(1) We discussed in class the first order energy shifts $\Delta^{(1)}_K$ from “relativistic kinetic energy.” We also discussed $\Delta^{(1)}_{LS}$ from the spin-orbit interaction, but there was a problem in that it gave a nonzero result for $l = 0$ states if we blindly let an $l = 0$ in the numerator cancel an $l = 0$ in the denominator. In fact, $\Delta^{(1)}_{LS}$ from (5.3.31) is invalid for $l = 0$. However, if we take into account the spread of the electron wave function in the region of changing electric field, we are led to include the “Darwin term” in the Hamiltonian. The perturbation is

$$V_D = -\frac{1}{8m^2c^2} [p_i, [p_i, e\phi(r)]]$$

where a sum over $i$ is implied and $\phi(r)$ is the Coulomb potential. Find $\Delta^{(1)}_D$ and show that it is only nonzero for $l = 0$ states, and that it gives exactly the same result as blindly using (5.3.31) for $l = 0$. Then show that

$$\Delta^{(1)}_{nj} \equiv \Delta^{(1)}_K + \Delta^{(1)}_{LS} + \Delta^{(1)}_D = \frac{mc^2(Z\alpha)^4}{2n^3} \left[ 3 \frac{3}{4n} - \frac{1}{j + 1/2} \right]$$

Later on in this course we will compare this expression to the result of solving the Dirac equation in the presence of the Coulomb potential.

(2) These questions are meant to associate numbers with atomic hydrogen phenomena.

(a) The red $n = 3 \rightarrow 2$ Balmer transition has a wavelength $\lambda \approx 656$ nm. Calculate the wavelength difference $\Delta \lambda$ (in nm) between the $3p_{3/2} \rightarrow 2s_{1/2}$ and $3p_{1/2} \rightarrow 2s_{1/2}$ transitions due to the spin-orbit interaction. Comment on how you might measure this splitting.

(b) How large an electric field $E$ is needed so that the Stark splitting in the $n = 2$ level is the same as the correction from relativistic kinetic energy between the $2s$ and $2p$ levels? How easy or difficult is it to achieve an electric field of this magnitude in the laboratory?

(c) The Zeeman effect can be calculated with a “weak” or “strong” magnetic field, depending on the size of the energy shift relative to the spin-orbit splitting. Give examples of a weak and a strong field. How easy or difficult is it to achieve such a magnetic field?

(3) Compute the Stark effect for the $2s_{1/2}$ and $2p_{1/2}$ levels of hydrogen. Assume that the electric field $E$ is sufficiently weak so that $eEa_0$ is small compared to the fine structure, but take the Lamb shift $\delta$ into account (that is, ignore $2p_{3/2}$ in this calculation). Show that for $eEa_0 \ll \delta$, the energy shifts are quadratic in $E$, whereas for $eEa_0 \gg \delta$ they are linear in $E$.

(4) *Modern Quantum Mechanics*, Problem 5.18

(5) *Modern Quantum Mechanics*, Problem 5.19

(6) *Modern Quantum Mechanics*, Problem 5.20