

**ASTR-4240 — Gravitation & Cosmology**  
**PHYS-4240 — General Relativity**

**Class 27**  
**The Early Universe**

**Exercise (40 pts)**

**a. (10 pts)** — Calculate the critical density of the universe today,

$$\rho_{\text{crit},0} = \frac{3H_0^2}{8\pi G}, \quad (1)$$

assuming  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc s}^{-1}$ . Give your answer in  $\text{g cm}^{-3}$ .

**b. (10 pts)** — Calculate  $n_{\text{b}0}$ , the number density of baryons in the Universe today. Assume that the Universe is pure hydrogen and that  $\Omega_{\text{b}0} = 0.04$ .

**c. (10 pts)** — Find the mean time between scatterings as a function of  $a/a_0$ .

**d. (10 pts)** — What was the mean time between scatterings at the end of the radiation dominated era, when  $a/a_0 = 2.8 \times 10^{-4}$ ? Give your answer in years.

**Solution**

**a.** — First convert the Hubble constant to cgs units to find  $H_0 = 2.3 \times 10^{-18} \text{ s}^{-1}$ . Then you should find that  $\rho_{\text{crit},0} = 9.2 \times 10^{-30} \text{ g cm}^{-3}$ .

**b.** — The number density of baryons in the Universe today is

$$n_{\text{b}0} = \frac{\Omega_{\text{b}0} \rho_{\text{crit},0}}{m_{\text{proton}}} = 2.2 \times 10^{-7} \text{ cm}^{-3}. \quad (2)$$

**c.** — The mean time between scatterings is

$$\tau_s = 2.3 \times 10^{20} \left( \frac{a}{a_0} \right)^3 \text{ s}. \quad (3)$$

**d.** — Plugging  $a/a_0 = 2.8 \times 10^{-4}$  into the last expression gives  $\tau_s = 5.0 \times 10^9 \text{ s} = 150 \text{ yr}$ .