

Gravitation & Cosmology — ASTR-4240
General Relativity — PHYS-4961

Class 4
Spacetime Physics

Exercise (20 pts)

1. (10 pts) — Write out all of the components of the tensor

$$L^{\mu\nu} \equiv x^\mu p^\nu - x^\nu p^\mu, \quad (1)$$

where x^μ and p^μ are the position and 4-momentum of a point particle with rest mass m and ordinary velocity \mathbf{u} . Use ordinary units, i.e., do not set $c = 1$.

2. (10 pts) — Do any of the components of $L^{\mu\nu}$ describe a familiar physical quantity? Which ones and what is the quantity?

Solution

1. The tensor has Cartesian components

$$L^{\mu\nu} = \begin{pmatrix} 0 & -\gamma mc(x - u_x t) & -\gamma mc(y - u_y t) & -\gamma mc(z - u_z t) \\ \gamma mc(x - u_x t) & 0 & \ell_z & -\ell_y \\ \gamma mc(y - u_y t) & -\ell_z & 0 & \ell_x \\ \gamma mc(z - u_z t) & \ell_y & -\ell_x & 0 \end{pmatrix}, \quad (2)$$

where

$$\boldsymbol{\ell} \equiv \mathbf{x} \times \mathbf{p}. \quad (3)$$

2. Yes: $\boldsymbol{\ell}$ is the orbital angular momentum of the particle.