This exam has four questions and you are to work all of them. You must hand in your paper by the end of class time (11:50am) unless prior arrangements have already been made with the instructor.

Note that not all of the problems are worth the same number of points.

You may use your textbook, course notes, or any other reference you may have other than another human. You are welcome to use your calculator or computer, although the test is designed so that these are not absolutely necessary.

Good luck!

Problem 1: ________________

Problem 2: ________________

Problem 3: ________________

Problem 4: ________________

Total: ________________
Problem 1 (20 points). A disturbance propagates along a string with a shape given by

\[ y(x,t) = 5e^{-(x+t/5)^2} \]

where \( x \) measures length along the string, \( t \) is time, and all numerical values are in SI units.

a. (5 points) Show that, or explain why, this disturbance is a “wave.”

b. (5 points) What is the wave speed, and is it to the right or the left?

c. (5 points) Sketch the shape of the transverse displacement as a function of \( x \) at \( t = 0 \). Label the axes, including tick marks that indicate appropriate values of the displacement and \( x \) position.

d. (5 points) An observer is located at \( x = 0 \). Sketch the shape of the transverse displacement as a function of time. Again, label the axes appropriately.
Problem 2 (30 points). Two objects, each of mass $m$ and located at $x_1(t)$ and $x_2(t)$, move in one dimension. They are connected by a spring with force constant $k$, but otherwise free.

a. (10 points) Write the two (coupled differential) equations of motion for $x_1$ and $x_2$.

b. (10 points) Use the ansatz $x_1(t) = a_1 e^{i\omega t}$ and $x_2(t) = a_2 e^{i\omega t}$ to find the two (squared) normal mode frequencies $\omega^2$.

c. (10 points) Find the relationship between the amplitudes $a_1$ and $a_2$ for each of the two normal modes, and briefly describe the motion in each case.
Problem 3 (25 points). The next page shows the Lorentz Transform graph we used in class. Use this graph to answer the following questions.

a. (5 points) As we did in class, use some line with a fixed value of \( x' \) to determine the speed \( u \) of the \( (x', t') \) frame in the \( (x, t) \) frame.

b. (10 points) Draw the “world line” of a particle that travels with speed \( v_0 = 1/2 \) in the \( (x', t') \) frame. *Hint: Identify two points that are separated by \( \Delta x' = v_0 \Delta t' \).*

c. (10 points) Find the speed of this particle as observed from the \( (x, t) \) frame. This is called “addition of velocities” and you can check your answer using Eq.20-12 in your textbook.
Graph for use with Problem 3
Problem 4 (25 points). A monatomic ideal gas is in a 1m³ container, at a pressure of 1Pa and a temperature of 100K.

a. (5 points) Find the number of atoms in the container.

b. (5 points) The volume expands against a piston to 2m³ under constant pressure. What is the final temperature of the gas?

c. (10 points) Find the work done on the piston by the ideal gas in this expansion.

d. (5 points) Did heat enter or leave the gas during the expansion, or was no heat transferred? Briefly explain your answer.
1. This is a wave since it is of the form \( y(x, t) = f(x - vt) \) where \( v = -1/5 \), so the speed is 0.2 m/sec to the left. At \( t = 0 \) the displacement is \( y(x, 0) = 5e^{-x^2} \), a Gaussian centered at \( x = 0 \), with height 5m and falling on either side to \( 1/e \) at \( x = \pm 1m \). At \( x = 0 \), the observer sees \( y(0, t) = 5e^{-(t/5)^2} \), a Gaussian which rises to 5m and then falls, with \( 1/e \) heights at \( t = \pm 5\sec \).

2. The only force on either mass is from the spring between them, and this force is just \( k \) times the difference \( x_2 - x_1 \). This gives the equations

\[
\begin{align*}
m\ddot{x}_1 &= +k(x_2 - x_1) \\
m\ddot{x}_2 &= -k(x_2 - x_1)
\end{align*}
\Rightarrow \quad
\begin{align*}
-m\omega^2 a_1 &= +k(a_2 - a_1) \\
-m\omega^2 a_2 &= -k(a_2 - a_1)
\end{align*}
\Rightarrow \quad
ka_1 + (m\omega^2 - k)a_2 = 0
\]

Zeroing the determinant gives \((m\omega^2 - k)^2 - k^2 = m\omega^2(m\omega^2 - 2k) = 0\). Solutions are \( m\omega^2 = 0 \) \((a_1 = a_2)\) and \( m\omega^2 = 2k \) \((a_1 = -a_2)\). The latter has the two masses oscillating against each other with \( \omega = \sqrt{2k/m} \). In the former the two masses move together and the spring is un-stretched always. But \( \omega^2 = \pm 0? \) Note that \( m(\ddot{x}_1 + \ddot{x}_2) = 0 \), so the center of mass can move with constant speed \( v_0 \). That is \( x_1(t) + x_2(t) = v_0 t \) corresponds to \( \omega = 0 \).

3. Just as in class, \( u = 1/2 \). Then, find the point with \((x', t') = (1, 2)\) and connect it by a line to the origin. Find a convenient point on this line and read off an \((x, t) = (\Delta x, \Delta t)\). You should find \( v = (1/2 + 1/2)/(1 + 1/2 \cdot 1/2) = 4/5 \).

4. From \( PV = NkT \), \( N = (1 \cdot 1/100 \text{ J/K})/k \). Under expansion, \( P \) and \( N \) do not change, but \( V \) doubles, so \( T \) also doubles, to 200K. The work done on the piston is positive since the piston moves in the direction of the force on it. Therefore

\[
W = \int_{V_1}^{V_2} PdV = P(V_2 - V_1) = 1 \cdot (2 - 1) = 1 \text{ J}
\]

The first law says that the change in internal energy \( \Delta U = Q - W \) where \( Q \) is the heat transferred into the gas and \( W \) is the work done on the piston. The temperature went up so \( \Delta U > 0 \) and \( W > 0 \), so \( Q > 0 \) and heat was added to the gas during the expansion. It is \textit{not} enough to say that temperature went up, so heat must have been added, since in principle a temperature rise can also come from work being done on the gas.