Midterm Exam #1    Thursday, 28 Sept 2006

This exam has four questions and you are to work all of them. You must hand in your paper by the end of class time (1:50pm) unless prior arrangements have already been made with the instructor.

You may use your textbook, course notes, or any other reference you may have other than another human. You are welcome to use your calculator or computer, although the test is designed so that these are not necessary.

Good luck!

Problem 1: ________________

Problem 2: ________________

Problem 3: ________________

Problem 4: ________________

Total: ________________
Problem 1 (25 points). A child of mass $m$ rides on a playground swing. The length of the chain connecting the seat to the overhead bar is $L$. The kid is able to get the swing so that he is stationary when the chain is stretched to its length and at 90° to the vertical.

a. (5 points) Draw a free body diagram of the child when the swing is at the bottom of its arc and the chain is vertical. Label all the forces on the child. *Hint: There are two forces.* You can ignore the mass of the chain and the seat.

b. (8 points) Determine an expression for the kinetic energy of the child when the swing is at the bottom of its arc, in terms of only $m$, $L$, and the acceleration $g$ due to gravity.

c. (12 points) Use a and b to find an expression for the tension $T$ in the chain, when the swing is at the bottom of its arc, only in terms of $m$ and $g$. *Hint: What kind of motion is the child undergoing at the bottom of the arc?*
Problem 2 (25 points). This question concerns a particle of mass $m$ moving in one dimension $x$. The particle is acted on by a force $F$ which is derived from a potential energy function $U(x)$. Consider the total mechanical energy of the particle $E = \frac{1}{2}mv^2 + U(x)$.

a. (10 points) Assuming that $F$ is the only force acting on the particle, prove that $dE/dt = 0$. Hint: We did this together in class.

b. (10 points) Now assume that in addition to the force $F$, a drag force $f = -bv$ also acts on the particle. Now $dE/dt$ is no longer zero. Derive a new expression for $dE/dt$. Hint: Remember that Newton’s second law is better written as $\sum F = ma$.

c. (5 points) Explain in a few words what your expression in (b) means, physically, is happening to the energy. Include an explanation for the sign of $dE/dt$. 
Problem 3 (25 points). Consider a thin rod of length $L$. Although its cross section does not change, the mass density of the rod increases linearly as you go from one end of the rod to the other. That is, the mass per unit length is $\rho(x) = ax$ where $a$ is a constant and $x$ measures the distance from one end. *Hint: Remember that $\int x^n dx = x^{n+1}/(n + 1)$.*

a. (5 points) What are the dimensions of $a$, in terms of length, mass, and time.

b. (10 points) Find the mass $M$ of the rod in terms of $a$ and $L$.

c. (10 points) Find the position $x_{CM}$ of the center of mass of the rod, in terms of $a$ and $L$. 
Problem 4 (25 points). A satellite of mass $m$ circularly orbits a planet of mass $M$.

a. (10 points) Assume the orbit radius is $R$. Find an expression for the total mechanical energy of the satellite only in terms of $m$, $M$, $R$, and Newton’s gravitational constant $G$.

b. (5 points) The satellite is moved to a circular orbit of radius $2R$. Does the total mechanical energy of the satellite increase, decrease, or remain the same? Explain your answer, briefly.

c. (10 points) Determine the amount by which the total mechanical energy of the satellite changes, again only in terms of $m$, $M$, $R$, and $G$. Is the change positive or negative?