Curve Fitting to a Trajectory

The point of this laboratory is to fit data to a model curve. Your data will be taken from a video clip of a ball flying freely through the air, given some initial velocity vector. In the end you should have a measurement of g, the acceleration due to gravity, along with some uncertainty for your measurement. That is, you should conclude by reporting $g \pm \delta g$.

Work in pairs if you like, but this laboratory can be done independently. Refer to the posted "Studio Physics I" activity, in the right hand column of the laboratory web page for our course. There you can download the video clip. You can also download the activity description. Use this description to see how to take data off of the clip, using LoggerPro. You will end up with three columns of numbers ("Time", "x", and "y") each with seven rows, i.e. seven "data points". Store your data in whatever form you like, even if you just write it down in your lab book.

We derived equations for projectile motion in class. (See also Section 4-3 of your textbook.) These are

$$\begin{aligned} x(t) &= v_{x_0} t \\ y(t) &= v_{y_0} t - \frac{1}{2} g t^2 \\ \text{and} \qquad y &= \frac{v_{y_0}}{v_{x_0}} x - \frac{1}{2} \frac{g}{v_{x_0}^2} x^2 \end{aligned}$$

where we have arranged things so that $x = y = 0$ at $t = 0$. You can easily incorporate this constraint by subtracting the first of your seven data rows from the other six.

Now plot your data as x versus t, y versus t, or y versus x. You can use Excel, Matlab, some other plotting program of your choice, or simply make a plot on the graph paper in your lab book. Try drawing curves through those points according to the equations above. A linear "curve" is simple. For a quadratic like $y = ax + bx^2$ you can pick two of your six (x, y) data points, insert them into the quadratic equation, and solve for a and b. Different pairs of data points will give you (slightly) different values of a and b, and you can use that to estimate your uncertainties in a and b. (Of course, you can do the same thing with y versus t data.)

Use your fitted values to determine v_{x_0}, v_{y_0}, and most importantly g. You will have to give some thought as to how to combine the uncertainties to come up with a value for δg.

It is possible to use "fitting programs" to come up with values of a and b - in fact, the Studio Physics I writeup describes how to use something inside LoggerPro or Excel - but I don't recommend using them right now. For one thing, you cannot get an uncertainty easily out of these programs. For another, I think you are better off trying it yourself first, before handing it over to a fitting program. Nevertheless, fitting programs are important tools in data analysis. Most of them work by a technique called "minimization of least squares." If you Google "least squares" you will find decent references in WikiPedia, MathWorld, and other places. Of course, this is covered in just about any textbook on experimental data analysis.