Probability Theory and Applications

Homework 4 - Continuous Random Variables

The starred problems (*) are assigned. The rest are practice. Assignment is due in class on Friday October 21. No late assignments.

1. (* b only) The actual amount of coffee (in grams) in a 230-gram jar filled by a certain machine is a random variable whose probability density is given by

$$f(x) = \begin{cases} 0 & x \leq 227.5 \\ \frac{1}{5} & 227.5 < x < 232.5 \\ 0 & x \geq 232.5 \end{cases}$$

   Find the probabilities that such a 230 gram jar filled by this machine will contain

   (a) at most 228.65 grams of coffee;

   (b) * anywhere from 229.34 to 231.66 grams of coffee;

   (c) at least 229.85 grams of coffee.

2. (*b only) The total lifetime (in years) of five-year-old dogs of a certain breed is a random variable whose cumulative distribution function is given by

$$F(x) = \begin{cases} 0 & x \leq 5 \\ 1 - \frac{25}{x^2} & x > 5 \end{cases}$$

   Find the probabilities that such a five-year-old dog will live

   (a) beyond 8 years;

   (b) * less than 8 years;

   (c) anywhere from 12 to 15 years;

3. *A contractor's profit on a construction job can be looked upon as a continuous random variable having the probability density

$$f(x) = \begin{cases} \frac{1}{18}(x+1) & -1 < x < 5 \\ 0 & elsewhere \end{cases}$$

   where the units are in \$1,000, what is her expected profit? What is the variance of her profit?

4. *The p.d.f. of the random variable Z is given by

$$f(z) = \begin{cases} kze^{-z^2} & z > 0 \\ 0 & z <= 0 \end{cases}$$

   Find k and draw the graph of this probability density function.

5. The c.d.f of the mixed random variable Z is given by

$$F(z) = \begin{cases} 0 & z < -2 \\ \frac{z+4}{8} & -2 \leq z \leq 2 \\ 1 & z \geq 2 \end{cases}$$

Find $P(Z = -2),\ P(Z = 2),\ P(-2 < Z < 1),\ P(0 \leq Z \leq 2)$.

6. The tread wear (in thousands of kilometers) which car owners get with certain kind of tire is a random variable whose probability density function is given by

$$f(x) = \begin{cases} \frac{1}{30}e^{-\frac{x}{30}} & for\ x > 0 \\ 0 & for\ x \leq 0 \end{cases}$$

What's the name of this distribution? Find the probability that one of these tires will last

(a) at most 19,000 kilometers;

(b) anywhere from 29,000 to 38,000 kilometers;

(c) at least 48,000 kilometers.

7. *In a certain city the daily consumption of water (in millions of liters) is a random variable whose probability density is given by

$$f(x) = \begin{cases} \frac{1}{9}xe^{-\frac{x}{3}} & for\ x > 0 \\ 0 & for\ x \leq 0 \end{cases}$$

What's the name of this distribution? What are the probabilities that on a given day

(a) the water consumption in this city is no more than 6 million liters;

(b) the water supply is inadequate if the daily capacity of this city is 6 million liters.

8. Lonely Maytag Repair person problem: A Maytag washer requires repairs on the average once every 7 years. Assume that the times between repairs are exponentially distributed.

(a) What is the probability that the washer will last at least three years in between repairs?

(b) Given that washer has not needed repair in the last 5 years, what is the probability that washer will not need to be repaired in the next 3 years?

9. *If Z is a random variable having the standard normal distribution, find the probabilities that this random variable will take on a value

(a) greater that 1.14; (b) less than -0.36;
(c) between -0.46 and -0.09; (d) between -0.58 and 1.12.

10. *If X is normally distributed with $\mu = 10$ and $\sigma = 4$. Find

(a) $\mathrm{P}(8 \leq x \leq 14.5)$;

(b) $\mathrm{P}(2x + 15 \leq 16)$;

(c) The number c such that $\mathrm{P}(X \leq c) = .80$;

(d) The number d such that $\mathrm{P}(|X - 10| \leq d) = .80$.