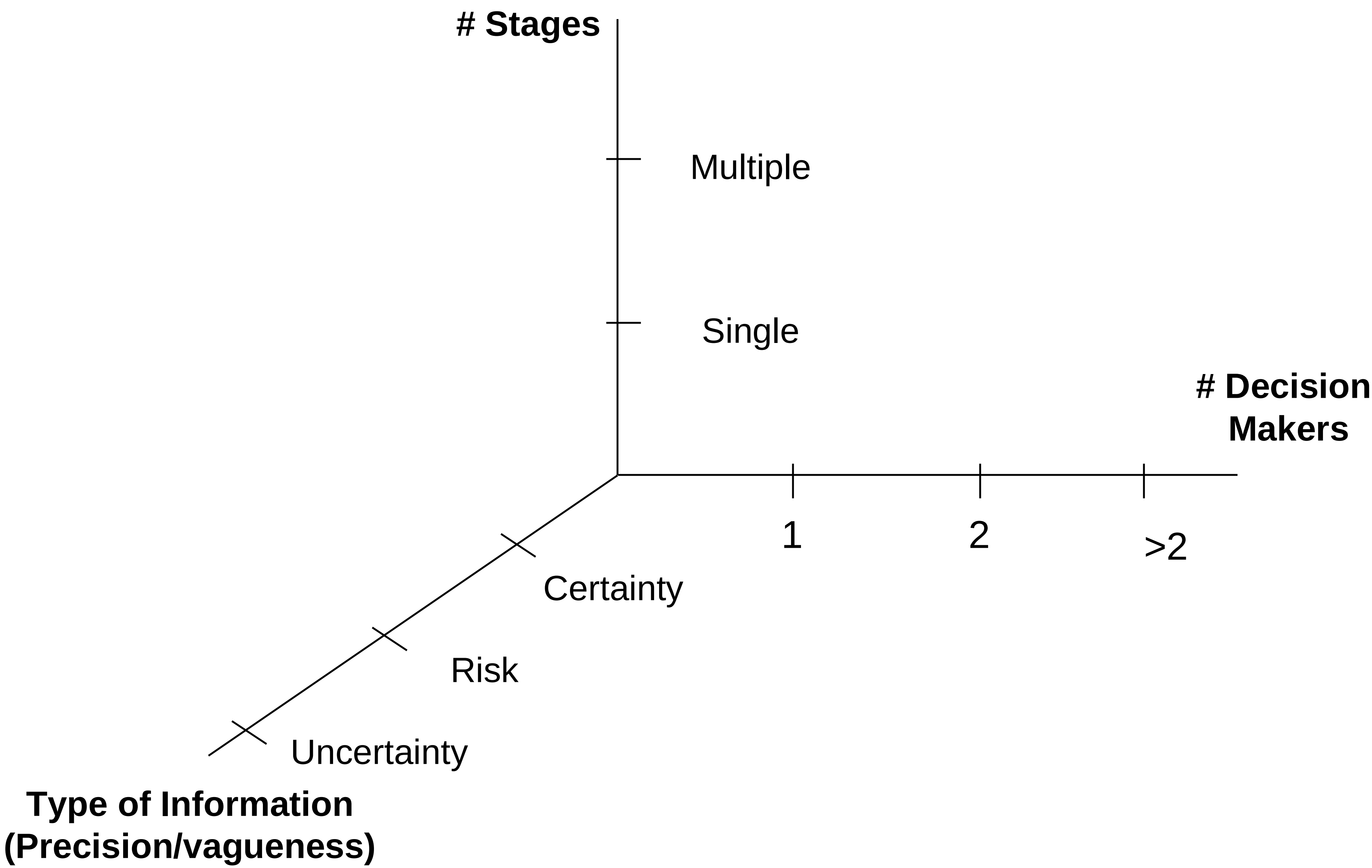
Decision Making



Rational Behavior

- DM Strives to Maximize Utility

$X = \{x_j\}$ States $1 \leq \mathrm{j} \leq n$

$A = \{a_i\}$ Alternatives $1 \leq \mathrm{i} \leq m$

$R \subset A \times X = \{a_i, x_j\}$ Reachability Relation

(x_j is the effect of a_i)

$O \subset X \times X = \{x_j, x_k\}$ Preference Ordering Relation

Cardinal Ordering: $\mathrm{V}(x_j)$

Quality of the Information

- Certainty
  - All information is deterministic; DM selects alternative a_i that optimizes the utility:

$$V(x_j) \text{ such that } (a_i, x_j) \in R$$

- Risk
  - Information is probabilistic
  - R is not deterministic: $R_{i,j} = P_{i,j} = prob(x_j | a_i)$
  - DM selects alternative a_i that optimizes the *expected utility*

$$E[a_i] = \sum_{j=1}^{n} P(x_j | a_i) V(x_j)$$

Quality of the Information

- Uncertainty
  - DM strives to maximize *expected utility* but probabilities are unknown
  - DM uses different strategies ranging from optimistic to pessimistic
    - MaxiMax
    - MaxiMin
    - Lagrangian
    - Minimum Regret

DM Under Uncertainty

- **MaxiMax** (MiniMin) - *[Optimistic & Risky]*
  - Maximizes the maximum achievable profit (minimizes the minimum loss) for any event
- **MaxiMin** (MiniMax) - *[Pessimistic & Conservative]*
  - Maximizes the minimum achievable profit (minimizes the maximum loss) for any event
- **Langrangian** - *[Equal Likelihood]*
  - Consider any event as equally likely
- **Minimum Regret**
  - Minimizes maximum regret for each event
  - Regret = (Best value for each event-Value)

Example

- A farmer needs to decide which of the three crops he should plant.
- The profit from each crop depends on the rainfall during the growing season

	Average Profit		
Rainfall	Crop A	Crop B	Crop C
Substantial	7000	2500	4000
Moderate	3500	3500	4000
Light	1000	4000	3000

Example

- **MaxiMax**

Selected: Crop A

  – Maximizes the maximum achievable profit for any event

  – A: 7000 B: 4000 C: 4000

	Average Profit		
Rainfall	Crop A	Crop B	Crop C
Substantial	**7000**	2500	**4000**
Moderate	3500	3500	**4000**
Light	1000	**4000**	3000

EVENTS

Example (cont)

- **MaxiMin** ***Selected: Crop C***
  - Maximizes the minimum achievable profit for any event
  - A: 1000 B: 2500 C: 3000

	Average Profit		
Rainfall	Crop A	Crop B	Crop C
Substantial	7000	**2500**	4000
Moderate	3500	3500	4000
Light	**1000**	4000	**3000**

EVENTS

Example

- **Lagrangian** ***Selected: Crop A***
  - Consider each event as equally likely: (Prob = 1/3)
  - A: 11,500/3 B: 10,000/3 C: 11,000/3

	Average Profit		
Rainfall	Crop A	Crop B	Crop C
Substantial	7000	2500	4000
Moderate	3500	3500	4000
Light	1000	4000	3000

EVENTS

Example (cont.)

- **Minimum Regret**

Selected: Crop A or C

  – Minimizes maximum regret for *each* event
  – Regret = (Best value for each event - Value)

EVENTS

	Average Profit		
Rainfall	Crop A	Crop B	Crop C
Substantial	**7000**	2500	4000
Moderate	3500	3500	**4000**
Light	1000	**4000**	3000

EVENTS

	Regret		
Rainfall	Crop A	Crop B	Crop C
Substantial	0	4500	**3000**
Moderate	500	500	0
Light	**3000**	0	1000

Fuzzy DM

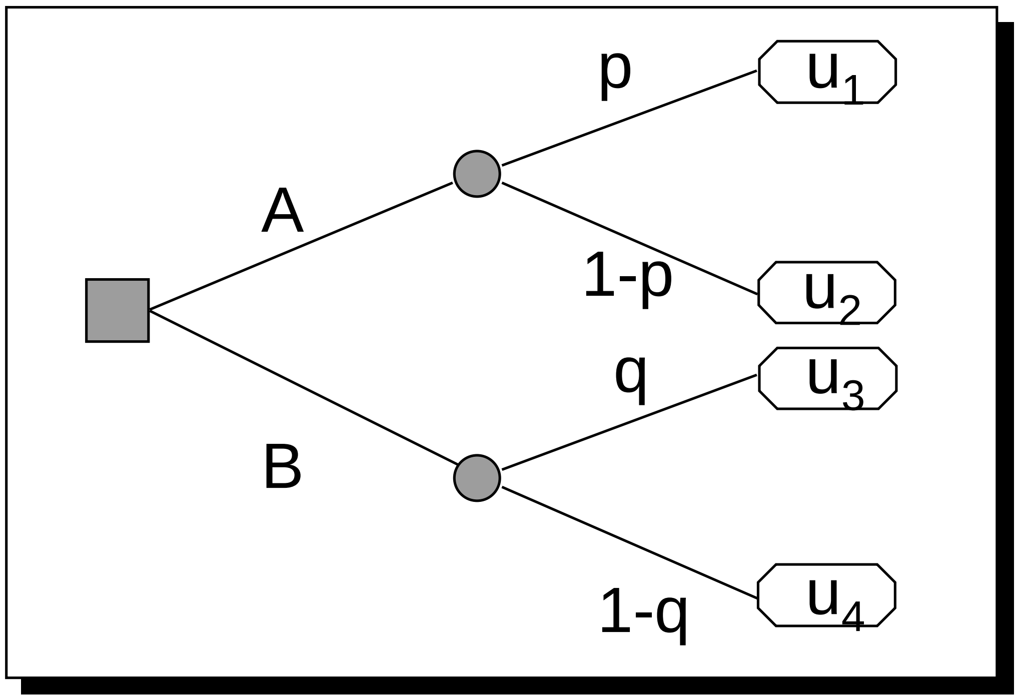
- Focus on Single Stage DM
- Binary Choice Example
  – Task Force Shooting Scenario
- Multiple Choice - Multi-Criteria Example
  – Investment Problem

Fuzzy DM - Single Stage - Binary Choice

- Decision tree with one decision box with two branches (A and B) - (see next slide)
- Crisp Case:
  - Compute *Expected Utility Value* (***EUV***) for each of the two decisions
  - Pick the one with largest value
- Fuzzy Case:
  - Compute *Fuzzy Expected Utility Value* (***FEUV***) for each of the two decisions, using Fuzzy Arithmetics (results are fuzzy values)
  - Determine degree of dominance

DM - Single Stage - Binary Choice

- Decision tree with one decision box with two branches (A and B)



$$EUV[A] = p\,V(u1) + (1-p)\,V(u2)$$

$$EUV[B] = q\,V(u3) + (1-q)\,V(u4)$$

Naval Task Force Shooting Scenario

- The Commander of small task force is cruising just off an enemy coastline
- An airplane is approaching from the direction of the coastline
- The commander has to decide whether or not to fire on the plane
- See Decision Tree to analyze all eight possible outcomes

Shooting Scenario - Four Probabilities

- p1: Probability that plane will be shot down (killed) if shot at (same for friend or enemy)

  p1 = P(Killed | Shoot at)

- p2: Probability that plane is friendly

  p2 = P(friendly plane)

- p3: Probability that enemy plane would score a hit on the task force after being shot (but not killed)

  p3 = P(Hit | Shoot at)

- p4: Probability that enemy plane would score a hit on the task force if not shot at

  p4 = P(Hit | Shoot at)

Shooting Scenario - Four Probabilities

- EU[Shooting]:

  EU[S] = p2 [(p1 u1) + (1-p1) u2] +

  + (1-p2){(u3 p1) +(1-p1)[(u4p3 + u5(1-p3)]}

- EU[Not-Shooting]:

  EU[N] = p2 u6 + (1-p2)[(u7 p4) +(1-p4)u8]

- Difference Z = S - N
  - Positive Z -> Shoot; Negative Z -> Do Not Shoot

Shooting Scenario - Four Probabilities

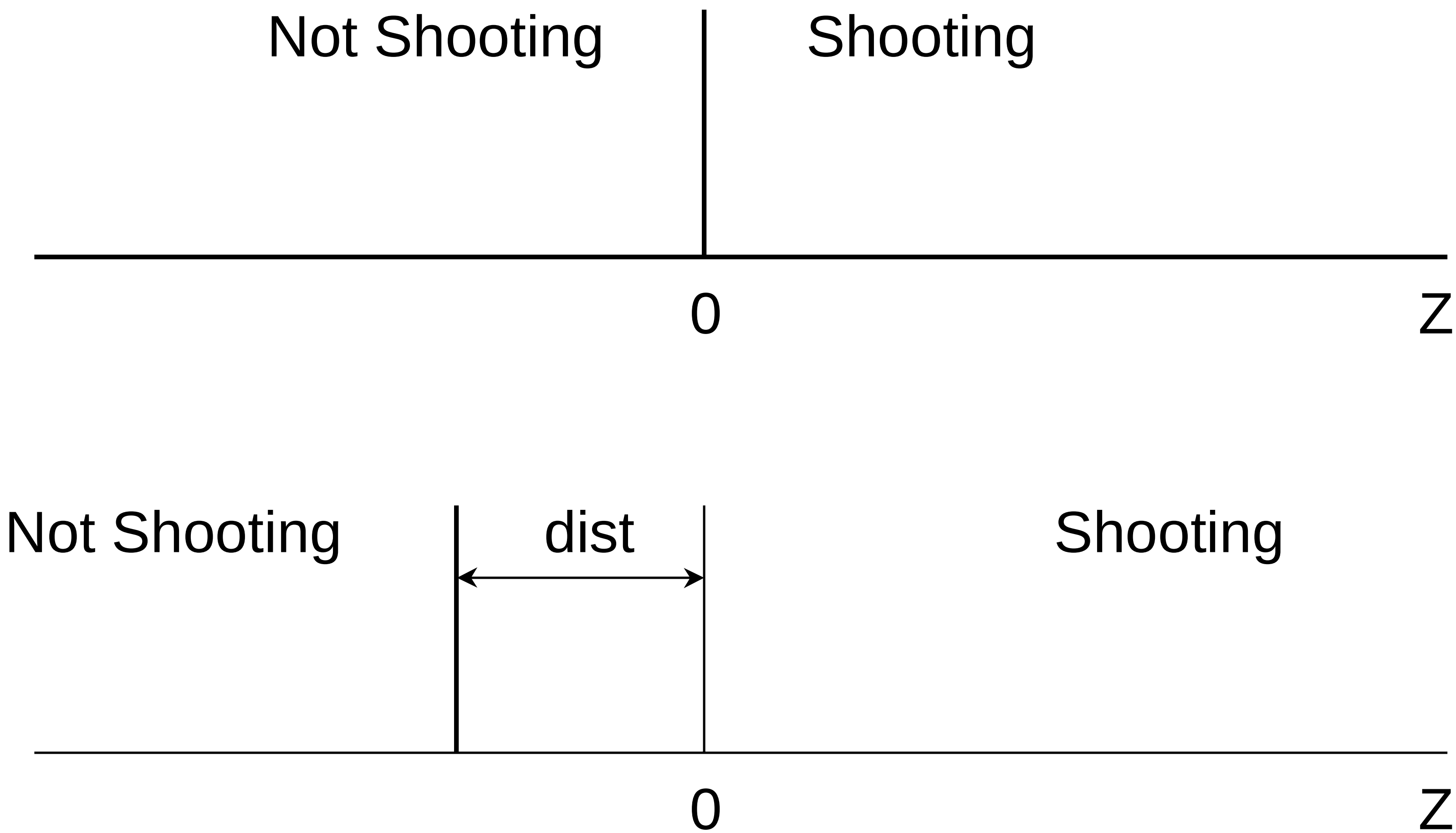
- Crisp Case
  - p1 = 0.70 u1 = -10.0 u5 = - 1.2
  - p2 = 0.80 u2 = - 3.3 u6 = 0.0
  - p3 = 0.20 u3 = 5.8 u7 = -28.8
  - p4 = 0.35 u4 = -26.9 u8 = - 2.5

EU[S] = p2 [(p1 u1) + (1-p1) u2] +
+ (1-p2){(u3 p1) +(1-p1)[(u4p3 + u5(1-p3)]} = **-5.960**

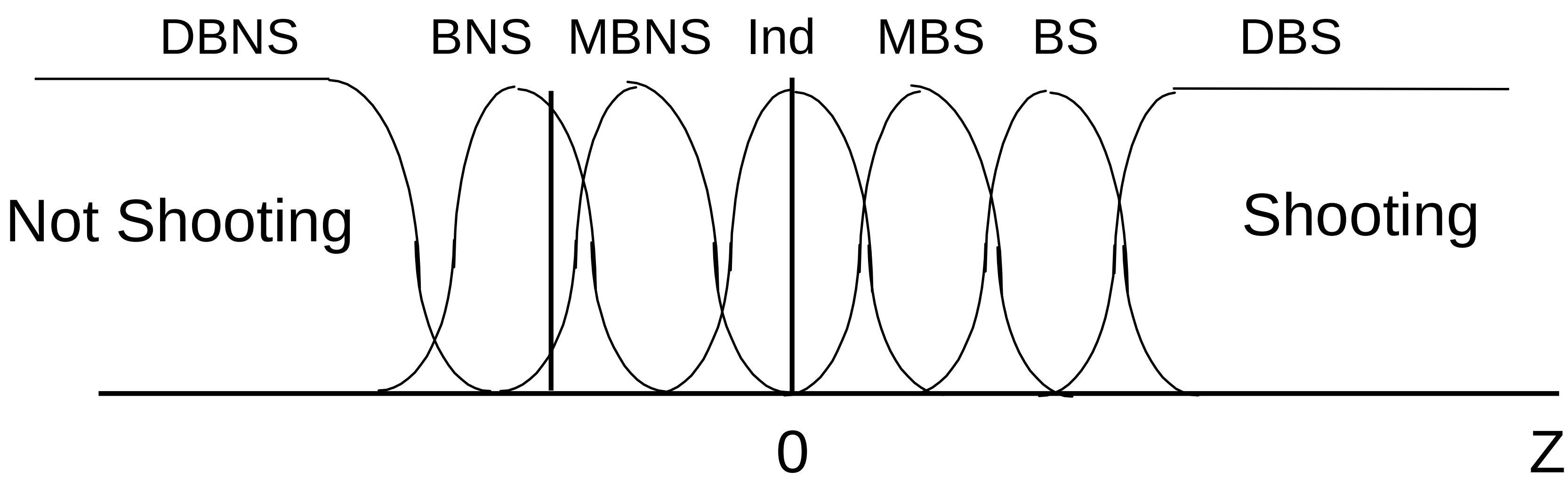
EU[N] = p2 u6 + (1-p2)[(u7 p4) +(1-p4)u8] = **-2.341**

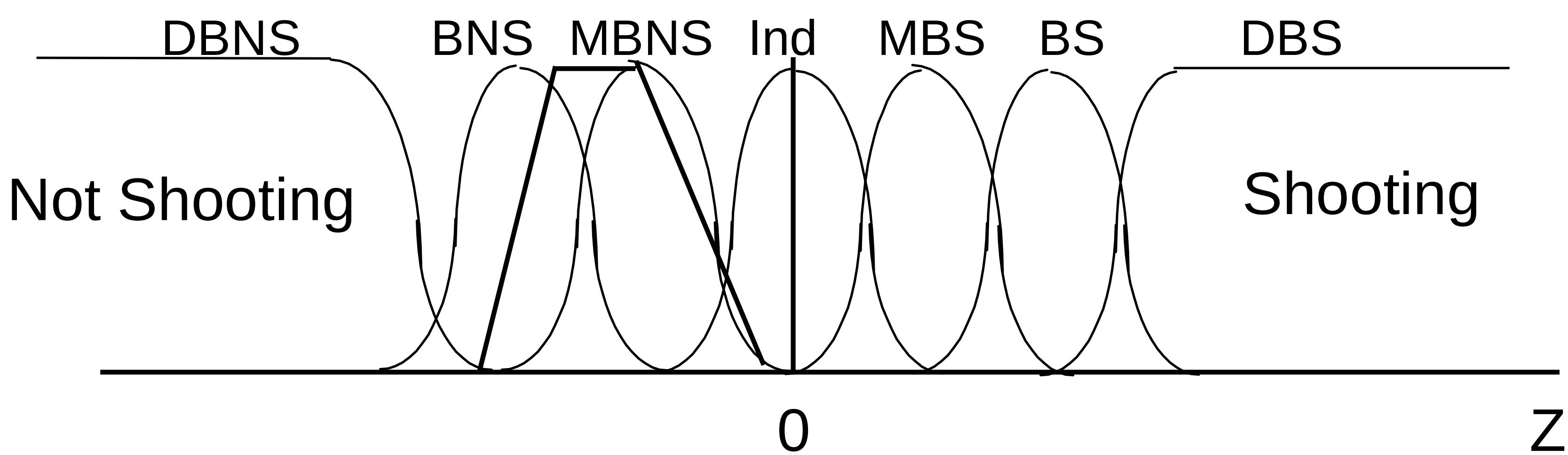
- Difference Z = S - N = **-3.614** -> Do Not Shoot

Shooting Scenario: Degree of Confidence in Decision



Shooting Scenario: Degree of Confidence in Decision





Shooting Scenario - Degree of Confidence

- Crisp Case
  - When utilities and probabilities are crisp, Z is a singleton (crisp value)
  - Confidence level obtained by evaluating the term set of labels with value of Z
- Fuzzy Case **See Hard-copies**
  - When utilities and/or probabilities are fuzzy values [see hardcopies for sample of values], Z is a fuzzy value
  - Confidence level obtained by obtaining the Linguistic Approximation of Z - LA[Z] - using the term set of labels as the target language

DM - Single Stage - Multiple Choices Multi-Criteria

- **Example: Investment Problem**
- Note:
  - This is a TOY problem, in which we invest all the capital in one choice.
  - In a real problem we would be considering a diversified portfolio and we would be asking the percentages of the portfolio components.
  - Ratings of alternatives under criteria reflect 1980 market conditions:
    - double digit inflation, recession, etc.
  - The performance of the alternatives would be VERY different today

DM - Single Stage - Multiple Choices Multi-Criteria - Example

- Investment example is illustrated in handout paper (page 719)
- Five Alternative Investment Choices:
  - a_1 = Commodity market
  - a_2 = stock market
  - a_3 = gold/diamonds
  - a_4 = real estate
  - a_5 = long-term-bonds
- 4 Weights on Criteria to evaluate the investment choices (reflecting the individual investor's situation/attitude):
  - w_1 = risk of losing capital
  - w_2 = erosion of capital by inflation
  - w_3 = interests/dividends returned by investment
  - w_4 = liquidity

DM - Single Stage - Multiple Choices Multi-Criteria

- Ratings:
  - Matrix [$\mathbf{r}_{\mathbf{i,j}}$] of Linguistic Values to rate each alternative A_i under each criterion C_j
  - Ratings and Weights indicated in Tables I & II (page 719)
- Investment Criteria Evaluation:
  - Vector [$\alpha_{\mathbf{j}}$] of Linguistic Values to represent DM's attributes (investment criteria)
  - Ratings and Weights termsets (and semantics) indicated in handout table using 4-tuple representation

DM - Single Stage - Multiple Choices Multi-Criteria

- Steps to solve Investment Problem:
  1) Compute Suitability of each alternative: S_i
  2) Compute Dominance Relation R_δ
  3) Compute Dominance Vector W_δ
  4) Compute Weighted Different Z_k
  5) Obtain Linguistic Approximation of Z_k $LA[Z_k]$

Suitability of each criteria

– Aggregation operator:

- By taking a weighted sum we are allowing compensation among different performances
- We could use some other aggregation operator
- For this example:

$$S_i = \sum_{j=1}^{4} a_j r_{i,j}$$

– where $\mathbf{r_{i,j}}$ is the linguistic value of performance of alternative A_i under each criterion C_j

$\alpha_{\mathbf{j}}$ is the linguistic value representing the DM's attitude (investment criteria)

Degree of Dominance

- Let the degree of dominance of A over B be:

$$\delta(A,B) = Sup_x\left(\mu_{\leq A}(x), \mu_B(x)\right) = \Pi(\leq A, B)$$

where

$$\mu_{\leq A}(x) = \begin{cases} 1 & x < x^* \\ \mu_A(x) & x \geq x^* \end{cases}$$

$$\text{and } \; x^* = \min\{x \in X \mid \mu_A(x) = 1\}$$

Dominance Relation R_δ

$$R_\delta(i, j) = \left[\delta(a_i, a_j)\right]$$

where a_i and a_j are the suitability of alternatives i, and j,

Dominance Vector W_δ

$$W_\delta(i) = Minimum_j \left\{R_\delta(i, j)\right\}$$

which represents the overall degree to which alternative a_i dominates the other ones

DM - Single Stage - Multiple Choices Multi-Criteria - Example

- Compute Suitability Values S_i (Fig 6, page 719)
- Compute Dominance Relation R_δ

$$\begin{bmatrix} 1.0 & 1.0 & .81 & .49 & 1.0 \\ 1.0 & 1.0 & .90 & .58 & 1.0 \\ 1.0 & 1.0 & 1.0 & .85 & 1.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ .77 & .79 & .63 & .34 & 1.0 \end{bmatrix}$$

- Compute Dominance Vector W_δ

$$\begin{bmatrix} .49 & .58 & .85 & 1.0 & .34 \end{bmatrix}$$

Weighted Difference Z_k

For each Z_k such that W(k) = 1

$$Z_k = a_k - \frac{\sum_{i=1, i \neq k}^{n} W(i) a_i}{\sum_{i=1, i \neq k}^{n} W(i)}$$

which is the overall degree to which alternative a_k dominates an aggregate of all other ones

DM - Single Stage - Multiple Choices Multi-Criteria

- Compute Weighted Difference Z_k
  - See Figure 7 page 720

- Obtain Linguistic Approximation of

  a_k : Real Estate (a_4)

  P: From Indifferent to Marginally better than Z_k