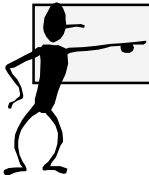


# Outline

- Motivation
- Fuzzy Sets Basic Concepts
  - Characteristic Function (Membership Function)
  - Examples
  - Notation
  - Semantics and Interpretations
  - Related crisp sets
    - » Support, Bandwidth, Core,  $\alpha$ -level cut
    - » Decomposition Theorem
  - Features, Properties, and More Definitions
    - » Convexity, Normality, Fuzzy Singletons
    - » Cardinality, Measure of Fuzziness, First Moment
    - » MF parametric formulation
  - Fuzzy Logic Operations
    - » Intersection, Union, Complementation
    - » Numerical Examples
    - » T-norms and T-conorms



# Numerical Example

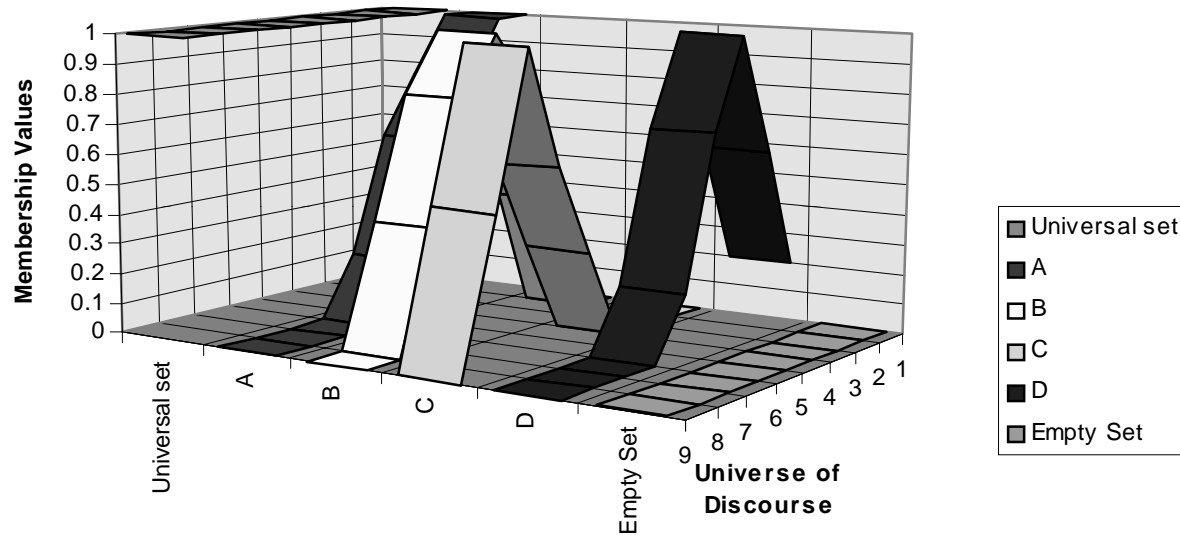
Fuzzy Logic Operations,  
Feature Extractions



# Example: Interpolation

X	1	2	3	4	5	6	7	8	9
Universal set	1	1	1	1	1	1	1	1	1
A	1	1	0.8	0.6	0.2	0	0	0	0
B	0	0	0.4	0.8	1	0.8	0.4	0	0
C	0	0	0	0	0.3	0.6	1	0.5	0
D	0.2	0.6	1	0.7	0.2	0	0	0	0
Empty Set	0	0	0	0	0	0	0	0	0

Interpolated Membership Functions



# Example: Logical Operations

Given:

X	1	2	3	4	5	6	7	8	9
Universal set	1	1	1	1	1	1	1	1	1
A	1	1	0.8	0.6	0.2	0	0	0	0
B	0	0	0.4	0.8	1	0.8	0.4	0	0
C	0	0	0	0	0.3	0.6	1	0.5	0
D	0.2	0.6	1	0.7	0.2	0	0	0	0
Empty Set	0	0	0	0	0	0	0	0	0

Obtain:

X	1	2	3	4	5	6	7	8	9
not A and B									
not A or B									
B and C									
B and D									
B and C and D									
B and (C or D)									
not B or not C									

# Example: Logical Operations (cont)

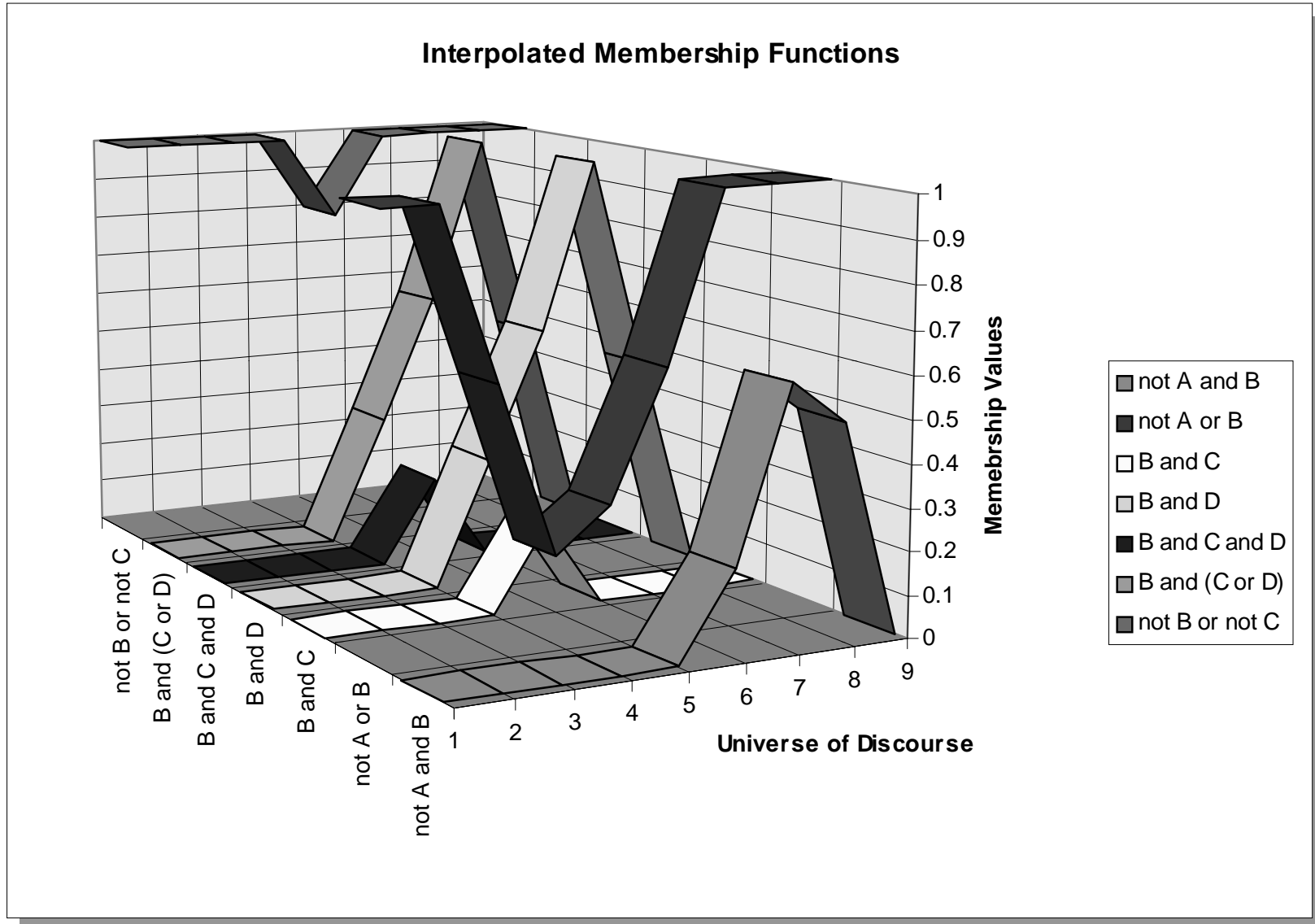
Given:

X	1	2	3	4	5	6	7	8	9
Universal set	1	1	1	1	1	1	1	1	1
A	1	1	0.8	0.6	0.2	0	0	0	0
B	0	0	0.4	0.8	1	0.8	0.4	0	0
C	0	0	0	0	0.3	0.6	1	0.5	0
D	0.2	0.6	1	0.7	0.2	0	0	0	0
Empty Set	0	0	0	0	0	0	0	0	0

Obtain:

X	1	2	3	4	5	6	7	8	9
not A and B	0	0	0.2	0.4	0.8	0.8	0.4	0	0
not A or B	0	0	0.4	0.8	1	1	1	1	1
B and C	0	0	0	0	0.3	0.6	0.4	0	0
B and D	0	0	0.4	0.7	0.2	0	0	0	0
B and C and D	0	0	0	0	0.2	0	0	0	0
B and (C or D)	0	0	0.4	0.7	0.3	0.6	0.4	0	0
not B or not C	1	1	1	1	0.7	0.4	0.6	1	1

# Visualization of Results



# Example: Features Extraction

Given:

X	1	2	3	4	5	6	7	8	9
Universal set	1	1	1	1	1	1	1	1	1
A	1	1	0.8	0.6	0.2	0	0	0	0
B	0	0	0.4	0.8	1	0.8	0.4	0	0
C	0	0	0	0	0.3	0.6	1	0.5	0
D	0.2	0.6	1	0.7	0.2	0	0	0	0
Empty Set	0	0	0	0	0	0	0	0	0

Obtain:

Support  
Core  
Bandwidth  
0.3  $\alpha$ -cut

*Sets*

(Sigma) Cardinality  
1<sup>st</sup> Moment (Centroid)  
Measure of Fuzziness

*Features*

# Example: Associated Sets

Given:

X	1	2	3	4	5	6	7	8	9
Universal set	1	1	1	1	1	1	1	1	1
A	1	1	0.8	0.6	0.2	0	0	0	0
B	0	0	0.4	0.8	1	0.8	0.4	0	0
C	0	0	0	0	0.3	0.6	1	0.5	0
D	0.2	0.6	1	0.7	0.2	0	0	0	0
Empty Set	0	0	0	0	0	0	0	0	0

Obtain:

	Support	Core	Bandwidth	0.3 $\alpha$ -cut
Universal Set	{1,2,3,4,5,6,7,8,9}	{1,2,3,4,5,6,7,8,9}	{1,2,3,4,5,6,7,8,9}	{1,2,3,4,5,6,7,8,9}
A	{1,2,3,4,5}	{1,2}	{1,2,3,4}	{1,2,3,4}
B	{3,4,5,6,7}	{5}	{4,5,6}	{3,4,5,6,7}
C	{5,6,7,8}	{7}	{6,7,8}	{5,6,7,8}
D	{1,2,3,4,5}	{3}	{2,3,4}	{2,3,4}
Empty Set	{}	{}	{}	{}

# Bandwidth and Measure of Fuzziness

X	1	2	3	4	5	6	7	8	9
B	0	0	0.4	0.8	1	0.8	0.4	0	0
Bandwidth(B)	0	0	0	1	1	1	0	0	0
B - Bandwidth(B)	0	0	0.4	-0.2	0	-0.2	0.4	0	0
B-Bandwidth(B)	0	0	0.4	0.2	0	0.2	0.4	0	0

$$\begin{aligned}
 \text{Measure of Fuzziness (B)} &= \\
 &= \text{Sigma Count}( | B - \text{Bandwidth(B)} | ) = \\
 &= 1.2
 \end{aligned}$$

# Bandwidth and Measure of Fuzziness

