

**Astrophysics — ASTR-4220**

**Problem Set 4**  
**Due Monday, November 28**

**Problem 1 — 10 pts**

- a. (5 pts)** — What is the cross section for orbiting collisions when singly-ionized carbon scatters off atomic oxygen? Assume that the center of mass energy is  $E_{\text{cm}} = \frac{1}{2}\mu v^2$ , where  $\mu$  is the reduced mass and  $v$  is the mean value of the relative collision speed.
- b. (5 pts)** — Compare the cross section you found in (a) to the geometrical cross section,  $\sigma_g = 10^{-15} \text{ cm}^2$ . The latter is a large cross section, in the sense that few quantum mechanical cross sections are much larger than  $\sigma_g$ . Is the cross section for orbiting collisions large or small? In physical terms, briefly explain this result.

**Problem 2 — 10 pts**

Consider the rotational transitions of the SiO molecule in the ground vibrational state, treated as a rigid rotor. For the 5 transitions of longest wavelength, make a table giving (i) the transition in standard spectroscopic notation (see Table 5.2); and (ii) the wavelength in mm.

**Problem 3 — 15 pts**

Consider a pencil beam of photons with frequencies in  $(\nu, \nu + d\nu)$  and directions in the infinitesimal solid angle  $d\Omega$  centered on the  $z$  direction. For an infinitesimal time interval  $dt$ , the beam is allowed to pass through an infinitesimal patch  $dA$  perpendicular to the  $z$  direction. It follows that, at the end of the time interval, the photons fill a cylinder of length  $cdt$  and cross section  $dA$ .

- a. (5 pts)** — Find the number of photons,  $dN$ , in the cylinder in terms of the intensity  $I_\nu$  and quantities defined above.
- b. (5 pts)** — Now find  $dN$  another way, by (i) figuring out the volume of phase space occupied by the photons; and (ii) writing  $dN$  in terms of the occupation number,  $n_\gamma$ .
- c. (5 pts)** — Combine the results of (a) and (b) to prove eq. (6.11) on p. 55.

**Problem 4 — 10 pts**

Derive expression (9.17) on p. 78.