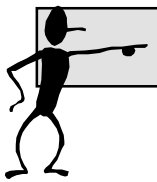
Outline

- Motivation
- Fuzzy Sets Basic Concepts
  - **Characteristic Function (Membership Function)**
  - **Examples**
  - **Notation**
  - **Semantics and Interpretations**
  - **Related crips sets**
    - » **Support, Bandwidth, Core, α-level cut**
    - » **Decomposition Theorem**
  - **Features, Properties, and More Definitions**
    - » **Convexity, Normality, Fuzzy Singletons**
    - » **Cardinality, Measure of Fuzziness, First Moment**
    - » **MF parametric formulation**
  - **Fuzzy Logic Operations**
    - » **Intersection, Union, Complementation**
    - » **Numerical Examples**
    - » **T-norms and T-conorms**



Membership Function (MF) Formulation

Triangular MF:

$$trimf\ (x;a,b,c) = \max\left(\min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right)$$

Trapezoidal MF:

$$trapmf\ (x;a,b,c,d) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right)$$

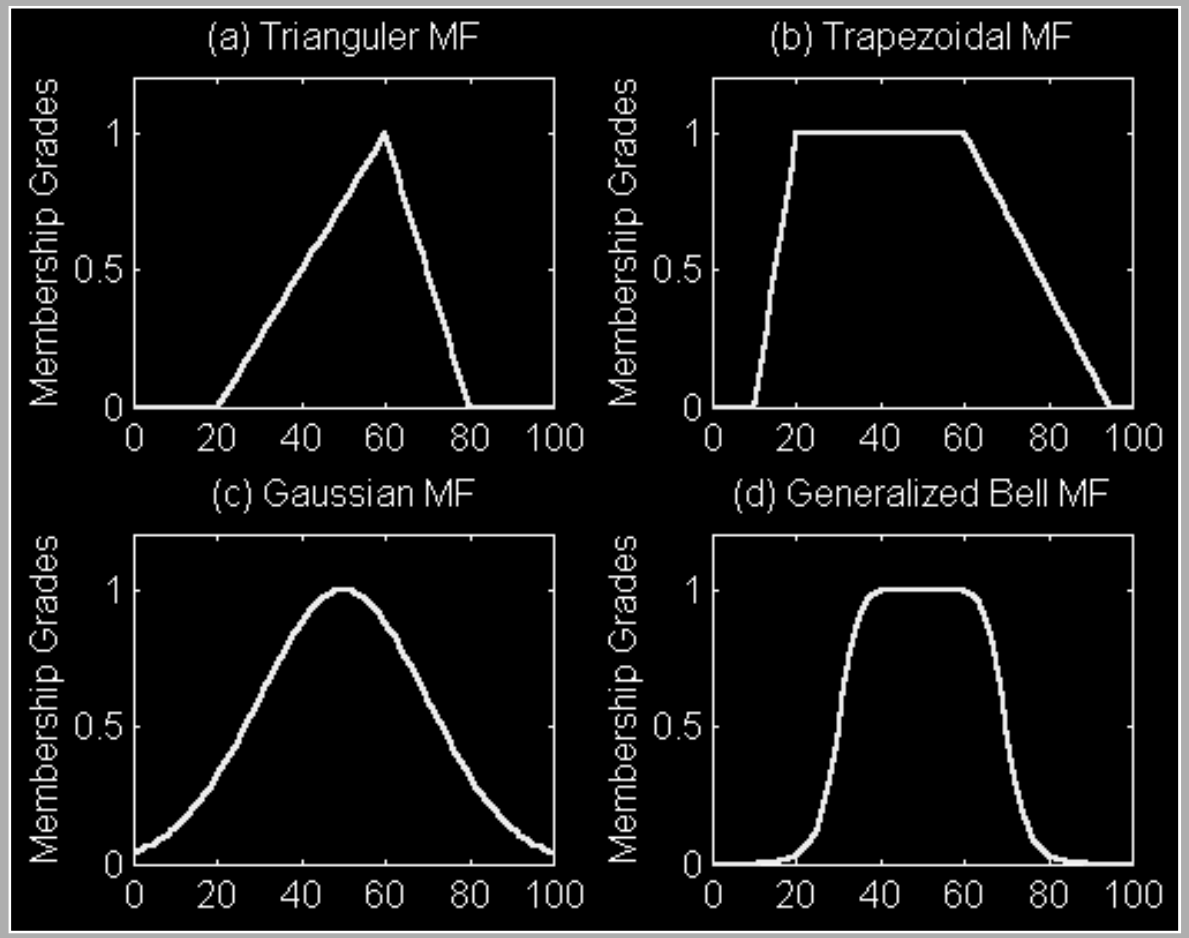
Gaussian MF:

$$gaussmf\ (x;a,b,c) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}$$

Generalized bell MF:

$$gbellmf\ (x;a,b,c) = \frac{1}{1+\left|\frac{x-c}{b}\right|^{2b}}$$

MF Formulation



disp_mf.m

MF Formulation

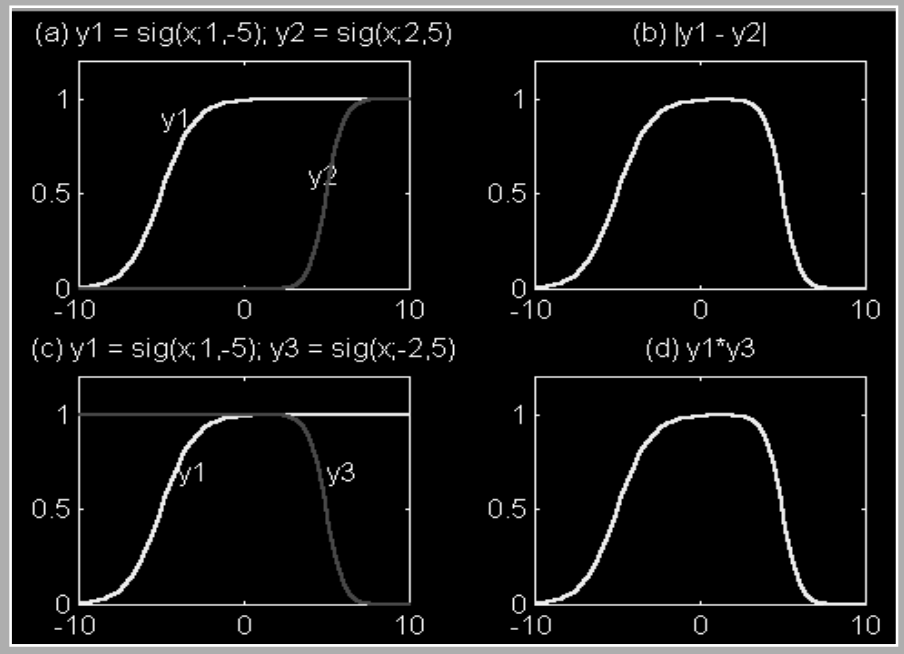
Sigmoidal MF:

$$sigmf\ (x\ ;a\ ,b\ ,c\ ) = \frac{1}{1+e^{\ -a\,(\,x\,-\,c\,)}}$$

Extensions:

Abs. difference of two sig. MF

Product of two sig. MF



disp_sig.m

MF Formulation

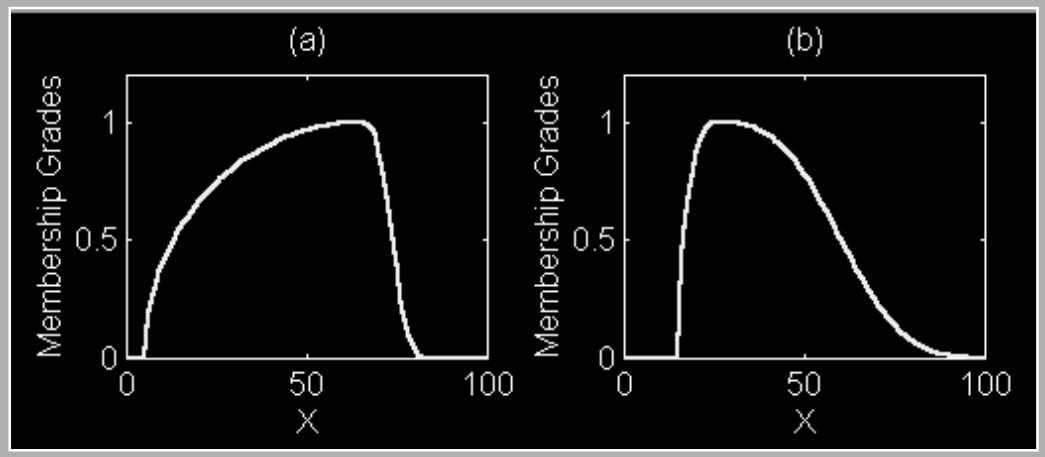
L-R MF:

$$LR(x;c,\alpha,\beta) = \begin{cases} F_L\left(\frac{c-x}{\alpha}\right), x < c \\ F_R\left(\frac{x-c}{\beta}\right), x \geq c \end{cases}$$

Example:

$$F_L(x) = \sqrt{\max(0, 1-x^2)} \qquad F_R(x) = \exp(-|x|^3)$$

c=65
a=60
b=10

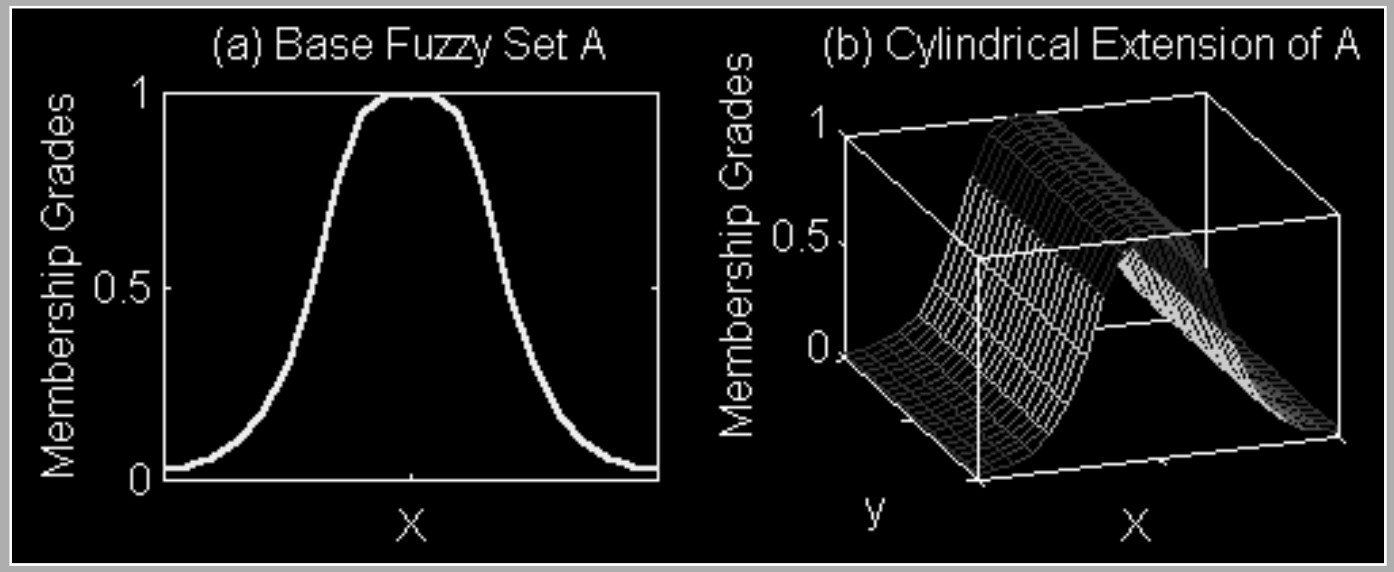


c=25
a=10
b=40

difflr.m

Cylindrical Extension

Base set A **Cylindrical Ext. of A**



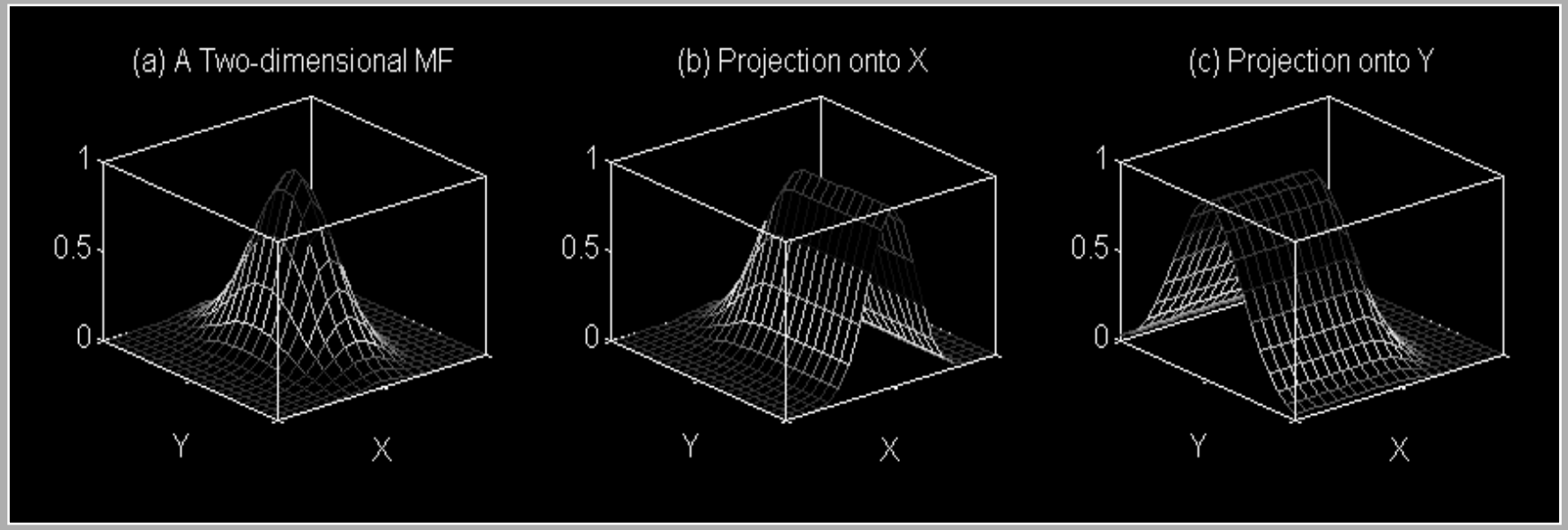
cyl_ext.m

2D MF Projection

Two-dimensional MF

Projection onto X

Projection onto Y



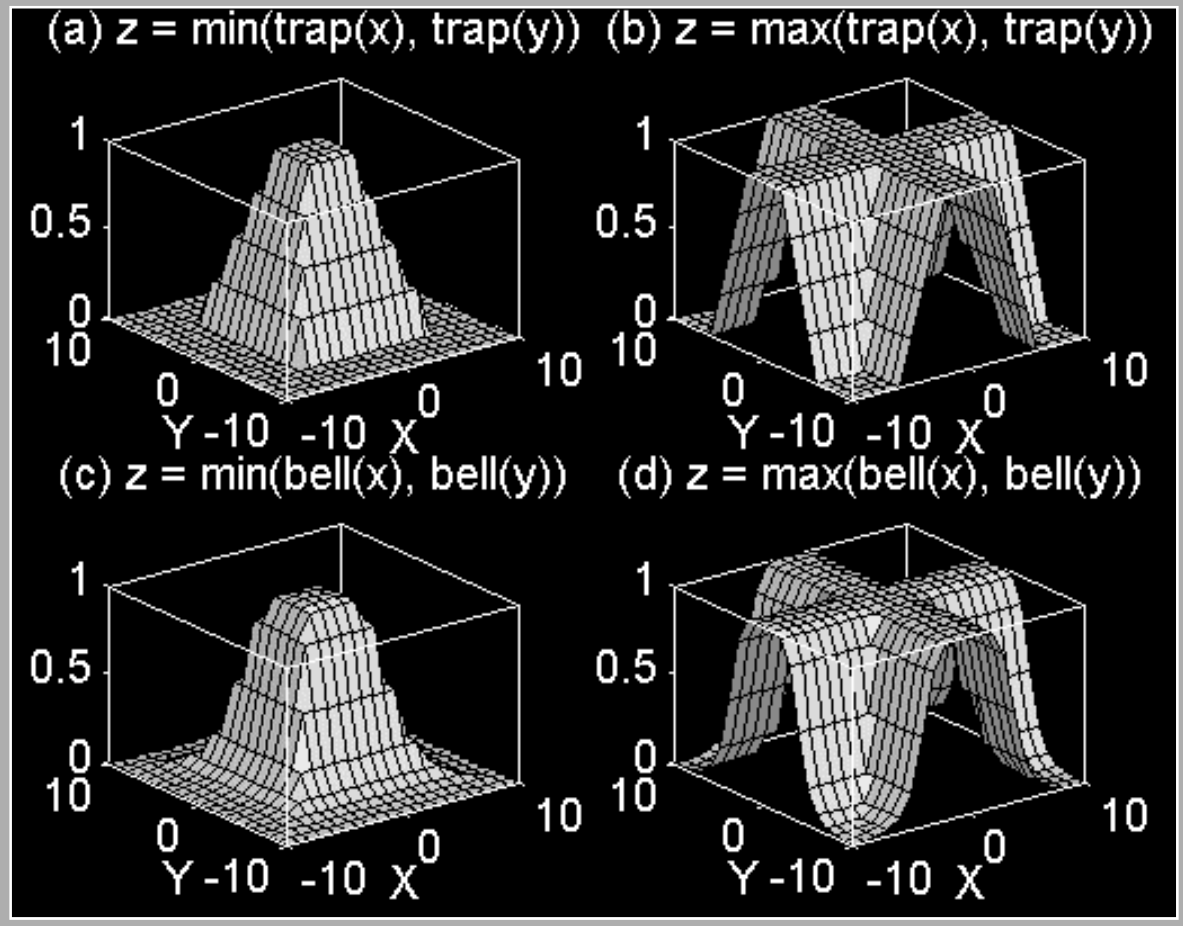
$\mu_R(x, y)$

project.m

$$\mu_A(x) = \max_y \mu_R(x, y)$$

$$\mu_B(y) = \max_x \mu_R(x, y)$$

2D MFs


(a) z = min(trap(x), trap(y))
(b) z = max(trap(x), trap(y))
(c) z = min(bell(x), bell(y))
(d) z = max(bell(x), bell(y))
1
0.5
0
10
0
Y -10
-10
X 0
10

2dmf.m

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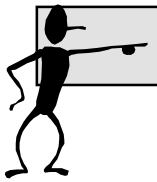
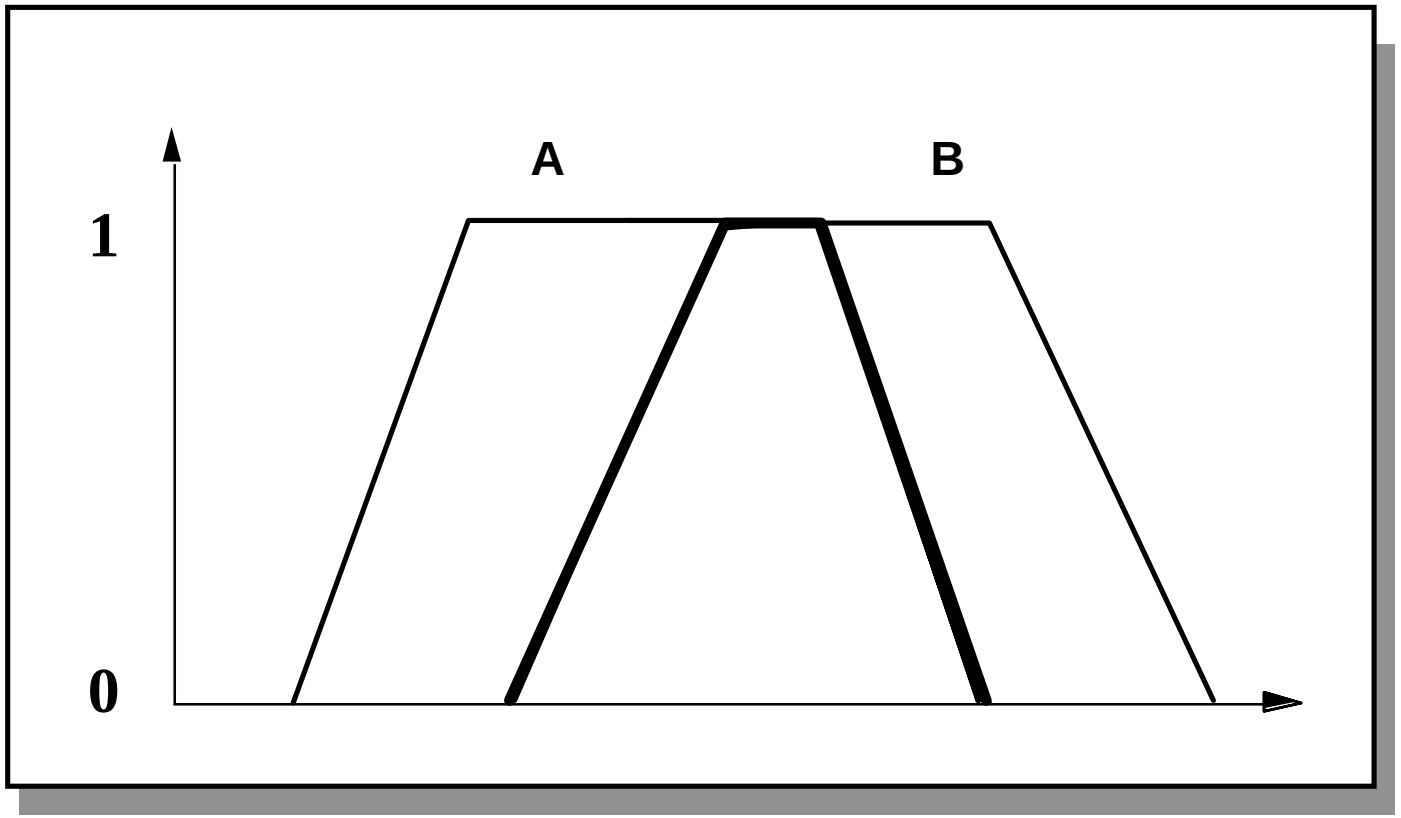
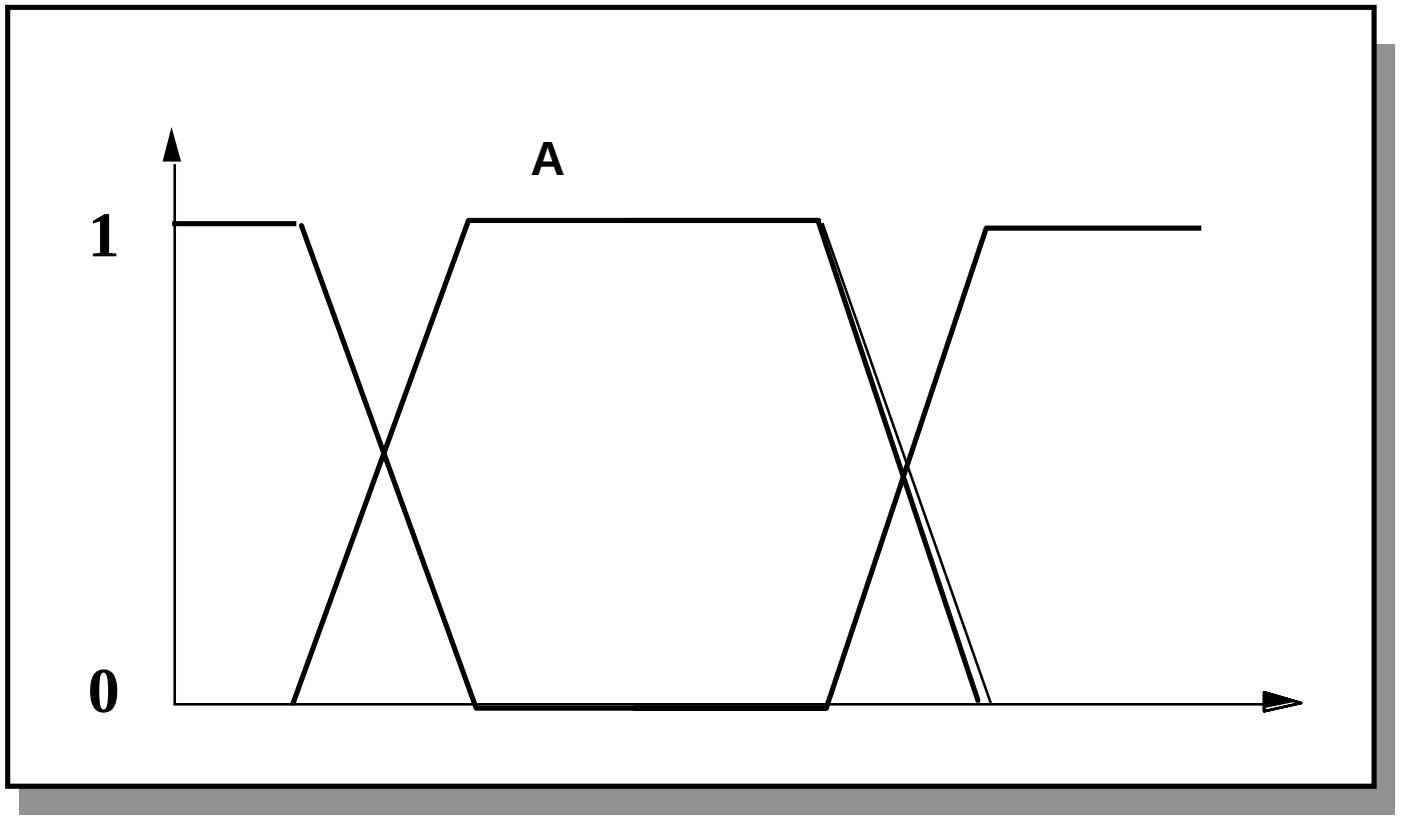


Intersection of Fuzzy Sets



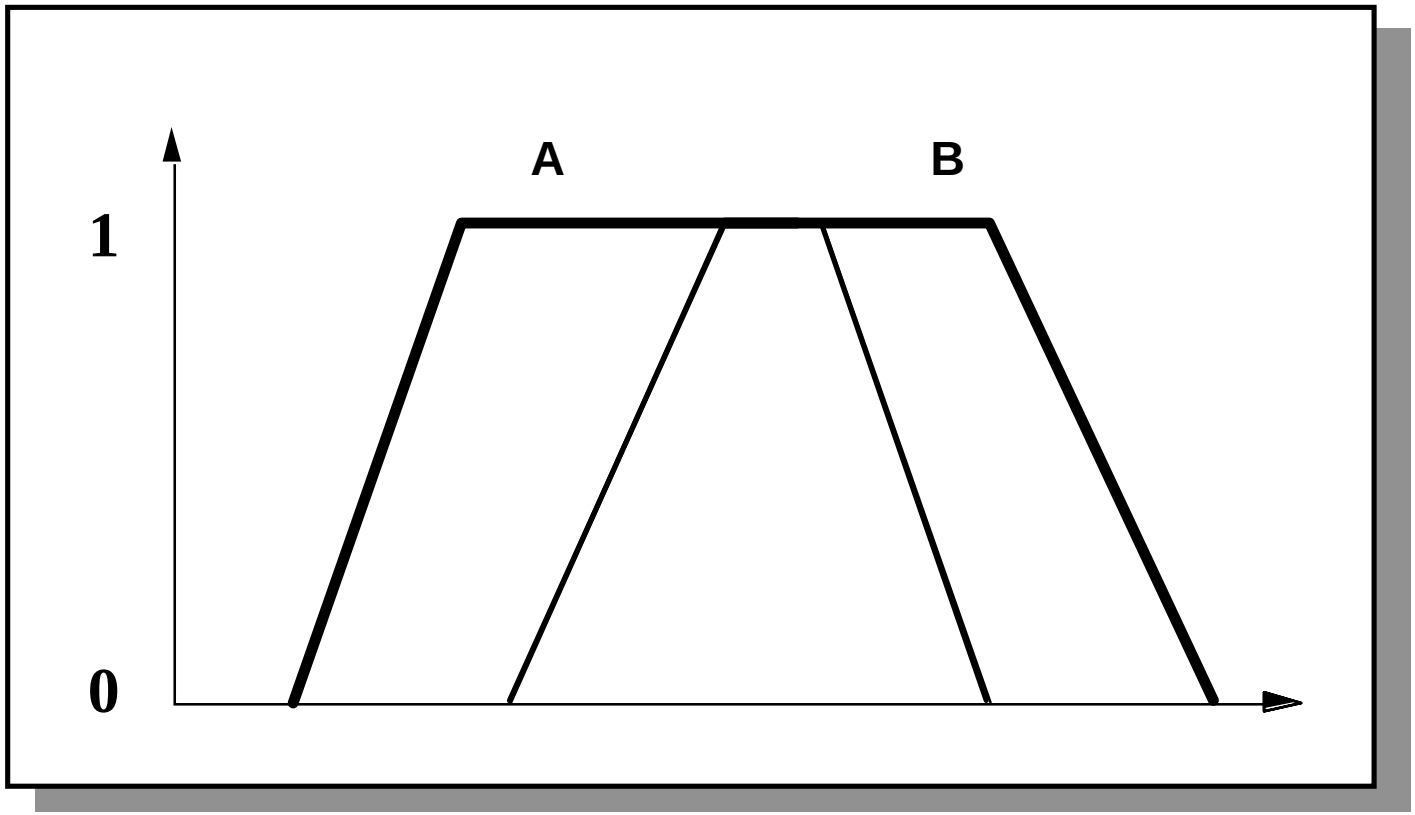
$$(A \cap B)(x) = \min (A(x), B(x))$$

Intersection of Fuzzy Sets



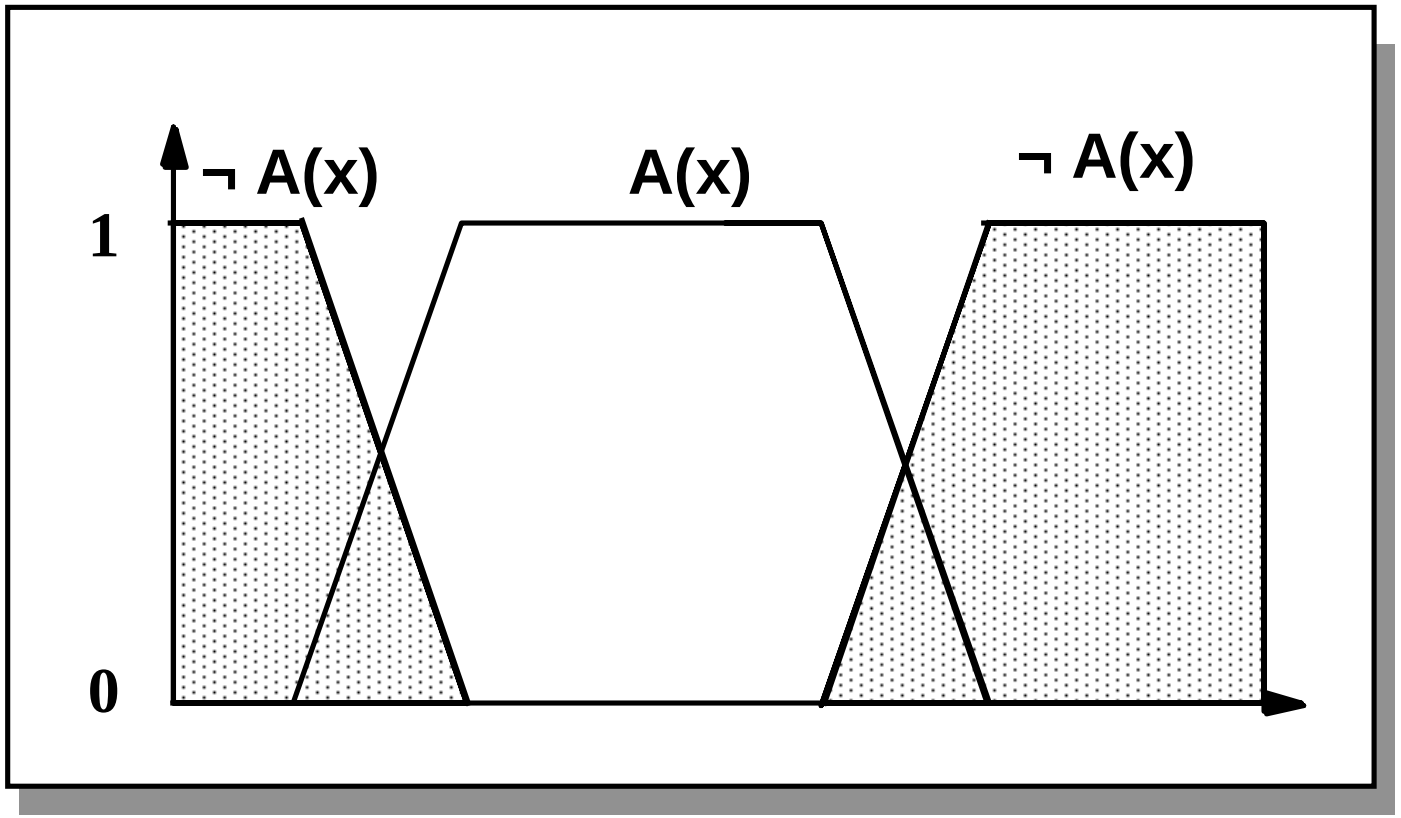
$$(A \cap B)(x) = \min (A(x), B(x))$$

Union of Fuzzy Sets



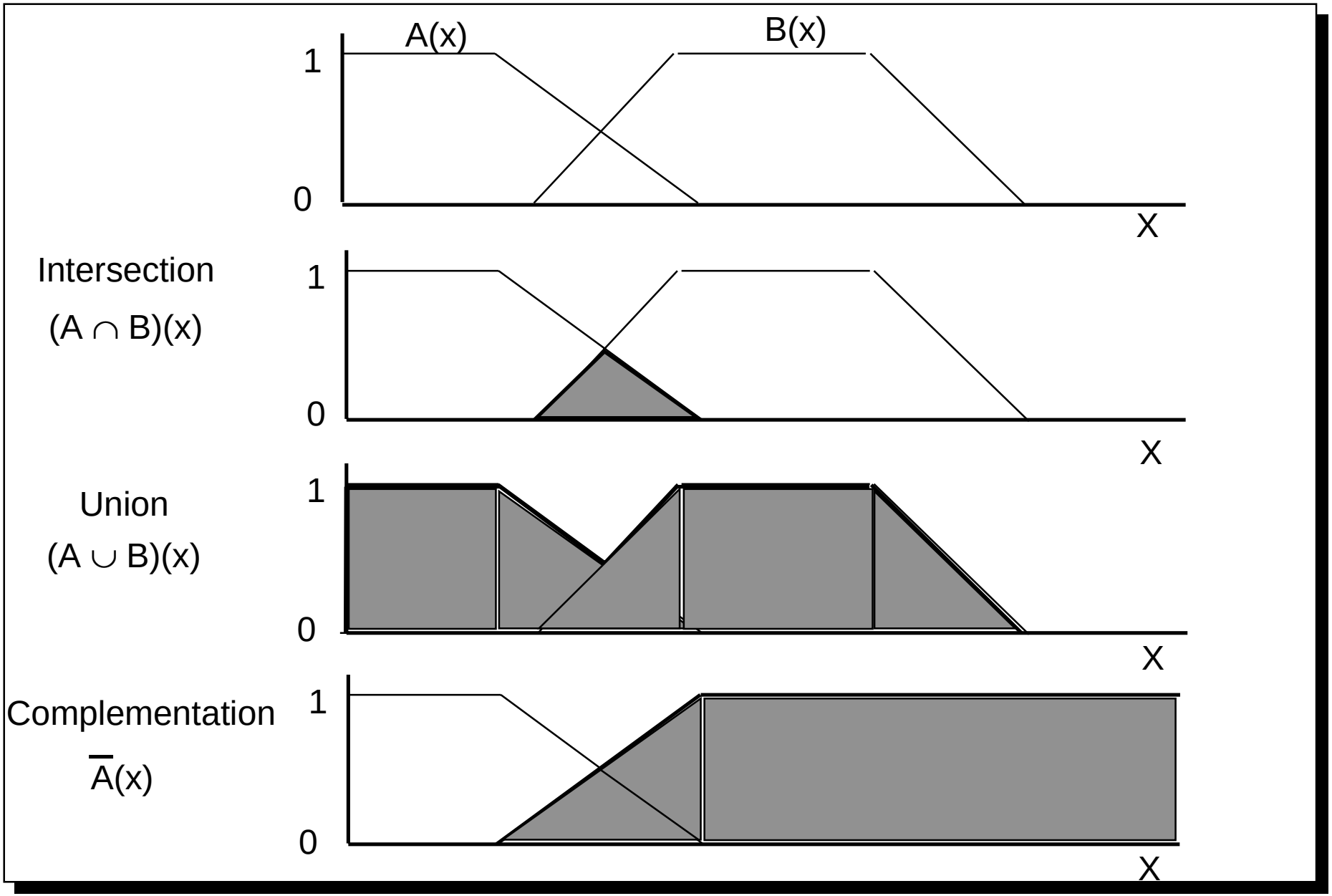
$$(A \cup B)(x) = \max (A(x), B(x))$$

Complement of a Fuzzy Set



$$\overline{A}\,(x) \equiv \neg A(x) = 1 - A(x)$$

Fuzzy Set Operations



Fuzzy Set Operations

- Zadeh's Original Definitions

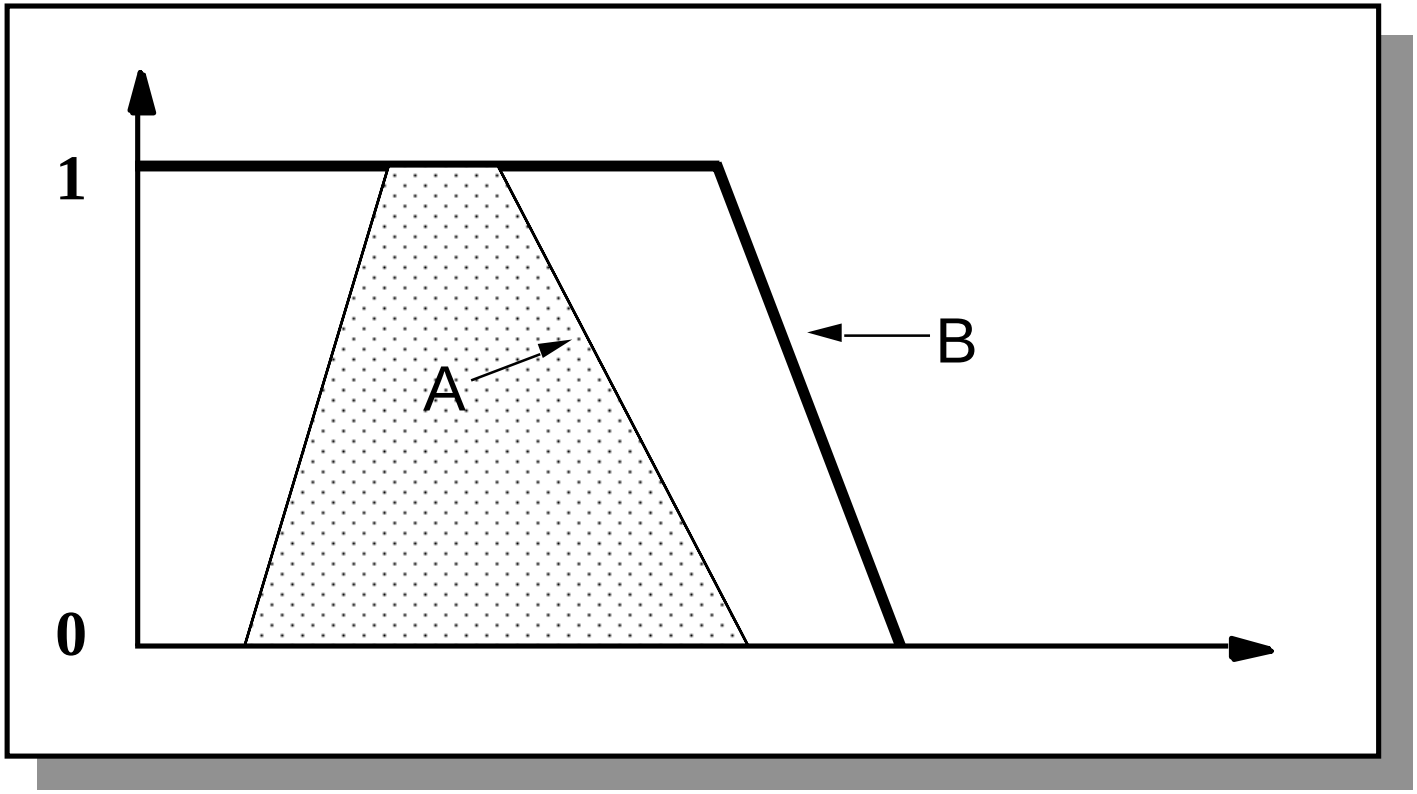
Intersection:	$(A \cap B)(x) = \min [A(x), B(x)]$
Union:	$(A \cup B)(x) = \max [A(x), B(x)]$
Complementation	$\bar{A}(x) = 1 - A(x)$

- Other Definitions

Intersection:	$(A \cap B)(x) = \text{T-norm} [A(x), B(x)]$
Union:	$(A \cup B)(x) = \text{T-Conorm} [A(x), B(x)]$

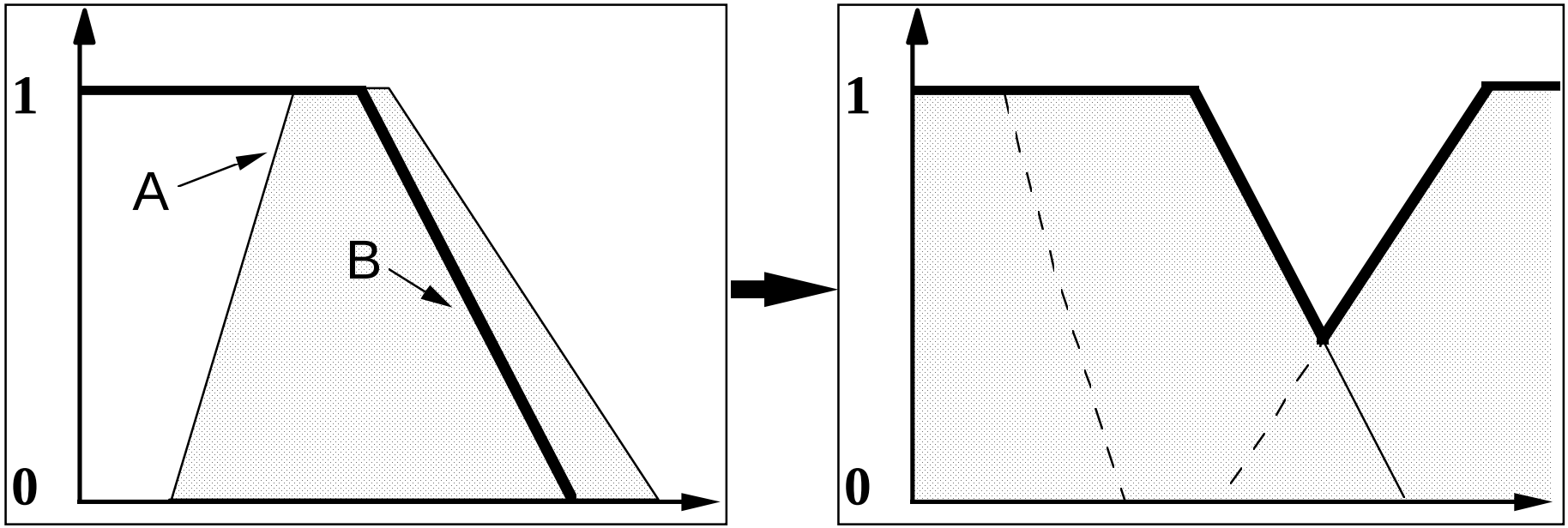
- Isomorphisms between fuzzy sets, algebra, and logics.
- Original definitions form a Brouwerian lattice:
  {Boolean ring properties} but not {excluded middle, law of non-contradiction}
- Beside (*min*, *max*) no other pair (T-norms, T-conorm) satisfies distributivity

Fuzzy Set Inclusion



$$A(x) \leq B(x)$$

Degree of Inclusion



$$(A \rightarrow B)(x) = \max [\, 1-A(x), B(x) \,]$$