

Name: \_\_\_\_\_

PHYS1100 Section 04

Physics I "Majors"

Fall 2006

Midterm Exam #2      Thursday, 2 Nov 2006

This exam has four questions and you are to work all of them. *Not all the problems carry the same number of points.* You must hand in your paper by the end of class time (1:50pm) unless prior arrangements have already been made with the instructor.

You may use your textbook, course notes, or any other reference you may have other than another human. You are welcome to use your calculator or computer, although the test is designed so that these are not necessary.

Good luck!

Problem 1: \_\_\_\_\_

Problem 2: \_\_\_\_\_

Problem 3: \_\_\_\_\_

Problem 4: \_\_\_\_\_

**Total:**      \_\_\_\_\_

**Problem 1 (30 points).** Refer to Fig.9-15 in your textbook for this problem.

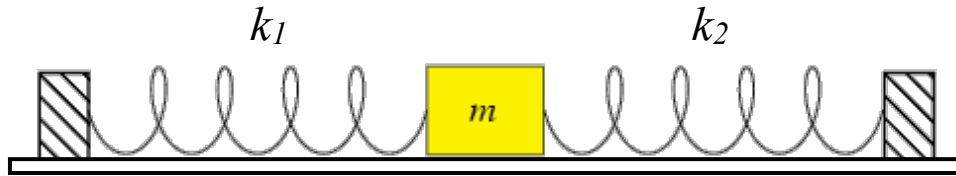
**a.** (5 points) Figure (a) shows that the rotational inertia for a “hoop” around a cylinder axis is  $I = MR^2$  where  $M$  is the mass of the entire hoop, and  $R$  is its radius. Explain *in words* how this follows from the definition of rotational inertia.

**b.** (5 points) If the hoop in (a) has a uniform density  $\rho$ , a width  $\ell$ , and a thickness  $dr \ll R$ , write an equation for the mass  $dm$  of the hoop in terms of  $\rho$ ,  $\ell$ ,  $dr$ , and  $R$ .

**c.** (15 points) Figure (b) shows for an annular cylinder with inner radius  $R_1$  and an outer radius  $R_2$ , one has  $I = \frac{1}{2}M(R_1^2 + R_2^2)$  where  $M$  is the mass of the cylinder. Derive this formula by integration, using the results of parts (a) and (b) above. *Hint:*  $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)$ .

**d.** (5 points) For a solid, uniform cylinder about its axis, Figure (c), one has  $I = \frac{1}{2}MR^2$  regardless of the cylinder length  $L$ . Derive this result using parts (a), (b), and/or (c) above.

**Problem 2 (20 points).** A mass  $m$  slides horizontally on a frictionless surface. It is attached to vertical pylons on the left and the right, by springs with spring constant  $k_1$  and  $k_2$  respectively. The position of  $m$  is given by  $x$ , where  $x = 0$  is the equilibrium position.



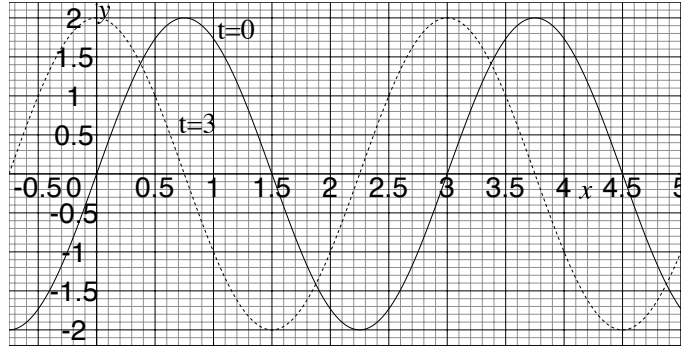
**a.** (5 points) Draw a free body diagram of the mass  $m$ . Include *all* forces on the mass.

**b.** (10 points) Write down the equation of motion for the mass  $m$  for movement in the horizontal (i.e.  $x$ ) direction. Your equation should contain only  $m$ ,  $k_1$ ,  $k_2$ , and appropriate powers and derivatives of  $x$ .

**c.** (5 points) Express the natural frequency of oscillation for the mass  $m$  in terms of  $m$ ,  $k_1$ , and  $k_2$ . Be very clear with what you mean by “frequency.”

**Problem 3 (20 points).** Andrew stands on a turntable like the one we used in class. He holds a small, spinning wheel, like a gyroscope. The axis of the gyroscope is vertical, but Andrew is not rotating himself. He then takes the gyroscope and inverts it, so that the axis is still vertical, but it rotates in the opposite direction. This makes Andrew start rotating. His rotational inertia is  $I_A$ , and that of the gyroscope is  $I_G$ . If the gyroscope is spinning with angular velocity  $\omega$ , find the angular velocity  $\Omega$  of Andrew, in terms of  $I_A$ ,  $I_G$ , and  $\omega$ . You may assume that  $I_A$  is much, much larger than  $I_G$ .

**Problem 4 (30 points).** The figure on the right shows a sinusoidal wave  $y(x, t)$  on a string at a time  $t = 0$  (solid line) and also at time  $t = 3$  sec (dashed line). (The  $t = 0$  curve is the one which passes through the origin.) The wave is traveling to the right. The axes are labeled in meters.



a. (2 points each) Read from the graph the values of the following quantities, including units. (You can leave your answer in terms of  $\pi$  where appropriate.)

- Wavelength  $\lambda =$
- Speed  $v =$
- Frequency  $f =$
- Amplitude  $y_m =$
- Wavenumber  $k =$

b. (10 points) Write the equation for  $y(x, t)$ .

c. (10 points) The same string at the same tension is now fixed at both ends, 6 m apart. Find the wavelengths and frequencies of *the first three* standing waves of the string.