Gravitation & Cosmology — ASTR-4240
General Relativity — PHYS-4961

Class 3
Tidal Forces

Exercise (30 pts)

On July 7, 1992 Comet Shoemaker Levy 9 broke apart as it passed about $96,000\,\mathrm{km}$ from the center of Jupiter. Assume that, prior to breakup, the comet was a spherical mass of radius $R = 0.5\,\mathrm{km}$ moving only under the gravitational influence of Jupiter. The mass of Jupiter is $M_\mathrm{J} = 5.69 \times 10^{28}\,\mathrm{g}$. Neglect the gravitational effects of the Sun and other bodies. Assume that the comet was held together entirely by its own gravity.

1. (10 pts) — Consider a chunk of rock on the side of the comet facing Jupiter. In the frame of the comet, what are the effective forces on the rock? Just describe them in words.

2. (10 pts) — Denote the mass of the rock by m' and the mass of the comet by M. Write down an expression for the critical distance from Jupiter, r_crit, where the rock is pulled away from the comet.

3. (10 pts) — Using your answer from Part 2, estimate the mass and density of the comet.

Solution

1. — In a freely-falling reference frame attached to the comet, the only forces on the rock are the gravitational force by the comet and the tidal force by Jupiter.

2. — The rock is barely held onto the comet when the two forces balance:

$$\frac{GMm'}{R^2} = 2R \cdot \frac{GM_\mathrm{J}m'}{r_\mathrm{crit}^3}.$$

Solving for r_crit gives the criterion for breakup:

$$r_\mathrm{crit} = R\left(\frac{2M_\mathrm{J}}{M}\right)^{1/3}.$$

3. — Solve the last expression for M to find

$$M = 2M_\mathrm{J}(R/r_\mathrm{crit})^3.$$

Plugging in the data gives $\boxed{M = 1.6 \times 10^{13}\,\mathrm{g}.}$ The density is $\boxed{\rho = 0.03\,\mathrm{g\,cm^{-3}}.}$