Lecture 9:Expectation of our favorite distributions

σ_1^2

Probability Theory and Applications

Fall 2005

September 30

Geometric Distribution

Discrete waiting time distribution

How many Bernoulli trials until the first success?

x = number of trials until first success

p = probability of success

$$f(x) = (1 - p)^{x-1} p \qquad x = 1, 2, 3, \ldots$$

$$E(X) = \frac{1}{p}$$

$$\mathrm{var}(X) = \frac{(1-p)}{p^2}$$

Negative Binomial

Generalization of Geometric

X=# of Bernoulli trials to get r^th success

$$Given,\ p(success) = p$$

$$\underbrace{f(x) = \binom{x-1}{r-1} p^{r}(1-p)^{x-r}\, p}_{Binomial(n=x-1,\,p)} \qquad x = r, r+1, \ldots$$

$$E(X) = \frac{r}{p} \qquad \operatorname{var}(X) = \frac{r(1-p)}{p^{2}}$$

Binomial Distribution

The RV *X* is the number of success in n Bernoulli trials.

Parameters:

p=prob. of success   n =# of trials

p.d.f.

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x} \qquad x = 0,1,\ldots,n$$

E(X) Binomial

$$E(X) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1} (1-p)^{n-x}$$

Change variables

$y = x-1 \qquad m = n-1$

$$= np \sum_{y=0}^{m} \binom{m}{y} p^y (1-p)^{m-y}$$

$$= np$$

Challenge Binomial Variance

The variance of a binomial is

$$Variance\ np(1-p)$$

Try computing it. Hint compute

$$E(X^2)$$

$$\mathrm{var}(X) = E(X^2) - E(X)^2$$

Problem

A multiple-choice test consists of eight questions and three answers to each question (of which one is correct). If a student answers each question by rolling a balance die and checking the first answer if he gets a 1 or 2, the second answer if he gets a 3 or 4, and the third answer if hegets a 5 or 6. The number of correct answers is a random variable Y. What is the pdf of Y. What is mean and variance of Y?

Answer

Y ~bin(n=8,p=1/3)

Expected average score if you guess

E(Y)=n*p =8/3

Variance Y = n*p*(1-p)= 8/3*2/3=16/9

Standard deviation in Y = 4/3

Discrete Uniform

A player rolls six sided die and receives the # of dollars corresponding to the roll of the die. What amount should the player pay for rolling to make the game fair?

Discrete Uniform

Let X be discrete uniform on 1..n

$$E(X)=\sum_{x=1}^{n}\frac{x}{n}=\frac{n(n+1)}{2n}=\frac{n+1}{2}$$

$$E(X^2)=\sum_{x=1}^{n}\frac{x^2}{n}=\frac{n(n+1)(2n+1)}{6n}=\frac{(n+1)(2n+1)}{6}$$

$$\mathrm{var}(x)=E(X^2)-E(X)^2$$

$$=\frac{(n+1)(2n+1)}{6}-\left(\frac{n+1}{2}\right)^2=\frac{n^2-1}{12}$$

In last problem

X is discrete uniform on 1..6

E(X)=7/2=$3.50

Var(X)=35/12

Standard deviation = $2.97

Fair game if pay $3.50