

Gravitational and Inertial Mass

We have made a big deal in class, how “gravitational” mass differs (in principle) from “inertial” mass. It was Newton’s genius to equate the two, and it was Einstein’s genius to see why this had to be true. (Einstein’s reasoning is called the “equivalence principle.”)

Your goal in this laboratory is to test the extent to which these two masses are the same. You’ll need to consider sources of uncertainty carefully, otherwise you are liable to discover that gravitational mass and inertial mass are not the same thing!

We will do the pendulum later in our class, but I’ll cover the basics here. You should also see your textbook, “The Simple Pendulum”, in Section 17-5. You might also consult *The Pendulum: Rich physics from a simple system*, by Robert A. Nelson and M. G. Olson, Am.J.Phys. 54(1986)112.

See Figure 17-10 in your textbook:

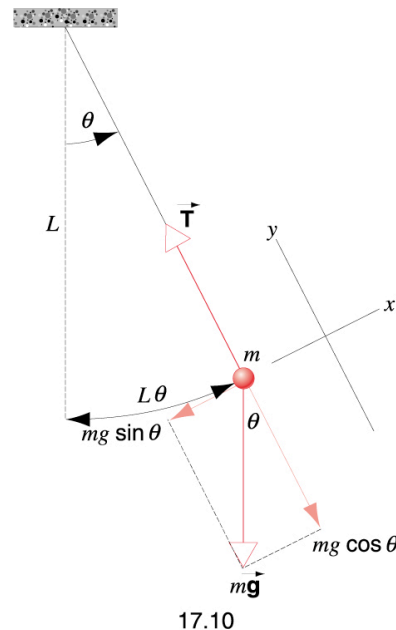


Figure 17-10. The simple pendulum. The forces acting on the pendulum are the tension \vec{T} and the gravitational force $m\vec{g}$, which is resolved into its radial and tangential components. We choose the x axis to be in the tangential direction and the y axis to be in the radial direction at this particular time.

The x coordinate will measure the position along the arc of the pendulum bob, that is $x = L\theta$. The only force in the x direction is the component of gravity along the arc, which is $-m_G g \sin \theta$, where we emphasize that m_G is the “gravitational” mass. The equation of motion, that is Newton’s second law, says that this force must equal $m_I a = m_I \ddot{x}$ where m_I is the “inertial” mass. Therefore

$$-m_G g \sin \theta = m_I \ddot{x} \quad \Rightarrow \quad \ddot{\theta} = - \left(\frac{m_G}{m_I} \right) \left(\frac{g}{L} \right) \sin \theta$$

is the differential equation we need to solve. Unfortunately, this equation is impossible to solve exactly, and we must resort to approximation.

If we assume that the angle θ never gets very large, then we can write $\sin \theta \approx \theta$. This turns the differential equation into $\ddot{\theta} = -\omega^2 \theta$ where $\omega = ((m_G/m_I)(g/L))^{1/2}$. This equation is easy to solve. All you need to do is recognize that if you take the derivative of cosine (or sine) twice, then you get the same thing back but with a minus sign out front! The chain rule takes care of putting two factors of ω out front if we just write

$$\theta(t) = A \cos \omega t$$

which is called “simple harmonic motion”. The pendulum always returns to the same place when a time $T = 2\pi/\omega$ elapses. This is called the *period* of the pendulum, and it is what you will measure in this experiment. In other words

$$T = 2\pi\sqrt{\left(\frac{m_I}{m_G}\right)\left(\frac{L}{g}\right)}$$

Your experimental setup is simple. You have a string+mass pendulum system, where you can change the value of the mass. Let the pendulum swing many times and measure the total time, then divide by the number of swings to get the period with a small uncertainty. (See Exercise 30 in Chapter 17.)

First confirm that you get a good value for g by assuming $m_I = m_G$, measuring the length L , and finding a precise value for the period T . Then, try measuring the period for many different values of the pendulum bob mass. Do you see any evidence of a mass-dependent period? You might get particularly precise values for $(m_G/m_I)g$ if you measure T as a function of L and determine the slope on a graph of T^2 *vs.* L .

You might try making a plot of $(m_G/m_I)g$ as a function of the pendulum bob mass. If this plot shows any slope, then it is an indication that (m_G/m_I) changes as a function of mass. In the end, come up with the largest value with which you are comfortable for the quantity

$$\Delta \equiv \frac{m_G - m_I}{m_G} = 1 - \frac{m_I}{m_G}$$

This called an “upper limit” for Δ and is the way that physicists report the result of a measurement that yielded “zero”.

Of course, if you convince yourself that your measurement for Δ shows a nonzero value outside the limits of your uncertainties, then you’ve discovered that the equivalence principle is wrong! Go to Stockholm to accept your Nobel Prize.