Astrophysics — ASTR-4220
Class 36
Molecular Clouds

Exercise (15 pts)

1. (3 pts) — Consider a cloud of dust particles with number density $n_{\rm dust}$, mass $m_{\rm dust}$, and absorption cross section $\sigma_{\rm dust}(\nu)$. Write down the linear opacity $\alpha_\nu\big(\mathrm{cm}^{-1}\big)$.

2. (3 pts) — Recall Kirchoff's Law, which says that $S_\nu = B_\nu$ in LTE. Use Kirchoff's Law and the result of (1) to write down the emissivity j_ν (erg cm^{-3} s^{-1} Hz^{-1} sr^{-1}).

3. (3 pts) — Suppose that the cloud has volume V. What is its luminosity $L_\nu\big(\mathrm{erg\ s^{-1}\ Hz^{-1}}\big)$?

4. (3 pts) — If the dust emits isotropically and the cloud is optically thin, what is the flux F_ν $\big(\mathrm{erg\ cm^{-2}\ s^{-1}\ Hz^{-1}}\big)$ measured by an observer at distance D?

5. (3 pts) — Prove Draine's relation, that the total mass of dust in the cloud is

$$M_{\rm dust} = \frac{F_\nu D^2}{\kappa_\nu B_\nu}, \tag{1}$$

where $\kappa_\nu \equiv \sigma_{\rm dust}(\nu)/m_{\rm dust}$ is the dust cross section per unit mass.

Solution

1. —

$$\alpha_\nu = n_{\rm dust}\sigma_{\rm dust}(\nu). \tag{2}$$

2. —

$$j_\nu = \alpha_\nu\, B_\nu = n_{\rm dust}\, \sigma_{\rm dust}(\nu)\, B_\nu. \tag{3}$$

3. —

$$L_\nu = 4\pi j_\nu\, V = 4\pi\, n_{\rm dust}\, \sigma_{\rm dust}(\nu)\, V\, B_\nu. \tag{4}$$

4. —

$$F_\nu = \frac{L_\nu}{4\pi D^2} = \frac{n_{\rm dust}\, \sigma_{\rm dust}(\nu)\, V\, B_\nu}{D^2} \tag{5}$$

5. — Setting $\sigma_{\rm dust}(\nu) = m_{\rm dust}\kappa_\nu$ in the last expression gives

$$F_\nu = \frac{n_{\rm dust} m_{\rm dust} V\, \kappa_\nu\, B_\nu}{D^2} \tag{6}$$

and since $n_{\rm dust} m_{\rm dust} V = M_{\rm dust}$ this is Draine's result.