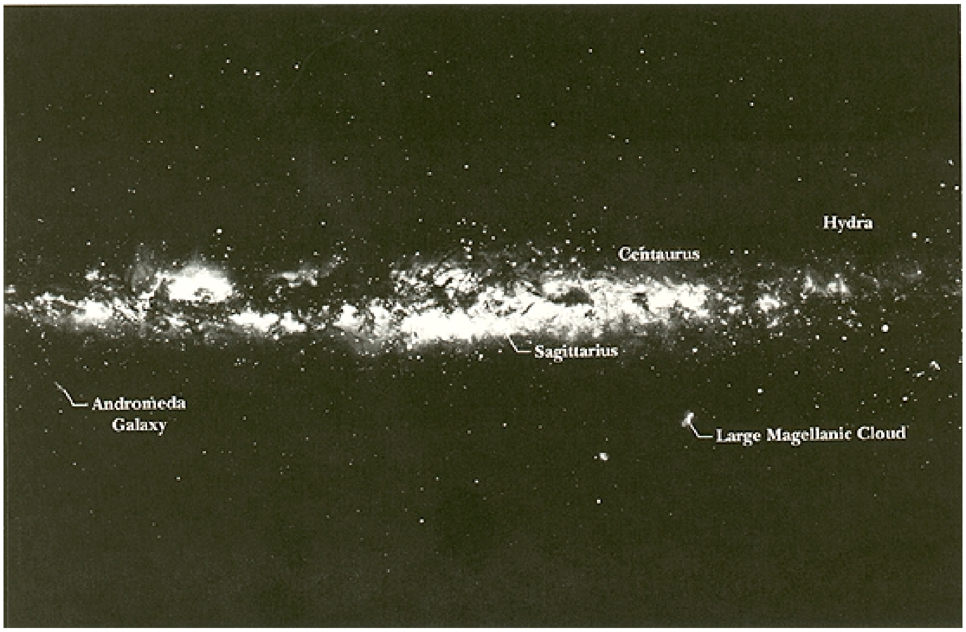
14 Gravitation



The Milky Way galaxy is a disk-shaped collection of dust, planets, and billions of stars, including our Sun and solar system. The force that binds it or any other galaxy together is the same force that holds Earth's moon in orbit and you on Earth—the gravitational force. That force is also responsible for one of nature's strangest objects, the black hole, a star that has completely collapsed onto itself. The gravitational force near a black hole is so strong that not even light can escape it.

If that is the case, how can a black hole be detected?

The answer is in this chapter.

14-1 The World and the Gravitational Force

The drawing that opens this chapter shows our view of the Milky Way galaxy. We are near the edge of the disk of the galaxy, about 26 000 light-years $(2.5\times10^{20}\ \text{m})$ from its center, which in the drawing lies in the star collection known as Sagittarius. Our galaxy is a member of the Local Group of galaxies, which includes the Andromeda galaxy (Fig. 14-1) at a distance of 2.3×10^6 light-years, and several closer dwarf galaxies, such as the Large Magellanic Cloud shown in the opening drawing.

The Local Group is part of the Local Supercluster of galaxies. Measurements taken during and since the 1980s suggest that the Local Supercluster and the supercluster consisting of the clusters Hydra and Centaurus are all moving toward an exceptionally massive region called the Great Attractor. This region appears to be about 300 million light-years away, on the opposite side of the Milky Way from us, past the clusters Hydra and Centaurus.

The force that binds together these progressively larger structures, from star to galaxy to supercluster, and may be drawing them all toward the Great Attractor, is the gravitational force. That force not only holds you on Earth but also reaches out across intergalactic space.

Fig. 14-1: The Andromeda galaxy. Located 2.3×10^6 light-years from us, and faintly visible to the naked eye, it is very similar to our home galaxy, the Milky Way.

14-2 Newton's Law of Gravitation

Physicists like to study seemingly unrelated phenomena to show that a relationship can be found if they are examined closely enough. This search for unification has been going on for centuries. In 1665, the 23-year-old Isaac Newton made a basic contribution to physics when he showed that the force that holds the Moon in its orbit is the same force that makes an apple fall. We take this so much for granted now that it is not easy for us to comprehend the ancient belief that the motions of earthbound bodies and heavenly bodies were different in kind and were governed by different laws.

Newton concluded that not only does Earth attract an apple and the Moon but every body in the universe attracts every other body; this tendency of bodies to move toward each other is called **gravitation.** Newton's conclusion takes a little getting used to, because the familiar attraction of Earth for earthbound bodies is so great that it overwhelms the attraction that earthbound bodies have for each other. For example, Earth attracts an apple with a force magnitude of about $0.8\ \text{N}$. You also attract a nearby apple (and it attracts you), but the force of attraction has less magnitude than the weight of a speck of dust.

Quantitatively, Newton proposed a *force law* that we call **Newton's law of gravitation:** Every particle attracts any other particle with a **gravitational force** whose magnitude is given by

$$\left|\vec{F}\right| = \left|\vec{F}_{2\to1}\right| = \left|\vec{F}_{1\to2}\right| = G\frac{m_1 m_2}{r^2} \qquad \text{(Newton's law of gravitation).} \qquad (14\text{-}1)$$

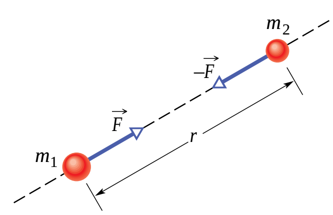


Fig. 14-2: Two particles, of masses m_1 and m_2 and with separation r, attract each other according to Newton's law of gravitation, Eq. 14-1. The forces of attraction, $\vec{F}$ and $-\vec{F}$, are equal in magnitude and in opposite directions.

Here m_1 and m_2 are the masses of the particles, r is the distance between them, and G is the **gravitational constant,** whose value is now known to be

$$\begin{aligned} G &= 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2 \\ &= 6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2\,. \end{aligned} \qquad (14\text{-}2)$$

As Fig. 14-2 shows, a particle m_2 attracts a particle m_1 with a gravitational force $\vec{F}$ that is directed toward particle m_2, and particle m_1 attracts particle m_2 with a gravitational force $-\vec{F}$ that is directed toward m_1. The forces $\vec{F}$ and $-\vec{F}$ form a third-law force pair; they are opposite in direction but equal in magnitude. They depend on the separation of the two particles, but not on their location: the particles could be in a deep cave or in deep space. Also forces $\vec{F}$ and $-\vec{F}$ are not altered by the presence of other bodies, even if those bodies lie between the two particles we are considering.

The strength of the gravitational force—that is, how strongly two particles with given masses at a given separation attract each other—depends on the value of the gravitational constant G. If G—by some miracle—were suddenly multiplied by a factor of 10, you would be crushed to the floor by Earth's attraction. If G were divided by this factor, Earth's attraction would be weak enough that you could jump over a building.

Although Newton's law of gravitation applies strictly to particles, we can also apply it to real objects as long as the sizes of the objects are small compared to the distance between them. The Moon and Earth are far enough apart so that, to a good approximation, we can treat them both as particles—but what about an apple and Earth? From the point of view of the apple, the broad and level Earth, stretching out to the horizon beneath the apple, certainly does not look like a particle.

Newton solved the apple-Earth problem by proving an important theorem called the *shell theorem:*

➤A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated at its center.

Earth can be thought of as a nest of such shells, one within another, and each attracting a particle outside Earth's surface as if the mass of that shell were at the center of the shell. Thus, from the apple's point of view, Earth *does* behave like a particle, one that is located at the center of Earth and has a mass equal to that of Earth.

Suppose, as in Fig. 14-3, that Earth pulls down on an apple with a force of magnitude 0.80 N. The apple must then pull up on Earth with a force of magnitude 0.80 N, which we take to act at the center of Earth. Although the forces are matched in magnitude, they produce different accelerations when the apple is released. For the apple, the acceleration is about 9.8 m/s^2, the familiar acceleration of a falling body near Earth's surface. For Earth, the acceleration measured in a reference frame attached to the center of mass of the apple-Earth system is only about $1 \times 10^{-25} \text{ m/s}^2$.

Fig. 14-3: The apple pulls up on Earth just as hard as Earth pulls down on the apple.

READING EXERCISE 14-1: A particle is to be placed, in turn, outside four objects, each of mass m: (1) a large uniform solid sphere, (2) a large uniform spherical shell, (3) a small uniform solid sphere, and (4) a small uniform shell. In each situation, the distance between the particle and the center of the object is d. Rank the objects according to the magnitude of the gravitational force they exert on the particle, greatest first.

14-3 Gravitation and the Principle of Superposition

Given a group of particles, we find the net (or resultant) gravitational force on any one of them from the others by using the **principle of superposition.** This is a general principle that says a net effect is the sum of the individual effects. Here, the principle means that we first compute the gravitational force that acts on our selected particle due to each of the other particles, in turn. We then find the net force by adding these forces vectorially, as usual.

For n interacting particles, we can write the principle of superposition for gravitational forces as

$$\vec{F}_{1,\text{net}} = \vec{F}_{2\to1} + \vec{F}_{3\to1} + \vec{F}_{4\to1} + \vec{F}_{5\to1} + \cdots + \vec{F}_{n\to1}. \qquad (14\text{-}3)$$

Here $\vec{F}_{1,\text{net}}$ is the net force on particle 1 and, for example, $\vec{F}_{3\to1}$ is the force on particle 1 from particle 3. We can express this equation more compactly as a vector sum:

$$\vec{F}_{1,\text{net}} = \sum_{i=2}^{n} \vec{F}_{i\to1}. \qquad (14\text{-}4)$$

What about the gravitational force on a particle from a real extended object? The force is found by dividing the object into parts small enough to treat as particles and then using Eq. 14-4 to find the vector sum of the forces on the particle from all the parts. In the limiting case, we can divide the extended object into differential parts of mass dm, each of which produces only a differential force $d\vec{F}$ on the particle. In this limit, the sum of Eq. 14-4 becomes an integral and we have

$$\vec{F}_1 = \int d\vec{F}, \tag{14-5}$$

in which the integral is taken over the entire extended object and we drop the subscript "net." If the object is a uniform sphere or a spherical shell, we can avoid the integration of Eq. 14-5 by assuming that the object's mass is concentrated at the object's center and using Eq. 14-1.

Touchstone Example 14-3-1, at the end of this chapter, illustrates how to use what you learned in this section.

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14-4 Gravitation Near Earth's Surface

Let us assume that Earth is a uniform sphere of mass M. The magnitude of the gravitational force from Earth on a particle of mass m, located outside Earth a distance r from Earth's center, is then given by Eq. 14-1 as

$$|\vec{F}| = G\frac{Mm}{r^2} \tag{14-6}$$

If the particle is released, it will fall toward the center of Earth, as a result of the gravitational force $\vec{F}$, with a **gravitational acceleration** $\vec{a}_g$. Newton's second law tells us that magnitudes $\vec{F}$ and $\vec{a}_g$ are related by

$$|\vec{F}| = m|\vec{a}_g| \tag{14-7}$$

Now, substituting $|\vec{F}|$ from Eq. 14-6 into Eq. 14-7 and solving for $|\vec{a}_g|$, we find

$$|\vec{a}_g| = \frac{GM}{r^2} \tag{14-8}$$

Table 14-1 shows values of $|\vec{a}_g|$ computed for various altitudes above Earth's surface.

Since Section 6-3, we have assumed that Earth is an inertial frame by neglecting its actual rotation. This simplification has allowed us to assume that the local gravitational strength g of a particle is the same as the magnitude of the gravitational acceleration (which we now call $|\vec{a}_g|$). Furthermore, we assumed that g has the constant value of $9.8\ \text{m/s}^2$ over Earth's surface. However, the g we would measure differs from the $|\vec{a}_g|$ we would calculate with Eq. 14-8 for three reasons: (1) Earth is not uniform, (2) it is not a perfect sphere, and (3) it rotates. Moreover, because g differs from $|\vec{a}_g|$, the measured weight mg of the particle differs from the magnitude of the gravitational force on the particle as given by Eq. 14-6 for the same three reasons. Let us now examine those reasons.

1. ***Earth is not uniform.*** The density (mass per unit volume) of Earth varies radially as shown in Fig. 14-4, and the density of the crust (or outer section) of Earth varies from region to region over Earth's surface. Thus, g varies from region to region over the surface.

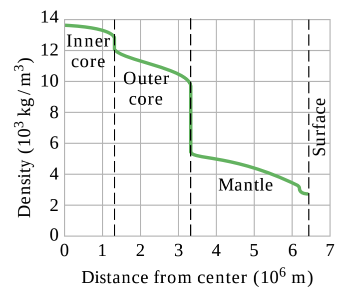


Fig. 14-4: The density of Earth as a function of distance from the center. The limits of the solid inner core, the largely liquid outer core, and the solid mantle are shown, but the crust of Earth is too thin to show clearly on this plot.

TABLE 14-1: Variation of $|\vec{a}_g|$ with Altitude

Altitude (km)	$\lvert\vec{a}_g\rvert$ (m/s^2)	Altitude Example
0	9.83	Mean Earth surface
8.8	9.80	Mt. Everest
36.6	9.71	Highest manned balloon
400	8.70	Space shuttle orbit
35 700	0.225	Communications satellite

2. ***Earth is not a sphere.*** Earth is approximately an ellipsoid, flattened at the poles and bulging at the equator. Its equatorial radius is greater than its polar radius by 21 km. Thus, a point at the poles is closer to the dense core of Earth than is a point on the equator. This is one reason the free-fall acceleration g increases as one proceeds, at sea level, from the equator toward either pole.

3. ***Earth is rotating.*** The rotation axis runs through the north and south poles of Earth. An object located on Earth's surface anywhere except at those poles must rotate in a circle about the rotation axis and thus must have a centripetal acceleration directed toward the center of the circle. This centripetal acceleration requires a centripetal net force that is also directed toward that center.

To see how Earth's rotation causes the local gravitational strength at the Earth's surface, g, to differ from $|\vec{a}_g|$, let us analyze a simple situation in which a crate of mass m is on a scale at the equator. Figure 14-5*a* shows this situation as viewed from a point in space above the north pole.

Figure 14-5*b*, a free-body diagram for the crate, shows the two forces on the crate, both acting along a radial axis r that extends from Earth's center. The normal force $\vec{N}$ on the crate from the scale is directed outward, in the positive direction of axis r. The gravitational force, represented with its equivalent $m\vec{a}_g$, is directed inward. Because the crate travels in a circle about the center of Earth as Earth turns, it has a centripetal acceleration $\vec{a}$ directed inward. From Eq. 11-20, we know the magnitude of this acceleration is equal to $\omega^2 R$, where ω is Earth's angular speed and R is the circle's radius (approximately Earth's radius). Thus, we can write Newton's Second Law in component form for the r axis $(F_{\text{net},r} = ma_r)$ as

$$\left|\vec{N} + m\vec{a}_g\right| = m(-\omega^2 R). \tag{14-9}$$

The magnitude $|\vec{N}|$ of the normal force is equal to the weight mg read on the scale. With mg substituted for $|\vec{N}|$ and with the fact that $\vec{N}$ and $\vec{a}_g$ point in opposite directions, Eq. 14-9 gives us

$$mg - m|\vec{a}_g| = -m(\omega^2 R), \tag{14-10}$$

or

$$mg = m|\vec{a}_g| - m(\omega^2 R),$$

which says

(measured weight) = (magnitude of gravitational force) – (mass times centripetal acceleration).

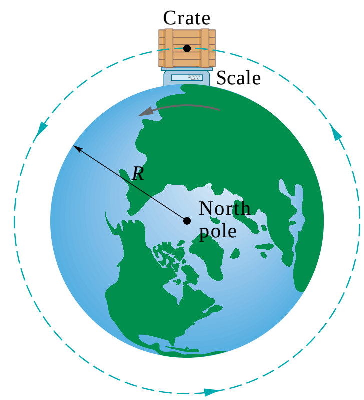
Thus, the measured weight is actually less than the magnitude of the gravitational force on the crate, because of Earth's rotation.

To find a corresponding expression for g and $|\vec{a}_g|$, we cancel m from Eq. 14-10 to write

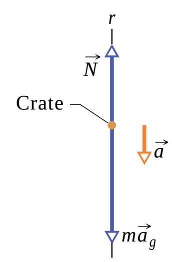
$$g = |\vec{a}_g| - \omega^2 R, \tag{14-11}$$

which says

(free-fall acceleration) = (local gravitational strength) – (centripetal acceleration).



(*a*)



(*b*)

Fig. 14-5: (*a*) A crate lies on a scale at Earth's equator, as seen along Earth's rotation axis from above the north pole. (*b*) A free-body diagram for the crate, with a radially outward r axis. The gravitational force on the crate is represented with its equivalent $m\vec{a}_g$. The normal force on the crate from the scale is $\vec{N}$. Because of Earth's rotation, the crate has a centripetal acceleration $\vec{a}$ that is directed toward Earth's center.

Thus, the measured free-fall acceleration is actually less than the local gravitational strength, because of Earth's rotation.

The difference between the local gravitational strength g and the magnitude of the gravitational acceleration $|\vec{a}_g|$ is equal to $\omega^2 R$ and is greatest on the equator (for one reason, the radius of the circle traveled by the crate is greatest there). To find the difference, we can use Eq. 11-5 $(\omega = \Delta\theta/\Delta t)$ and Earth's radius $R = 6.37 \times 10^6$ m. For one rotation of Earth, θ is 2π rad and the time period Δt is about 24 h. Using these values (and converting hours to seconds), we find that g is less than $|\vec{a}_g|$ by only about 0.034 m/s^2 (compared to 9.8 m/s^2). Therefore, neglecting the difference in g and $|\vec{a}_g|$ is often justified. Similarly, neglecting the difference between weight and the magnitude of the gravitational acceleration is also often justified.

14-5 Gravitation Inside Earth

Newton's shell theorem can also be applied to a situation in which a particle is located *inside* a uniform shell, to show the following:

➤A uniform shell of matter exerts no *net* gravitational force on a particle located inside it.

Caution: This statement does *not* mean that the gravitational forces on the particle from the various elements of the shell magically disappear. Rather, it means that the *sum* of the force vectors on the particle from all the elements is zero.

If the density of Earth were uniform, the gravitational force acting on a particle would be a maximum at Earth's surface and would decrease as the particle moved outward. If the particle were to move inward, perhaps down a deep mine shaft, the gravitational force would change for two reasons. (1) It would tend to increase because the particle would be moving closer to the center of Earth. (2) It would tend to decrease because the thickening shell of material lying outside the particle's radial position would not exert any net force on the particle.

For a uniform Earth, the second influence would prevail and the force on the particle would steadily decrease to zero as the particle approached the center of Earth. However, for the real (nonuniform) Earth, the force on the particle actually increases as the particle begins to descend. The force reaches a maximum at a certain depth; only then does it begin to decrease as the particle descends farther.

Touchstone Example 14-5-1, at the end of this chapter, illustrates how to use what you learned in this section.

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14-6 Gravitational Potential Energy

In Section 10-3, we discussed the gravitational potential energy of a particle-Earth system. We were careful to keep the particle near Earth's surface, so that we could regard the gravitational force as constant. We then chose some reference configuration of the system as having a gravitational potential energy of zero. Often, in this configuration the particle was on Earth's surface. For particles not on Earth's surface, the gravitational potential energy decreased when the separation between the particle and Earth decreased.

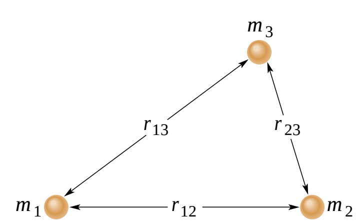

Fig. 14-6: Three particles form a system. (The separation for each pair of particles is labeled with a double subscript to indicate the particles.) The gravitational potential energy *of the system* is the sum of the gravitational potential energies of all three pairs of particles.

Here, we broaden our view and consider the gravitational potential energy U of two particles, of masses m and M, separated by a distance r. We again choose a reference configuration with U equal to zero. However, to simplify the equations, the separation distance r in the reference configuration is now large enough to be approximated as *infinite*. As before, the gravitational potential energy decreases when the separation decreases. Since $U = 0$ for $r = \infty$, the potential energy is negative for any finite separation and becomes progressively more negative as the particles move closer together.

With these facts in mind and as we shall justify next, we take the gravitational potential energy of the two-particle system to be

$$U = -\frac{GMm}{r} \quad \text{(gravitational potential energy).} \qquad (14\text{-}12)$$

Note that $U(r)$ approaches zero as r approaches infinity and that for any finite value of r, the value of $U(r)$ is negative.

The potential energy given by Eq. 14-12 is a property of the system of two particles rather than of either particle alone. There is no way to divide this energy and say that so much belongs to one particle and so much to the other. However, if $M >> m$, as is true for Earth (mass M) and a baseball (mass m), we often speak of "the potential energy of the baseball." We can get away with this because, when a baseball moves in the vicinity of Earth, changes in the potential energy of the baseball-Earth system appear almost entirely as changes in the kinetic energy of the baseball, since changes in the kinetic energy of Earth are too small to be measured. Similarly, in Section 14-8 we shall speak of "the potential energy of an artificial satellite" orbiting Earth, because the satellite's mass is so much smaller than Earth's mass. When we speak of the potential energy of bodies of comparable mass, however, we have to be careful to treat them as a system.

If our system contains more than two particles, we consider each pair of particles in turn, calculate the gravitational potential energy of that pair with Eq. 14-12 as if the other particles were not there, and then algebraically sum the results. Applying Eq. 14-12 to each of the three pairs of Fig. 14-6, for example, gives the potential energy of the system as

$$U = -\left(\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right). \qquad (14\text{-}13)$$

Proof of Equation 14-12

Let us shoot a baseball directly away from Earth along the path shown in Fig. 14-7. We want to find an expression for the gravitational potential energy U of the ball at point P along its path, at radial distance R from Earth's center. To do so, we first find the work W done on the ball by the gravitational force as the ball travels from point P to a great (infinite) distance from Earth. Because the gravitational force $\vec{F}(r)$ is a variable force (its magnitude depends on r), we must use the techniques of Section 9-5 to find the work. In vector notation, we can write

$$W = \int_R^\infty \vec{F}(r) \cdot d\vec{r}. \qquad (14\text{-}14)$$

The integral contains the scalar (or dot) product of the force $\vec{F}(r)$ and the differential displacement vector $d\vec{r}$ along the ball's path. We can expand that product as

$$\vec{F}(r) \cdot d\vec{r} = F(r)\, dr \cos\phi, \qquad (14\text{-}15)$$

where ϕ is the angle between the directions of $\vec{F}(r)$ and $d\vec{r}$. When we substitute 180° for ϕ and Eq. 14-1 for $F(r)$, Eq. 14-15 becomes

$$\vec{F}(r) \cdot d\vec{r} = -\frac{GMm}{r^2} dr$$

where M is Earth's mass and m is the mass of the ball.

Substituting this into Eq. 14-14 and integrating gives us

$$W = -GMm \int_R^\infty \frac{1}{r^2} dr = \left[\frac{GMm}{r}\right]_R^\infty = 0 - \frac{GMm}{R} = -\frac{GMm}{R}. \qquad (14\text{-}16)$$

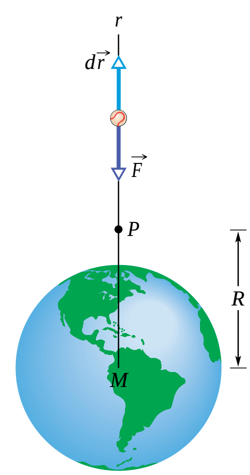

Fig. 14-7: A baseball is shot directly away from Earth, through point P at radial distance R from Earth's center. The gravitational force $\vec{F}$ and a differential displacement vector $d\vec{r}$ are shown, both directed along a radial r axis.

W in Eq. 14-16 is the work required to move the ball from point P (at distance R) to infinity. Equation 10-4 $(\Delta U = -W)$ tells us that we can also write that work in terms of potential energies as

$$U_\infty - U = -W.$$

The potential energy U_∞ at infinity is zero, and U is the potential energy at P. Thus, with Eq. 14-16 substituted for W, the previous equation becomes

$$U = W = -\frac{GMm}{R}$$

Switching R to r gives us Eq. 14-12, which we set out to prove.

Path Independence

In Fig. 14-8, we move a baseball from point A to point G along a path consisting of three radial lengths and three circular arcs (centered on Earth). We are interested in the total work W done by Earth's gravitational force $\vec{F}$ on the ball as it moves from A to G. The work done along each circular arc is zero, because the direction of $\vec{F}$ is perpendicular to the arc at every point. Thus, the only works done by $\vec{F}$ are along the three radial lengths, and the total work W is the sum of those works.

Now, suppose we mentally shrink the arcs to zero. We would then be moving the ball directly from A to G along a single radial length. Does that change W? No. Because no work was done along the arcs, eliminating them does not change the work. The path taken from A to G now is clearly different, but the work done by $\vec{F}$ is the same.

We discussed such a result in a general way in Section 10-2. Here is the point: The gravitational force is a conservative force. Thus, the work done by the gravitational force on a particle moving from an initial point i to a final point f is independent of the actual path taken between the points. From Eq. 10-4, the change ΔU in the gravitational potential energy from point i to point f is given by

$$\Delta U = U_f - U_i = -W. \tag{14-17}$$

Since the work W done by a conservative force is independent of the actual path taken, the change ΔU in gravitational potential energy is *also independent* of the actual path taken.

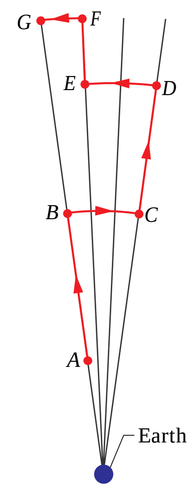

Fig. 14-8: Near Earth, a baseball is moved from point A to point G along a path consisting of radial lengths and circular arcs.

Potential Energy and Force

In the proof of Eq. 14-12, we derived the potential energy function $U(r)$ from the force function $\vec{F}(r)$. We should be able to go the other way—that is, to start from the potential energy function and derive the force function. Guided by Eq. 10-22, we can write the radial force component F_r as

$$\begin{aligned} F_r &= -\frac{dU}{dr} = -\frac{d}{dr}\left(-\frac{GMm}{r}\right) \\ &= -\frac{GMm}{r^2}. \end{aligned} \tag{14-18}$$

This is Newton's law of gravitation (Eq. 14-1). The minus sign indicates that the force on mass m points radially inward, toward mass M.

Escape Speed

If you fire a projectile upward, usually it will slow, stop momentarily, and return to Earth. There is, however, a certain minimum initial speed that will cause it to move upward forever, theoretically coming to rest only at infinity. This initial speed is called the (Earth) **escape speed.**

Consider a projectile of mass m, leaving the surface of a planet (or some other astronomical body or system) with escape speed *v*. It has a kinetic energy *K* given by $\frac{1}{2}mv^2$ and a potential energy *U* given by Eq. 14-12:

$$U = -\frac{GMm}{R}$$

in which *M* is the mass of the planet, and R is its radius.

When the projectile reaches infinity, it stops and thus has no kinetic energy. It also has no potential energy because this is our zero-potential-energy configuration. Its total energy at infinity is therefore zero. From the principle of conservation of energy, its total energy at the planet's surface must also have been zero, so

$$K + U = \tfrac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = 0$$

This yields

$$v = \sqrt{\frac{2GM}{R}}. \qquad (14\text{-}19)$$

The escape speed *v* does not depend on the direction in which a projectile is fired from a planet. However, attaining that speed is easier if the projectile is fired in the direction the launch site is moving as the planet rotates about its axis. For example, rockets are launched eastward at Cape Canaveral to take advantage of the Cape's eastward speed of 1500 km/h due to Earth's rotation.

Equation 14-19 can be applied to find the escape speed of a projectile from any astronomical body, provided we substitute the mass of the body for *M* and the radius of the body for *R*. Table 14-2 shows escape speeds from some astronomical bodies.

TABLE 14-2: Some Escape Speeds

Body	Mass (kg)	Radius (m)	Escape Speed (km/s)
Ceres[a]	1.17×10^{21}	3.8×10^{5}	0.64
Earth's moon	7.36×10^{22}	1.74×10^{6}	2.38
Earth	5.98×10^{24}	6.37×10^{6}	11.2
Jupiter	1.90×10^{27}	7.15×10^{7}	59.5
Sun	1.99×10^{30}	6.96×10^{8}	618
Sirius B[b]	2×10^{30}	1×10^{7}	5200
Neutron star[c]	2×10^{30}	1×10^{4}	2×10^{5}

[a]The most massive of the asteroids.
[b]A *white dwarf* (a star in a final stage of evolution) that is a companion of the bright star Sirius.
[c]The collapsed core of a star that remains after that star has exploded in a *supernova* event.

READING EXERCISE 14-2: You move a ball of mass *m* away from a sphere of mass *M*. (a) Does the gravitational potential energy of the ball-sphere system increase or decrease? (b) Is positive or negative work done by the gravitational force between the ball and the sphere?

Touchstone Example 14-6-1, at the end of this chapter, illustrates how to use what you learned in this section.

TE

14-7 Planets and Satellites: Kepler's Laws

The motions of the planets, as they seemingly wander against the background of the stars, have been a puzzle since the dawn of history. The "loop-the-loop" motion of Mars, shown in Fig. 14-9, was particularly baffling. Johannes Kepler (1571-1630), after a lifetime of study, worked out the empirical laws that govern these motions. Tycho Brahe (1546-1601), the last of the great astronomers to make observations without the help of a telescope, compiled the extensive data from which Kepler was able to derive the three laws of planetary motion that now bear his name. Later, Newton (1642-1727) showed that his law of gravitation leads to Kepler's laws.

In this section we discuss each of Kepler's laws in turn. Although here we apply the laws to planets orbiting the Sun, they hold equally well for satellites, either natural or artificial, orbiting Earth or any other massive central body.

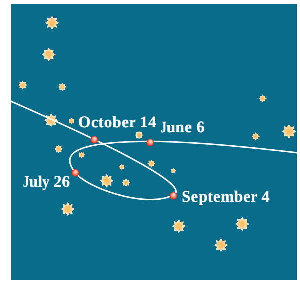

Fig. 14-9: The path of the planet Mars as it moved against a background of the constellation Capricorn during 1971. Its position on four selected days is marked. Both Mars and Earth are moving in orbits around the Sun so that we see the position of Mars relative to us; this sometimes results in an apparent loop in the path of Mars.

➤1. THE LAW OF ORBITS: All planets move in elliptical orbits, with the Sun at one focus.

Figure 14-10 shows a planet of mass m moving in such an orbit around the Sun, whose mass is M. We assume that $M >> m$, so that the center of mass of the planet-Sun system is approximately at the center of the Sun.

The orbit in Fig. 14-10 is described by giving its **semimajor axis** a and its **eccentricity** e, the latter defined so that ea is the distance from the center of the ellipse to either focus f or f'. *An eccentricity of zero corresponds to a circle,* in which the two foci merge to a single central point. The eccentricities of the planetary orbits are not large, so—sketched on paper—the orbits look circular. The eccentricity of the ellipse of Fig. 14-10, which has been exaggerated for clarity, is 0.74. The eccentricity of Earth's orbit is only 0.0167.

➤2. THE LAW OF AREAS: A line that connects a planet to the Sun sweeps out equal areas in the plane of the planet's orbit in equal times; that is, the rate dA/dt at which it sweeps out area A is constant.

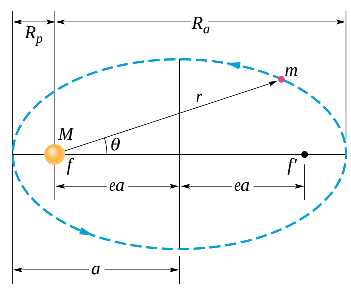

Fig. 14-10: A planet of mass m moving in an elliptical orbit around the Sun. The Sun, of mass M, is at one focus F of the ellipse. The other focus is F', which is located in empty space. Each focus is a distance ea from the ellipse's center, with e being the eccentricity of the ellipse. The semimajor axis a of the ellipse, the perihelion (nearest the Sun) distance R_p, and the aphelion (farthest from the Sun) distance R_a are also shown.

Qualitatively, this second law tells us that the planet will move most slowly when it is farthest from the Sun and most rapidly when it is nearest to the Sun. As it turns out, Kepler's second law is totally equivalent to the law of conservation of angular momentum. Let us prove it.

The area of the shaded wedge in Fig. 14-11*a* closely approximates the area swept out in time Δt by a line connecting the Sun and the planet, which are separated by a distance r. The area ΔA of the wedge is approximately the area of a triangle with base $r\,\Delta\theta$ and height r. Since the area of a triangle is one-half of the base times the height, $\Delta A >> \frac{1}{2} r^2\,\Delta\theta$. This expression for ΔA becomes more exact as Δt (hence $\Delta\theta$) approaches zero. The instantaneous rate at which area is being swept out is then

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \omega, \tag{14-20}$$

in which ω is the angular speed of the rotating line connecting Sun and planet.

Figure 14-11*b* shows the linear momentum $\vec{p}$ of the planet, along with its radial and perpendicular components. From Eq. 12-23 ($|\vec{L}| = r|\vec{p}_\perp|$), the magnitude of the angular momentum $\vec{L}$ of the planet about the Sun is given by the product of r and $p_\perp$, the component of $\vec{p}$ perpendicular to r. Here, for a planet of mass m,

$$\begin{aligned}|\vec{L}| &= r|\vec{p}_\perp| = (r)(m|\vec{v}_\perp|) = (r)(m\omega r) \\ &= mr^2\omega,\end{aligned} \tag{14-21}$$

where we have replaced $v_\perp$ with its equivalent ωr (Eq. 11-15). Eliminating $r^2\omega$ between Eqs. 14-20 and 14-21 leads to

$$\frac{dA}{dt} = \frac{|\vec{L}|}{2m} \tag{14-22}$$

If dA/dt is constant, as Kepler said it is, then Eq. 14-22 means that $|\vec{L}|$ must also be constant—angular momentum is conserved. Kepler's second law is indeed equivalent to the law of conservation of angular momentum.

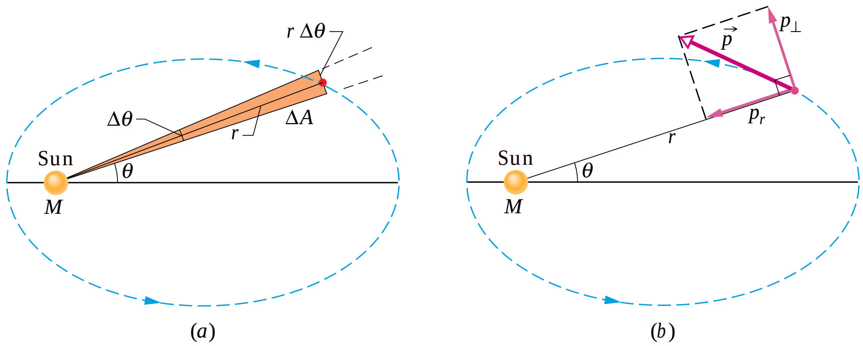

Fig. 14-11: (*a*) In time Δt, the line r connecting the planet to the Sun (of mass M) sweeps through an angle $\Delta\theta$, sweeping out an area ΔA (shaded). (*b*) The linear momentum $\vec{p}$ of the planet and its components.

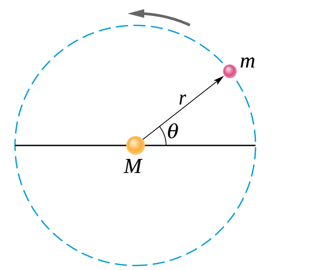

Fig. 14-12: A planet of mass m moving around the Sun in a circular orbit of radius r.

➤**3. THE LAW OF PERIODS:** The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

To see this, consider the circular orbit of Fig. 14-12, with radius r (the radius of a circle is equivalent to the semimajor axis of an ellipse). Applying Newton's Second Law, $(|\vec{F}| = m|\vec{a}|)$, to the orbiting planet in Fig. 14-12 yields

$$\frac{GMm}{r^2} = (m)(\omega^2 r). \qquad (14\text{-}23)$$

Here we have substituted from Eq.14-1 for the force magnitude $|\vec{F}|$ and used Eq. 11-20 to substitute $\omega^2 r$ for the centripetal acceleration. If we use Eq. 11-17 to replace ω with $2\pi/T$, where T is the period of the motion, we obtain Kepler's third law:

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \quad \text{(law of periods)}. \qquad (14\text{-}24)$$

The quantity in parentheses is a constant that depends only on the mass M of the central body about which the planet orbits.

Equation 14-24 holds also for elliptical orbits, provided we replace r with a, the semimajor axis of the ellipse. This law predicts that the ratio T^2/a^3 has essentially the same value for every planetary orbit around a given massive body. Table 14-3 shows how well it holds for the orbits of the planets of the solar system.

TABLE 14-3:
Kepler's Law of Periods for the Solar System

Planet	Semimajor Axis a (10^{10} m)	Period T (y)	T^2/a^3 (10^{-34} y^2/m^3)
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1.00	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84.0	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

READING EXERCISE 14-3: Satellite 1 is in a certain circular orbit about a planet, while satellite 2 is in a larger circular orbit. Which satellite has (a) the longer period and (b) the greater speed?

Touchstone Examples 14-7-1 and 14-7-2, at the end of this chapter, illustrate how to use what you learned in this section.

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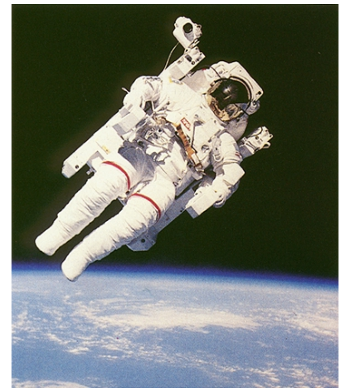
14-8 Satellites: Orbits and Energy



On February 7, 1984, at a height of 102 km above Hawaii and with a speed of about 29,000 km/h, Bruce McCandless stepped (untethered) into space from a space shuttle and became the first human satellite.

As a satellite orbits Earth on its elliptical path, both its speed, which fixes its kinetic energy *K*, and its distance from the center of Earth, which fixes its gravitational potential energy *U*, fluctuate with fixed periods. However, the mechanical energy *E* of the satellite remains constant. (Since the satellite's mass is so much smaller than Earth's mass, we assign *U* and *E* for the Earth-satellite system to the satellite alone.)

The potential energy of the system is given by Eq. 14-12 and is

$$U = -\frac{GMm}{r}$$

(with $U = 0$ for infinite separation). Here *r* is the radius of the orbit, assumed for the time being to be circular, and *M* and *m* are the masses of Earth and the satellite, respectively.

To find the kinetic energy of a satellite in a circular orbit, we write Newton's second law, in terms of vector magnitudes as $(|\vec{F}| = m|\vec{a}|)$, as

$$\frac{GMm}{r^2} = m\frac{v^2}{r} \qquad (14\text{-}25)$$

where v^2/r is the centripetal acceleration of the satellite. Then, from Eq. 14-25, the kinetic energy is

$$K = \tfrac{1}{2}mv^2 = \frac{GMm}{2r} \qquad (14\text{-}26)$$

which shows us that for a satellite in a circular orbit,

$$K = -\frac{U}{2} \quad \text{(circular orbit).} \qquad (14\text{-}27)$$

The total mechanical energy of the orbiting satellite is

$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r}$$

or

$$E = -\frac{GMm}{2r} \quad \text{(circular orbit).} \qquad (14\text{-}28)$$

This tells us that for a satellite in a circular orbit, the total energy *E* is the negative of the kinetic energy *K*:

$$E = -K \quad \text{(circular orbit).} \qquad (14\text{-}29)$$

For a satellite in an elliptical orbit of semimajor axis *a*, we can substitute *a* for *r* in Eq. 14-28 to find the mechanical energy as

$$E = -\frac{GMm}{2a} \quad \text{(elliptical orbit).} \qquad (14\text{-}30)$$

Equation 14-30 tells us that the total energy of an orbiting satellite depends only on the semimajor axis of its orbit and not on its eccentricity e. For example, four orbits with the same semimajor axis are shown in Fig. 14-13; the same satellite would have the same total mechanical energy E in all four orbits. Figure 14-14 shows the variation of K, U, and E with r for a satellite moving in a circular orbit about a massive central body.

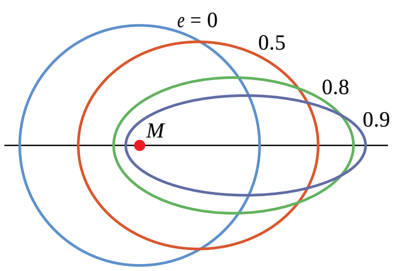


Fig. 14-13: Four orbits about an object of mass M. All four orbits have the same semimajor axis a and thus correspond to the same total mechanical energy E. Their eccentricities e are marked.

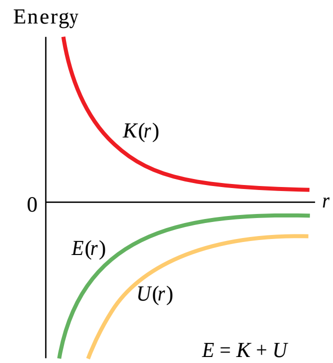
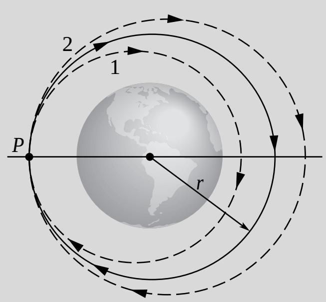


Fig. 14-14: The variation of kinetic energy K, potential energy U, and total energy E with radius r for a satellite in a circular orbit. For any value of r, the values of U and E are negative, the value of K is positive, and $E = -K$. As r approaches infinity, all three energy curves approach a value of zero.

READING EXERCISE 14-4: In the figure, a space shuttle is initially in a circular orbit of radius r about Earth. At point P, the pilot briefly fires a forward-pointing thruster to decrease the shuttle's kinetic energy K and mechanical energy E. (a) Which of the dashed elliptical orbits shown in the figure will the shuttle then take? (b) Is the orbital period T of the shuttle (the time to return to P) then greater than, less than, or the same as in the circular orbit?



14-9 Einstein and Gravitation

Principle of Equivalence

Albert Einstein once said: "I was…in the patent office at Bern when all of a sudden a thought occurred to me: 'If a person falls freely, he will not feel his own weight.' I was startled. This simple thought made a deep impression on me. It impelled me toward a theory of gravitation."

Thus Einstein tells us how he began to form his **general theory of relativity.** The fundamental postulate of this theory about gravitation (the gravitating of objects toward each other) is called the **principle of equivalence,** which says that gravitation and acceleration are equivalent. If a physicist were locked up in a small box as in Fig. 14-15, he would not be able to tell whether the box was at rest on Earth (and subject only to Earth's gravitational force), as in Fig. 14-15*a*, or accelerating through interstellar space at

$9.8\ \mathrm{m/s^2}$ (and subject only to the force producing that acceleration), as in Fig. 14-15*b*. In both situations he would feel the same and would read the same value for his weight on a scale. Moreover, if he watched an object fall past him, the object would have the same acceleration relative to him in both situations.

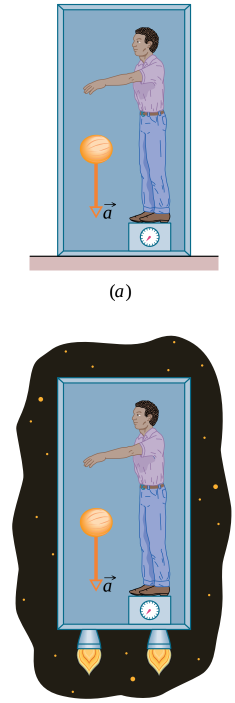


Fig. 14-15: (*a*) A physicist in a box resting on Earth sees a cantaloupe falling with acceleration $a = 9.8\ \mathrm{m/s^2}$. (*b*) If he and the box accelerate in deep space at $9.8\ \mathrm{m/s^2}$, the cantaloupe has the same acceleration relative to him. It is not possible, by doing experiments within the box, for the physicist to tell which situation he is in. For example, the platform scale on which he stands reads the same weight in both situations.

Curvature of Space

We have thus far explained gravitation as due to a force between masses. Einstein showed that, instead, gravitation is due to a curvature (or shape) of space that is caused by the masses. (As is discussed later in this book, space and time are entangled so the curvature of which Einstein spoke is really a curvature of *spacetime,* the combined four dimensions of our universe.)

Picturing how space (such as vacuum) can have curvature is difficult. An analogy might help: Suppose that from orbit we watch a race in which two boats begin on the equator with a separation of 20 km and head due south (Fig. 14-16*a*). To the sailors, the boats travel along flat, parallel paths. However, with time the boats draw together until, nearer the south pole, they touch. The sailors in the boats can interpret this drawing together in terms of a force acting on the boats. However, we can see that the boats draw together simply because of the curvature of Earth's surface. We can see this because we are viewing the race from "outside" that surface.

Figure 14-16*b* shows a similar race: Two horizontally separated apples are dropped from the same height above Earth. Although the apples may appear to travel along parallel paths, they actually move toward each other because they both fall toward Earth's center. We can interpret the motion of the apples in terms of the gravitational force on the apples from Earth. We can also interpret the motion in terms of a curvature of the space near Earth, due to the presence of Earth's mass. This time we cannot see the curvature because we cannot get "outside" the curved space, as we got "outside" the curved Earth in the boat example. However, we can depict the curvature with a drawing like Fig. 14-16*c*; there the apples would move along a surface that curves toward Earth because of Earth's mass.

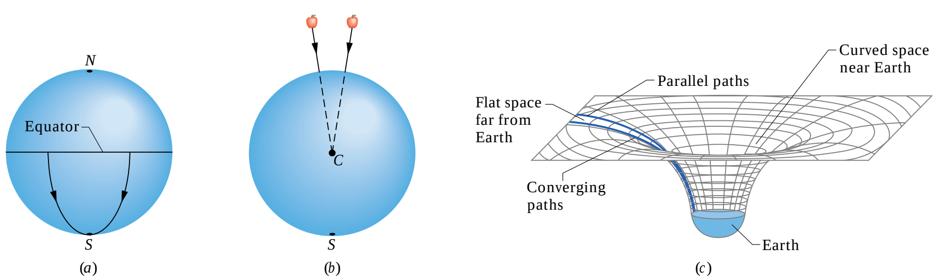
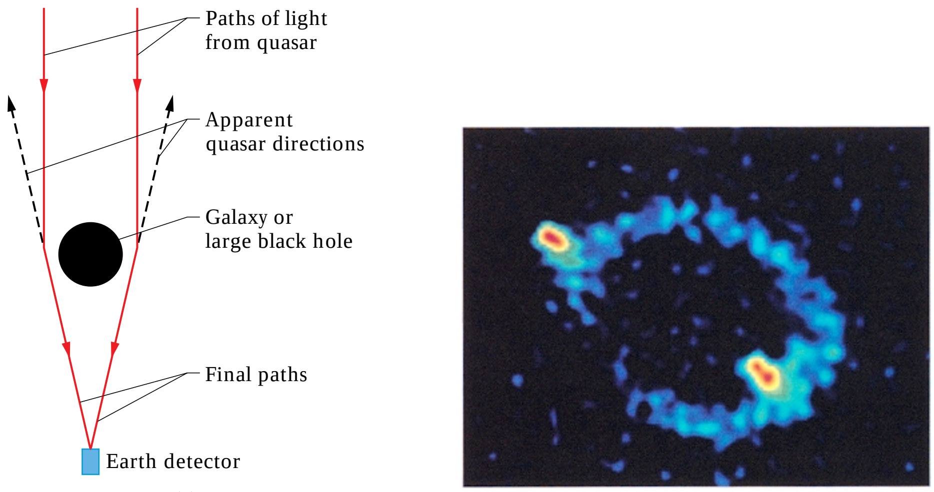


Fig. 14-16: (*a*) Two objects moving along lines of longitude toward the south pole converge because of the curvature of Earth's surface. (*b*) Two objects falling freely near Earth move along lines that converge toward the center of Earth because of the curvature of space near Earth. (*c*) Far from Earth (and other masses), space is flat and parallel paths remain parallel. Close to Earth, the parallel paths begin to converge because space is curved by Earth's mass.

When light passes near Earth, its path bends slightly because of the curvature of space there, an effect called *gravitational lensing.* When it passes a more massive structure, like a galaxy or a black hole having large mass, its path can be bent more. If such a massive structure is between us and a quasar (an extremely bright, extremely distant source of light), the light from the quasar can bend around the massive structure and toward us (Fig. 14-17*a*). Then, because the light seems to be coming to us from a number of slightly different directions in the sky, we see the same quasar in all those different directions. In some situations, the quasars we see blend together to form a giant luminous arc, which is called an *Einstein ring* (Fig. 14-17*b*).

Should we attribute gravitation to the curvature of spacetime due to the presence of masses or to a force between masses? Or should we attribute it to the actions of a type of fundamental particle called a *graviton,* as conjectured in some modern physics theories? We do not know.



(*a*) (*b*)

Fig. 14-17: (*a*) Light from a distant quasar follows curved paths around a galaxy or a large black hole because the mass of the galaxy or black hole has curved the adjacent space. If the light is detected, it appears to have originated along the backward extensions of the final paths (dashed lines). (*b*) The Einstein ring known as MG1131+0456 on the computer screen of a telescope. The source of the light (actually, radio waves, which are a form of invisible light) is far behind the large, unseen galaxy that produces the ring; a portion of the source appears as the two bright spots seen along the ring.