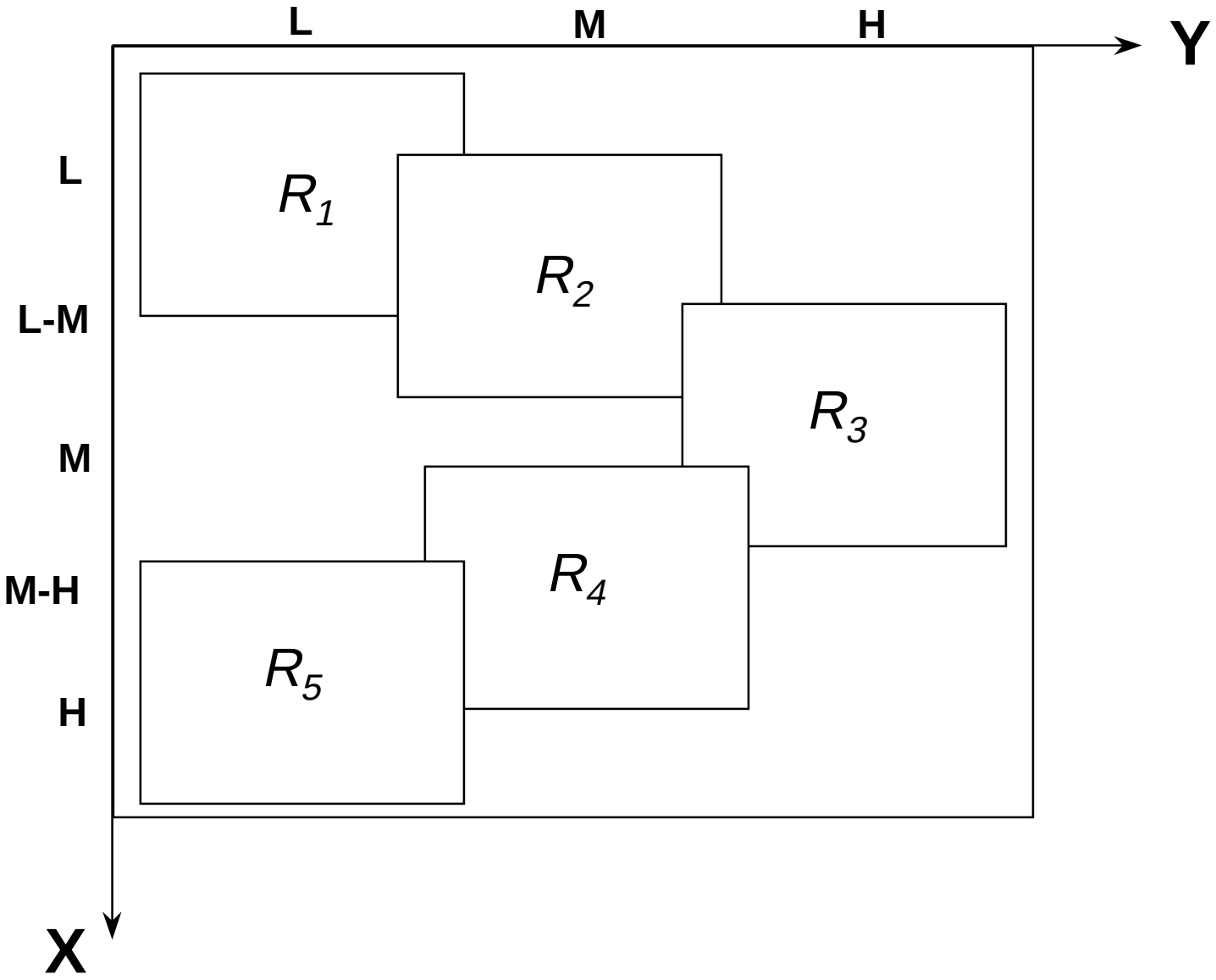
Corresponding Fuzzy Graph



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Modus Ponens Implementation: Three Methods

Method 1

a) Relation R is the Union of the Cartesian Products

b) Cylindrical Extension of Input I

c) Intersection with Relation R

d) Projection of Intersection

Method 2

a) Relation R is the Union of the Cartesian Products

b) Inner Product *<Min, Max>* between Input I and Relation R

Method 3

a) λ_i = degree of $rule_i$ applicability (T-norm of Possibilty Measures between Input I and $rule_i$ LHS)

b) Weigh $rule_i$ RHS using λ_i as weights

c) Union of Weighted Outputs

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Method 1a: Union of Fuzzy Relations

R_1		Y = L								
		1	**0.8**	**0.3**	**0.1**	**0**	**0**	**0**	**0**	**0**
X = L	**1**	**1.0**	0.8	0.3	0.1	0.0	0.0	0.0	0.0	0.0
	0.8	0.8	0.8	0.3	0.1	0.0	0.0	0.0	0.0	0.0
	0.3	0.3	0.3	0.3	0.1	0.0	0.0	0.0	0.0	0.0
	0.1	0.1	0.1	0.1	0.1	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

$$R_1(x,y) =$$
$$= [\ \mathrm{Low}(x)\ \times\ \mathrm{Low}(y)]$$
$$= Min\{\mathrm{Low}(x), \mathrm{Low}(y)\}$$

R_2		Y = M								
		0	**0**	**0.1**	**0.75**	**1**	**0.8**	**0.1**	**0**	**0**
X = L M	**0**	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.3	0.0	0.0	0.1	0.3	0.3	0.3	0.1	0.0	0.0
	1	0.0	0.0	0.1	0.8	**1.0**	0.8	0.1	0.0	0.0
	0.4	0.0	0.0	0.1	0.4	0.4	0.4	0.1	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

$$R_2(x,y) =$$
$$= [\ \mathrm{Low\text{-}Medium}(x)\ \times\ \mathrm{Medium}(y)]$$
$$= Min\{\mathrm{Low\text{-}Medium}(x), \mathrm{Medium}(y)\}$$

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Method 1a: Union of Fuzzy Relations (cont.)

R_3		Y = H								
		0	**0**	**0**	**0**	**0**	**0.1**	**0.3**	**0.8**	**1**
X = M	**0**	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.1	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1
	0.8	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.8	0.8
	1	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.8	**1.0**
	0.8	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.8	0.8
	0.1	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

$R_3(x,y) =$

$= [\ \text{Medium}(x)\ \text{x}\ \text{High}(y)]$

$= Min\{\text{Medium}(x), \text{High}(y)\}$

R4		Y = M								
		0	**0**	**0.1**	**0.8**	**1**	**0.8**	**0.1**	**0**	**0**
X = MH	**0**	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.4	0.0	0.0	0.1	0.4	0.4	0.4	0.1	0.0	0.0
	1	0.0	0.0	0.1	0.8	**1.0**	0.8	0.1	0.0	0.0
	0.3	0.0	0.0	0.1	0.3	0.3	0.3	0.1	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

$R_4(x,y) =$

$= [\ \text{Medium-High}(x)\ \text{x}\ \text{Medium}(y)]$

$= Min\{\text{Medium-High}(x), \text{Medium}(y)\}$

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Method 1a: Union of Fuzzy Relations (cont.)

R_5		Y = Ver_L								
		1	**0.64**	**0.09**	**0.01**	**0**	**0**	**0**	**0**	**0**
X = H	**0**	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.1	0.10	0.10	0.09	0.01	0.0	0.0	0.0	0.0	0.0
	0.3	0.30	0.30	0.09	0.01	0.0	0.0	0.0	0.0	0.0
	0.8	0.80	0.64	0.09	0.01	0.0	0.0	0.0	0.0	0.0
	1	**1.00**	0.64	0.09	0.01	0.0	0.0	0.0	0.0	0.0

$R_5(x,y) =$

$= [\ \mathrm{High}(x)\ \times\ \mathrm{VeryLow}(y)]$

$= Min\{\mathrm{High}(x), \mathrm{VeryLow}(y)\}$

R	Y								
X	**1.00**	0.80	0.30	0.10	0.0	0.0	0.0	0.0	0.0
	0.80	0.80	0.30	0.29	0.29	0.29	0.10	0.0	0.0
	0.30	0.30	0.30	0.75	**1.00**	0.75	0.10	0.10	0.10
	0.10	0.10	0.10	0.36	0.36	0.36	0.30	0.75	0.75
	0.00	0.00	0.00	0.00	0.00	0.10	0.30	0.80	**1.00**
	0.10	0.10	0.10	0.36	0.36	0.36	0.30	0.75	0.75
	0.30	0.30	0.10	0.75	**1.00**	0.75	0.10	0.10	0.10
	0.80	0.64	0.10	0.29	0.29	0.29	0.10	0.0	0.0
	1.00	0.64	0.09	0.01	0.0	0.0	0.0	0.0	0.0

$$\mathrm{R} = \bigcup_{i=1}^{5} R_i$$

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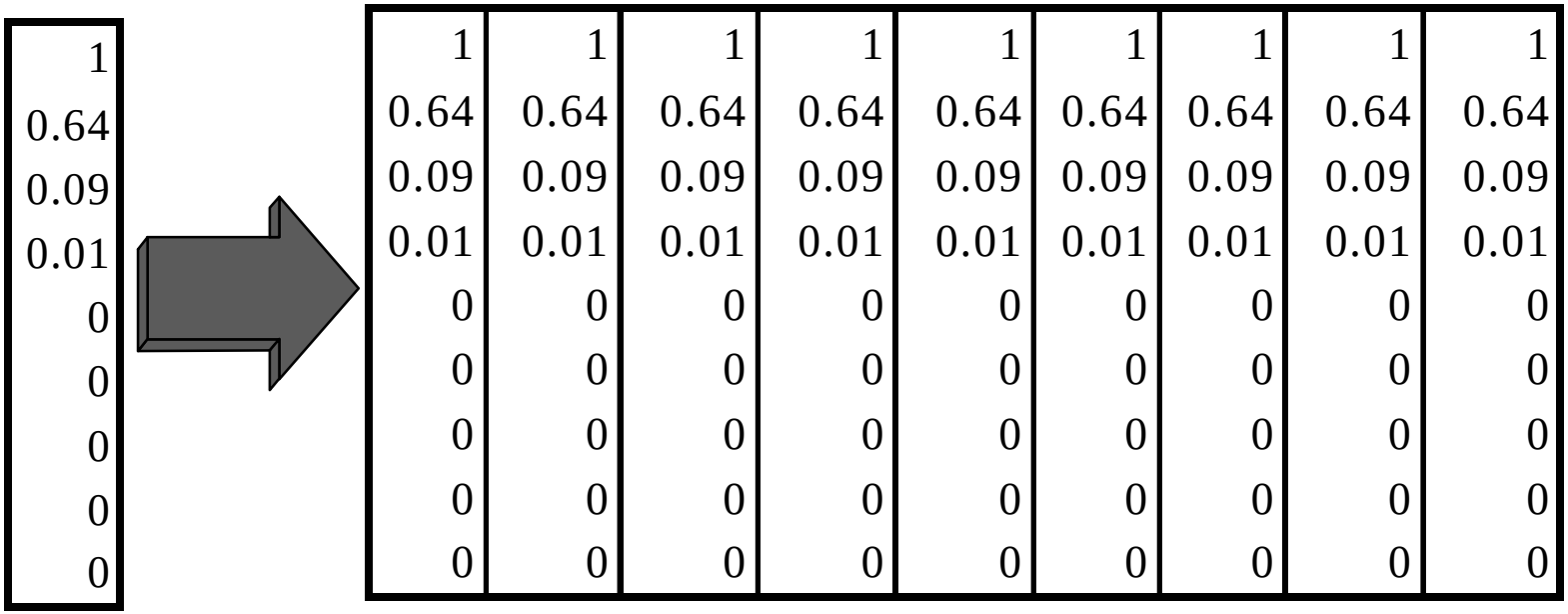
Method 1b: Input Cylindrical Extension

Example: $Y_1 = X_1$ o R

Cylindrical Extension



1	1	1	1	1	1	1	1	1
0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64
0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

X_1= Very Low

$\overline{X}_1$= Cylindrical Extension(Very Low)

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Method 1c: Intersection of Cyl. Ext. with Relation

Intersection

Cylindrical Extension

1	1	1	1	1	1	1	1	1
0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64
0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Relation

1	0.8	0.3	0.1	0	0	0	0	0
0.8	0.8	0.30	0.29	0.29	0.29	0.10	0	0
0.3	0.3	0.30	0.75	**1**	0.75	0.10	0.10	0.10
0.1	0.1	0.10	0.36	0.36	0.36	0.30	0.75	0.75
0	0	0	0	0	0.1	0.30	0.80	**1**
0.1	0.10	0.10	0.36	0.36	0.36	0.30	0.75	0.75
0.3	0.30	0.10	0.75	**1**	0.75	0.10	0.10	0.10
0.8	0.64	0.10	0.29	0.29	0.29	0.1	0	0
1	0.64	0.09	0.01	0	0	0	0	0

Minimum (pointwise)

Intersection

1	0.8	0.3	0.1	0	0	0	0	0
0.64	0.64	0.30	0.29	0.29	0.29	0.10	0	0
0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

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Method 1d: Projection of Intersection

Projection

Intersection

1	0.8	0.3	0.1	0	0	0	0	0
0.64	0.64	0.30	0.29	0.29	0.29	0.10	0	0
0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Maximum

Result:
$Y_1 = X_1 \text{ o } R$

1	0.8	0.30	0.29	0.29	0.29	0.1	0.09	0.09

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Method 2 b: Inner Product *<Min Max>*

Example: $Y_1 = X_1$ o R

Inner Product

X^T_1= Very Low

1	0.64	0.09	0.01	0	0	0	0	0

<Min Max> Inner Product

[similar to Vector-Matrix Multiplication, using <Min Max> instead of <x,+>]

R

1	0.8	0.3	0.1	0	0	0	0	0
0.8	0.8	0.30	0.29	0.29	0.29	0.10	0	0
0.3	0.3	0.30	0.75	**1**	0.75	0.10	0.10	0.10
0.1	0.1	0.10	0.36	0.36	0.36	0.30	0.75	0.75
0	0	0	0	0	0.1	0.30	0.80	**1**
0.1	0.10	0.10	0.36	0.36	0.36	0.30	0.75	0.75
0.3	0.30	0.10	0.75	**1**	0.75	0.10	0.10	0.10
0.8	0.64	0.10	0.29	0.29	0.29	0.1	0	0
1	0.64	0.09	0.01	0	0	0	0	0

Result:
$Y_1 = X_1$ o R

1	0.8	0.30	0.29	0.29	0.29	0.10	0.09	0.09

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Method 3 a: Degree of Rule Applicability (λ_i)

Example: $Y_1 = X_1$ o R

Degree of Rule Applicability

R	X	Y
R_1	L	L
R_1	L-M	M
R_3	M	H
R_4	M-H	M
R_5	H	Ver_L

Input: Very Low	1	0.6	0.09	0.01	0	0	0	0	0
X_1: Low	1	0.8	0.3	0.1	0	0	0	0	0
Ver_L AND L	1	0.6	0.09	0.01	0	0	0	0	0

Max → $\lambda_1 = 1$

Input: Very Low	1	0.64	0.09	0.01	0	0	0	0	0
X_2: Low-Medium	0	0.3	1	0.36	0	0	0	0	0
Ver_L AND L-M	0	0.3	0.09	0.01	0	0	0	0	0

Max → $\lambda_2 = 0.29$

Input: Very Low	1	0.6	0.09	0.01	0	0	0	0	0
X_3: Medium	0	0	0.1	0.75	1	0.75	0.1	0	0
Ver_L AND M	0	0	0.09	0.01	0	0	0	0	0

Max → $\lambda_3 = 0.09$

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Method 3 a: Degree of Rule Applicability (λ_i)(cont.)

Example: $Y_1 = X_1$ o R

Degree of Rule Applicability

R	X	Y
R_1	L	L
R_1	L-M	M
R_3	M	H
R_4	M-H	M
R_5	H	Ver_L

Input: Very Low	1	0.6	0.09	0.01	0	0	0	0	0
X_4: Medium-High	0	0	0	0	0	0.36	1	0.29	0
Ver_L AND M-H	0	0	0	0	0	0	0	0	0

Max → $\lambda_4 = 0$

Input: Very Low	1	0.6	0.09	0.01	0	0	0	0	0
X_5: High	0	0	0	0	0	0.1	0.3	0.8	1
Ver_L AND H	0	0	0	0	0	0	0	0	0

Max → $\lambda_5 = 0$

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Method 3 b: Weighting Rules Output

Example: $Y_1 = X_1$ o R

Weighting of Rules Output

R	X	Y
R_1	L	(L)
R_1	L-M	M
R_3	M	H
R_4	M-H	M
R_5	H	Ve r_L

1	0.8	0.3	0.1	0	0	0	0	0
1	**0.8**	**0.3**	**0.1**	**0**	**0**	**0**	**0**	**0**

$\lambda_1 = 1$

Weighting (min)

0	0	0.10	0.75	1	0.75	0.10	0	0
0	**0**	**0.10**	**0.29**	**0.29**	**0.29**	**0.10**	**0**	**0**

$\lambda_1 = 0.29$

Weighting (min)

0	0	0	0	0	0.1	0.3	0.8	1
0	**0**	**0**	**0**	**0**	**0.1**	**0.1**	**0.09**	**0.09**

$\lambda_1 = 0.09$

Weighting (min)

0	0	0.1	0.75	1	0.75	0.1	0	0
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**

$\lambda_1 = 0$

Weighting (min)

1	0.64	0.09	0.01	0	0	0	0	0
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**

$\lambda_1 = 0$

Weighting (min)

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Method 3 c: Union of weighted Outputs

Example: $Y_1 = X_1$ o R

Union of Weighted Outputs

Output of Rule r_i

r_1	1	0.8	0.3	0.1	0	0	0	0	0
r_2	0	0	0.10	0.29	0.29	0.29	0.10	0	0
r_3	0	0	0	0	0	0.1	0.1	0.09	0.09
r_4	0	0	0	0	0	0	0	0	0
r_5	0	0	0	0	0	0	0	0	0

$$Output = \bigcup_{i=1}^{5} Output(r_i)$$

1	0.8	0.3	0.29	0.29	0.29	0.10	0.09	0.09

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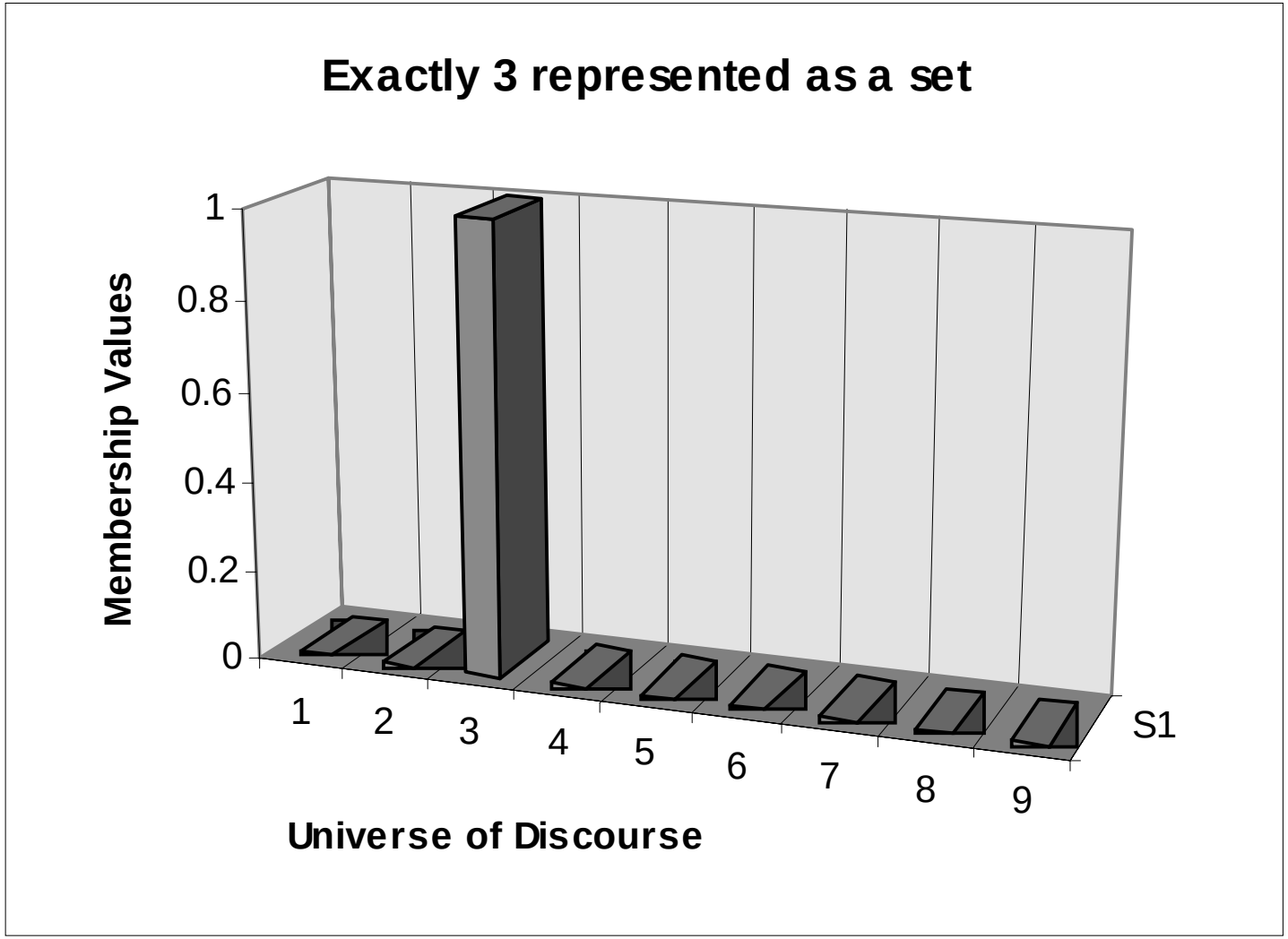
Example with Crisp Input (Exactly 3)

Universe of Discourse	1	2	3	4	5	6	7	8	9
Exactly Three	0	0	1	0	0	0	0	0	0



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Example with Crisp Input (Exactly 3)

Second Method

Input = Exactly 3

Output = Exactly 3 o R_{TOT}

Exactly 3

0	0	1	0	0	0	0	0	0

<min,max>

Note: **<min,max>** means the inner product using the min and max operator

This is similar to the vector-matrix multiplication, which is denoted by the inner product **<.,+>**

R_{TOT}

1	0.8	0.3	0.1	0	0	0	0	0
0.8	0.8	0.3	0.29	0.29	0.29	0.1	0	0
0.3	0.3	0.3	0.75	1	0.75	0.1	0.1	0.1
0.1	0.1	0.1	0.36	0.36	0.36	0.3	0.75	0.75
0	0	0	0	0	0.1	0.3	0.8	1
0.1	0.1	0.1	0.36	0.36	0.36	0.3	0.75	0.75
0.3	0.3	0.1	0.75	1	0.75	0.1	0.1	0.1
0.8	0.64	0.1	0.29	0.29	0.29	0.1	0	0
1	0.64	0.09	0.01	0	0	0	0	0

0.3	**0.3**	**0.3**	**0.75**	**1**	**0.75**	**0.1**	**0.1**	**0.1**

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Analysis of Storage Complexity for Generalized Modus Ponens - Legend

Legend:	**D**	Number of sampling of Xij, i.e. : | Xij| = D, for j = 1,..., n
	m	Number of sampling for Yi, i.e.: | Yi| = m, for i=1,..., r
	n	Number of inputs, i.e.: | Xi| = n, for i= 1,..., r
	t	Numbers of linguistic values of each state variable Xij
	r	Maximum numbers of rules in relation: r = t^n
	Ar	Maximum number of active rules in relation : Ar= 2^n (for each crisp input, assuming overlapping only among adjacent terms in termset)
	Rt	Union of Ri relations (for i=1,..., r), i.e. | Rt| = m D^n
	T	Rule Table: | T| = r(nd+m)

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Analysis of Storage Complexity for Generalized Modus Ponens - Table 1 (D=11)

For D = m =11 (Sampling Points)

Inputs	Values	Rules	Active Rules	Size of Cartesian Product	Size of Rule Table
n	t	r= t^n	Ar= 2^n	|Rt| = m D^n	|T| = (nD+m)r
2	3	9	4	1,331	297
2	5	25	4	1,331	825
2	7	49	4	1,331	1,617
2	9	81	4	1,331	2,673
3	3	27	8	14,641	1,188
3	5	125	8	14,641	5,500
3	7	343	8	14,641	15,092
3	9	729	8	14,641	32,076
4	3	81	16	161,051	4,455
4	5	625	16	161,051	34,375
4	7	2401	16	161,051	132,055
4	9	6561	16	161,051	360,855
5	3	243	32	1,771,561	16,038
5	5	3125	32	1,771,561	206,250
5	7	16807	32	1,771,561	1,109,262
5	9	59049	32	1,771,561	3,897,234
6	3	729	64	19,487,171	56,133
6	5	15625	64	19,487,171	1,203,125
6	7	117649	64	19,487,171	9,058,973
6	9	531441	64	19,487,171	40,920,957

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Analysis of Storage Complexity for Generalized Modus Ponens - Table 2 (D=21)

For D = m =21 (Sampling Points)

Inputs	Values	Rules	Active Rules	Size of Cartesian Product	Size of Rule Table
n	t	r= t^n	Ar= 2^n	|Rt| = m D^n	|T| = (nD+m)r
2	3	9	4	9,261	567
2	5	25	4	9,261	1,575
2	7	49	4	9,261	3,087
2	9	81	4	9,261	5,103
3	3	27	8	194,481	2,268
3	5	125	8	194,481	10,500
3	7	343	8	194,481	28,812
3	9	729	8	194,481	61,236
4	3	81	16	4,084,101	8,505
4	5	625	16	4,084,101	65,625
4	7	2401	16	4,084,101	252,105
4	9	6561	16	4,084,101	688,905
5	3	243	32	85,766,121	30,618
5	5	3125	32	85,766,121	393,750
5	7	16807	32	85,766,121	2,117,682
5	9	59049	32	85,766,121	7,440,174
6	3	729	64	1,801,088,541	107,163
6	5	15625	64	1,801,088,541	2,296,875
6	7	117649	64	1,801,088,541	17,294,403
6	9	531441	64	1,801,088,541	78,121,827

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