Solution to Multiple Choice Question 10 in Chapter 14, part (c)

A and D are circular orbits, with the radius of D smaller than that of A. C and D are elliptical, and the farthest distance form the Sun for each of them is the radius of orbit A. The distance of closest approach for C is the same as the radius of orbit D, but the distance of closest approach for B is within the orbit of D.

(c) On which orbit is the largest speed acquired.

For any given orbit, call R farthest point and r nearest point, and write L for the angular momentum. Then both

$$E = \frac{L^2}{2mR^2} - \frac{GMm}{R}$$

and

$$E = \frac{L^2}{2mr^2} - \frac{GMm}{r}$$

must be true.

Then

$$\frac{L^2}{2mR^2} - \frac{GMm}{R} = \frac{L^2}{2mr^2} - \frac{GMm}{r}$$

and

$$\frac{L^2}{2m}\left(\frac{1}{r^2} - \frac{1}{R^2}\right) = \frac{L^2}{2m}\left(\frac{1}{r} - \frac{1}{R}\right)\left(\frac{1}{r} + \frac{1}{R}\right) = GMm\left(\frac{1}{r} - \frac{1}{R}\right)$$

so that

$$L^2 = 2GMm^2\left(\frac{1}{r} + \frac{1}{R}\right)^{-1} = 2GMm^2\frac{Rr}{R+r}$$

If R is equal for two orbits, then smaller r has smaller angular momentum.

But at the point of nearest approach, $L = mvr$, so

$$v^2 = 2GM\frac{R}{r(R+r)}$$

so smaller r has larger velocity (assuming equal R).