(1) Consider scattering in one dimension $x$ from a potential $V(x)$ localized near $x = 0$. The initial state is a plane wave coming from the left, that is $\phi(x) \equiv \langle x | i \rangle = e^{ikx}/\sqrt{2\pi}$

(a) Find the scattering Green's function $G(x, x')$, defined in one dimension analogously with (6.2.3). You only need concern yourself with scattering from the past to the future.

(b) Take the special case of an attractive $\delta$-function potential, namely

$$V(x) = -\gamma \frac{\hbar^2}{2m} \delta(x) \quad \text{with} \quad \gamma > 0$$

Use the Lippman-Schwinger Equation to find the outgoing wave function $\psi(x) \equiv \langle x | \psi^{(+)} \rangle$.

(c) Determine the transmission and reflection coefficients $T(k)$ and $R(k)$, defined as

$$\psi(x) = T(k)\phi(x) \quad \text{for} \quad x > 0 \quad \text{and} \quad \psi(x) = \phi(x) + R(k)e^{-ikx}/\sqrt{2\pi} \quad \text{for} \quad x < 0$$

Show that $|T|^2 + |R|^2 = 1$, as must be the case.

(d) Confirm that you get the same result by using grade-school quantum mechanics and matching right and left going waves on the left with a right going wave on the right at $x = 0$. You’ll need to integrate the Schrödinger equation across $x = 0$ to match the derivatives.

(e) We showed last semester that this potential has one, and only one, bound state. Show that your results for $T(k)$ and $R(k)$ have bound state poles at the expected positions when $k$ is treated as a complex variable.

(2) Modern Quantum Mechanics, Problem 6.3

(3) Modern Quantum Mechanics, Problem 6.4. I recommend Chapter 10 of the NIST Digital Library of Mathematical Functions at http://dlmf.nist.gov/ as a good online reference for the properties of spherical Bessel functions.

(4) Modern Quantum Mechanics, Problem 6.12