Astrophysics — ASTR-4220
Class 18
Solar Neutrinos

Background

For the purposes of this exercise, assume that the Sun makes all of its energy via the proton-proton (p-p) cycle. That is, you should neglect the carbon-nitrogen cycle.[1]

According to Bahcall's Standard Model, the proton-proton cycle terminates via Branch I 85% of the time, Branch II 15% of the time, and Branch III 0.02% of the time. The quantity $Q_{\rm eff}$ is the average thermal energy added to the gas for each ^{4}He produced (Fig. 4.4). After correcting for neutrino losses, one finds that $Q_{\rm eff} = 26.2\,$MeV for Branch I,, 25.2 MeV for Branch II, and 19.1 MeV for Branch III.

Exercise (55 pts)

1. (5 pts) — How many p-p reactions are required to produce one ^{4}He nucleus via Branch I?

2. (5 pts) — How many p-p reactions are required to produce one ^{4}He nucleus via Branch II?

3. (10 pts) — Compute the average thermal energy released (MeV) for each proton consumed by the p-p reaction.

4. (10 pts) — For the temperature, density and composition at the center of the Sun ($T = 15 \times 10^6$, $\rho = 10^5\,\mathrm{kg\,m^{-3}}$, and $X_1 = 0.5$), the p-p reaction occurs at a rate $R_{\rm pp} = 5 \times 10^{13}\,\mathrm{m^{-3}\,s^{-1}}$. Use the answers from parts 1–3 to compute the energy generation rate, ϵ, at the center of the Sun. Give the answer in Watts per kilogram.

5. (10 pts) — For comparison, compute ϵ for an average human being, who consumes 2,000 kcal of food energy per day and weighs 60 kg. **Note:** 1 kcal= 4.2 kJ.

6. (5 pts) — The neutrinos created by hydrogen burning can be destroyed by interactions with electrons, protons, and heavier nuclei. The cross sections are all of order $10^{-48}\,\mathrm{m^2}$. What is the mean free path of a neutrino before being destroyed by a proton? Use conditions at the center of the Sun (see part 4). What can you say about the destruction of neutrinos inside the Sun?

[1] In fact this is an excellent approximation, since C-N accounts for only 1.6% of the production according to the Standard Model.

7. (5 pts) — Compute the flux of solar neutrinos at the Earth. Include neutrinos produced by all of the p-p branches.

8. (5 pts) — Ray Davis built a solar neutrino detector consisting of 610 tons of cleaning fluid (C_2Cl_4). It detected the radioactive ^{37}Ar produced by the reaction

$$\nu_e \ + \ ^{37}\text{Cl} \longrightarrow ^{37}\text{Ar} + e^+. \tag{1}$$

However this reaction only occurs for the high-energy neutrinos emitted by ^{8}B (Fig. 4.4). The cross section, averaged over the neutrino energy spectrum, is $\sigma = 1.06 \times 10^{-46}\,\text{m}^2$. How many reactions did Davis expect in his 610 ton detector, if he accumulated argon for one month? Assume that 24% of the chlorine in the tank is ^{37}Cl.

Solution

1. (5 pts) — Two.

2. (5 pts) — One.

3. (10 pts) — Use the data in Fig. 4.4, noting that one p-p reaction produces half a helium nucleus:

$$\bar{Q} = 0.85 \times \frac{1}{2} \times 26.2\,\text{MeV} \ + \ 0.15 \times 25.2\,\text{MeV} \ \approx \ 15\,\text{MeV}. \tag{2}$$

4. (10 pts) — The energy generation rate is

$$\epsilon_{\text{Sun}} = \frac{R_{\text{pp}}\,\bar{Q}}{\rho} = 1.2\,\text{mW/kg}. \tag{3}$$

5. (10 pts) — The human consumes

$$\Delta E = 2,000\,\text{kcal} \times \frac{4,200\,\text{kJ}}{\text{kcal}} = 8.4\,\text{MJ} \tag{4}$$

in a time interval of

$$\Delta t = 1\,\text{da} \times \frac{24\,\text{hr}}{\text{da}} \times \frac{3,600\,\text{s}}{\text{hr}} = 8.64 \times 10^4\,\text{s}. \tag{5}$$

If we assume that all of the energy goes into thermal energy (i.e., the human does no work), the energy generation rate is

$$\epsilon_{\text{human}} = \frac{\Delta E}{M\,\Delta t} = 1.6\,\text{W/kg}, \tag{6}$$

where $M = 60\,\text{kg}$ is the assumed mass.

6. (5 pts) — The proton number density is

$$n_\mathrm{p} = \frac{\rho\, X_1}{m_\mathrm{p}} = 3.0 \times 10^{31}\,\mathrm{m}^{-3} \tag{7}$$

and the mean free path is

$$\ell = \frac{1}{n_\mathrm{p}\sigma} = 3.3 \times 10^{16}\,\mathrm{m} \ \ \sim 3\,\mathrm{light\ years}. \tag{8}$$

The probability of an interaction inside the Sun is negligible.

7. (5 pts) — The number of neutrinos emitted per second is

$$\dot{N}_\mathrm{nu} = \frac{1\,L_\odot}{26\,\mathrm{MeV}} \times 2 = 1.8 \times 10^{38}\,\mathrm{s}^{-1}. \tag{9}$$

The total neutrino flux at Earth is

$$F_{\nu,\mathrm{tot}} = \frac{\dot{N}_\nu}{4\pi d^2} = 6.6 \times 10^{14}\,\mathrm{m}^{-2}\,\mathrm{s}^{-1}. \tag{10}$$

where $d = 1.5 \times 10^{11}$ m is the distance to the Sun.

8. (5 pts) — The mass of the detecting fluid is

$$M = 610\,\mathrm{ton} \times \frac{2000\,\ell\mathrm{b}}{\mathrm{ton}} \times \frac{1\,\mathrm{kg}}{2.2\,\ell\mathrm{b}} = 5.5 \times 10^5\,\mathrm{kg}. \tag{11}$$

The mass of one molecule of C_2Cl_4 is approximately

$$m = (2 \cdot 12 + 4 \cdot 37)\, m_\mathrm{p} = 2.9 \times 10^{-25}\,\mathrm{kg}, \tag{12}$$

where I assumed for this part only that all of the chlorine is ^{37}Cl. Then the number of ^{37}Cl nuclei in the detector is

$$N_\mathrm{Cl} = 4 \times 0.24 \times \frac{M}{m} = 1.8 \times 10^{30}. \tag{13}$$

The flux of ^{8}B neutrinos is

$$F_{\nu,\mathrm{B}} = F_{\nu,\mathrm{tot}} \times 2 \times 10^{-4} \times \frac{1}{2} = 6.6 \times 10^{10}\,\mathrm{m}^{-2}\,\mathrm{s}^{-1}. \tag{14}$$

The rate of captures per second in the tank is then

$$R = N_\mathrm{Cl}\, F_{\nu,\mathrm{B}}\, \sigma = 1.3 \times 10^{-5}\,\mathrm{s}^{-1}. \tag{15}$$

In one month (30 da) this translates to about 30 events.