Astrophysics — ASTR-4220
Class 3
Radiation Fields

Exercise (30 pts)

The table on the next page describes a Zero-Age model of the Sun, where "zero age" means the time when hydrogen-burning reactions begin. Your job is to decide whether LTE is a good approximation at the center of this model.

1. (10 pts) — Find the number density of electrons in m^{-3} at the center of the Sun. The plasma is completely ionized. Assume that it was composed entirely of hydrogen and helium with mass fractions $X = 0.75$ and $Y = 0.25$, respectively. For simplicity, assume that all the hydrogen was ^{1}H and all the helium was ^{4}He. The atomic mass unit is 1.661×10^{-27} kg.

2. (10 pts) — Find the photon mean free path at the center of the Sun in meters. Assume that the opacity is due entirely to scattering by free electrons (Thomson scattering) with cross section $\sigma = 0.665 \times 10^{-28}\,\mathrm{m}^2$.

3. (10 pts) — Evaluate the criterion for LTE at the center of the Sun. Use the table to estimate the temperature gradient, dT/dr, there. Is LTE a good approximation?

Solution

1. — The electron density is

$$n_\mathrm{e} = \frac{\rho X}{m_\mathrm{u}} + 2 \cdot \frac{\rho Y}{4 m_\mathrm{u}} \approx 4.74 \times 10^{31}\,\mathrm{m}^{-3}. \tag{1}$$

2. — The mean free path is

$$\ell = \frac{1}{n_\mathrm{e}\sigma} \approx 3.17 \times 10^{-4}\,\mathrm{m}. \tag{2}$$

3. — Use the first two rows in the table to find

$$\left|\frac{dT}{dr}\right| = \frac{(13.7 - 12.3) \times 10^6\,\mathrm{K}}{0.07 \times 10^9\,\mathrm{m}} \approx 2.00 \times 10^{-2}\,\mathrm{K\ m}^{-1} \tag{3}$$

and

$$\frac{\ell}{T}\left|\frac{dT}{dr}\right| \approx 5 \times 10^{-13}. \tag{4}$$

Local Thermodynamic Equilibrium is a *superb* approximation!

Zero-Age Model of the Solar Interior[1]

$M(r)$ $M_\odot$	r 10^{11} cm	T 10^6 K	ρ g cm^{-3}	$L(r)$ 10^{33} erg s^{-1}	ϵ erg g^{-1} s^{-1}	κ cm^2 g^{-1}
0	0.00	13.7	90	0.00	13.9	1.38
0.05	0.07	12.3	74	0.95	7.2	1.64
0.1	0.09	11.6	65	1.54	4.8	1.82
0.2	0.11	10.4	51	2.20	2.3	2.16
0.3	0.14	9.4	40	2.53	1.1	2.50
0.4	0.16	8.5	30.5	2.68	0.5	2.87
0.5	0.18	7.6	22.4	2.75	0.2	3.3
0.6	0.20	6.8	15.7	2.77	0.04	3.8
0.7	0.23	5.9	10.0	2.78	0.01	4.4
0.8	0.26	5.0	5.5	2.78	0	5.2
0.9	0.32	3.8	2.09	2.78	0	7.0
0.95	0.37	3.0	0.87	2.78	0	8.6
0.99	0.46	1.73	0.142	2.78	0	11.1
0.99968	0.60	0.62	0.0057	2.78	0	conv

Notes

[1] Adapted from D. D. Clayton, Principles of Stellar Evolution and Nucleosynthesis, McGraw-Hill, 1968, p. 483.