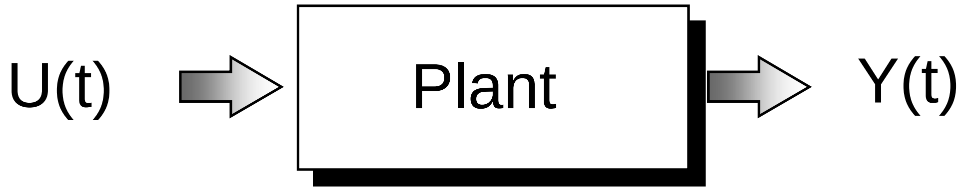
Control Theory

- Open Loop



- Plant Characteristics
  - Linearity
    - Linear or State variable repr.
      - State variable repr. $\dot{X} = AX + BU \qquad Y = CX + DU$
      - Transfer Function (observable & controllable part): Y(s)/U(s)
    - Non Linear
      - State Variable repr. $\dot{X} = f(X, U) \qquad Y = g(X, U)$
      - Linear Approximation (Linerarization at operational points, Describing Function, etc)

Control Theory

– Time

- Time Invariance (fixed coefficients)
- Time Variance (dynamic coefficients)

$$\dot{X} = f(X(t), U(t), t) \qquad Y = g(X(t), U(t), t)$$

– Granularity

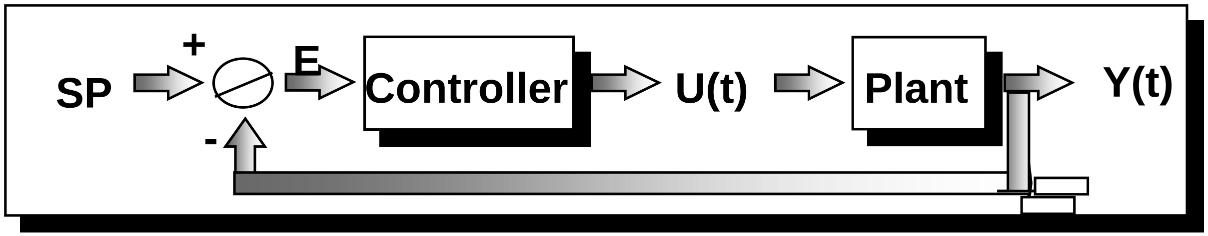
- lumped parameters (linear differential equations)
- distributed parameters (partial differential equations)

– Number of Inputs and Outputs

- SISO, SIMO, MISO, MIMO

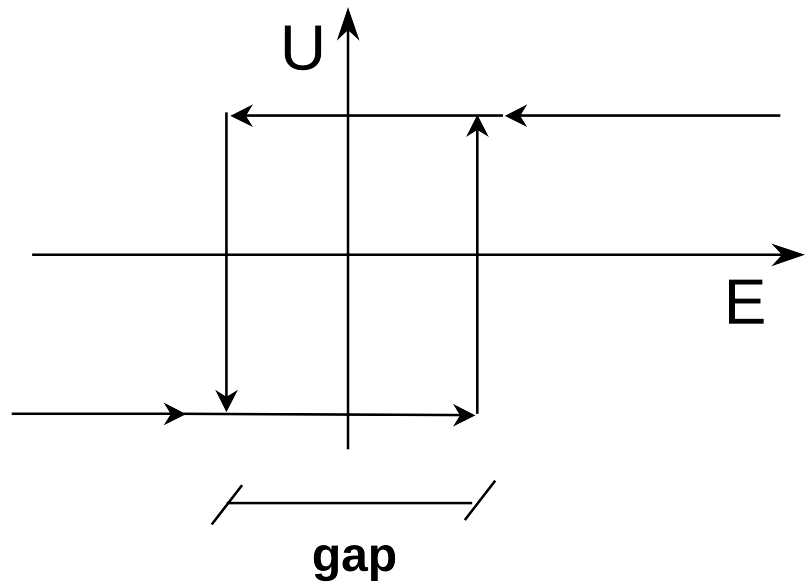
Control Theory

- Closed Loop (Output feedback)



- Bang-Bang Control

E = SP - Y
U = Sign(E) or
U = Sign(E+/- 1/2gap)

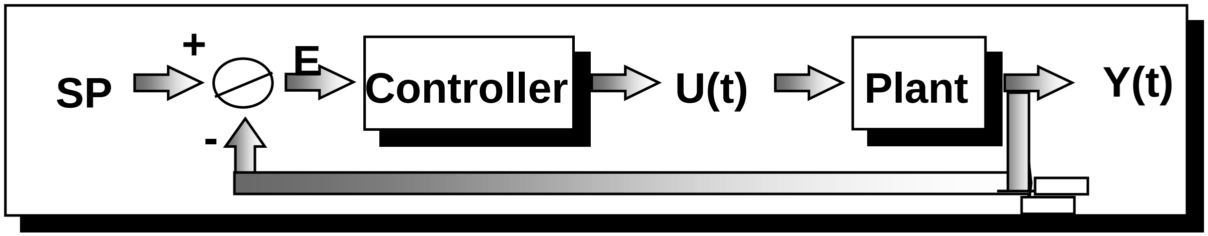


Issues:

- Limit Cycles and Overshooting
- Gap reduces cycling frequency

Control Theory

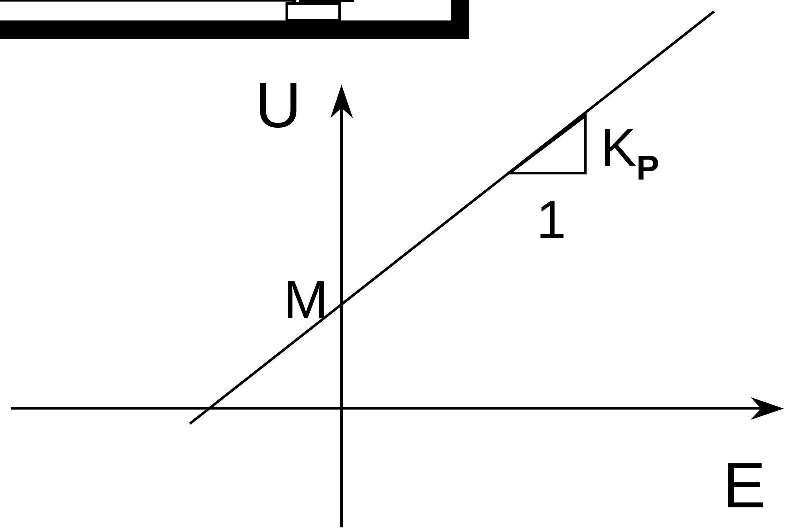
- Closed Loop (Output feedback)



- Proportional Control

$$E = SP - Y$$

$$U = K_P E + M$$



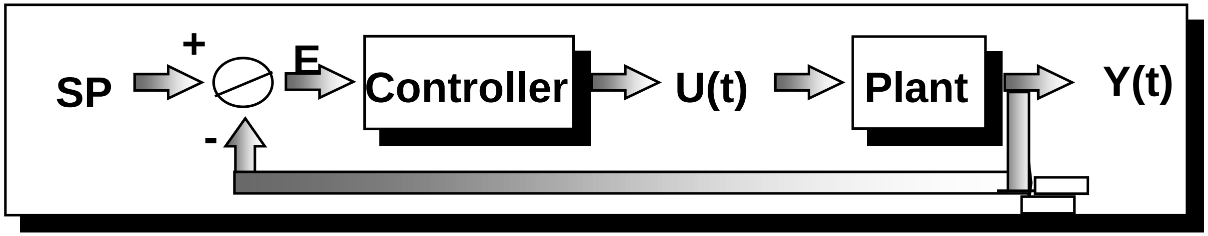
K_P = Proportional Gain (to control rise time)

M = Manual reset

Issues: Offset Error (SSE)

Control Theory

- Closed Loop (Output feedback)



- **Proportional Integral (PI) Control**

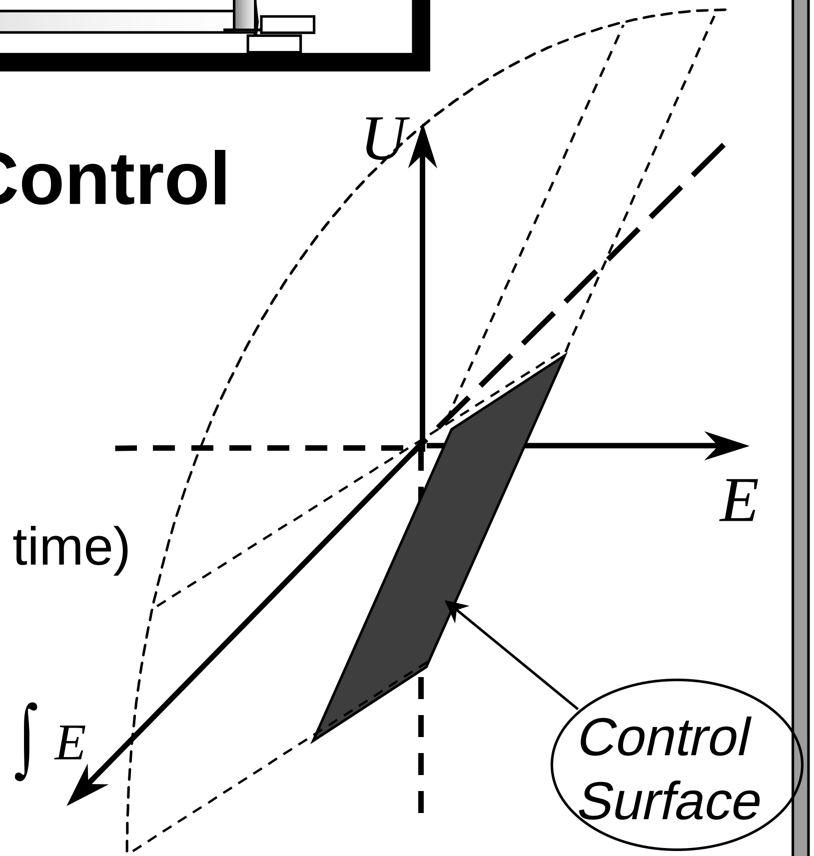
$$E = SP - Y$$
$$U = K_P E + \int K_I E dt$$

K_P = Proportional Gain (to control rise time)

K_I = Integral Gain (to control SSE)

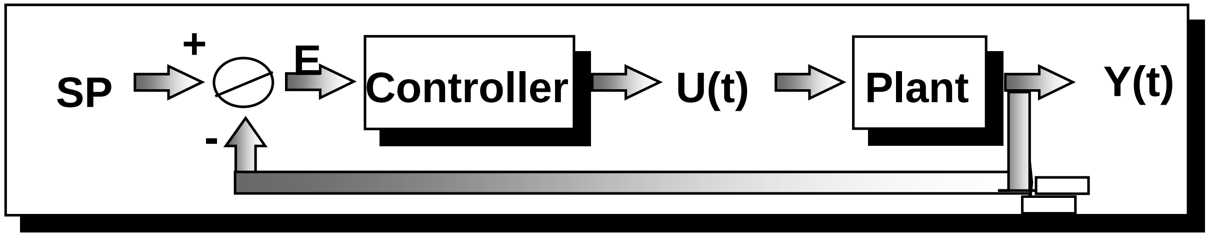
M = Manual reset

Issues: Slow in correcting SSE



Control Theory

- Closed Loop (Output feedback)



- **Proportional Integral Derivative (PID) Control**

$$E = SP - Y$$

$$U = K_P E + \int K_I E dt + K_D \frac{dE}{dt}$$

K_P = Proportional Gain (to control rise time)

K_I = Integral Gain (to control SSE)

K_D = Derivate Gain (to anticipate error)

Issues: Pure Derivative term not realizable (non-causal)

Control Theory

$$U = K_P E + \int K_I E dt + K_D \frac{dE}{dt}$$

Issues: Pure Derivative term not realizable (non-causal)

Usually solved by placing a pole with large negative value (-A):

$$\frac{U(s)}{E(s)} = K(1 + \frac{1}{T_I s} + T_D s)$$

$$\approx K(1 + \frac{1}{T_I s} + T_D \frac{s}{s + A})$$