(A) Use Gauss’ Law and the relationship between the electric field and the electric potential \( \phi(r) \) to show that

\[
\nabla^2 \phi = -4\pi \rho(r)
\]

where \( \nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \) and \( \rho(r) \) is the charge density distribution.

(B) Consider the function \( f(r) = 1/r = (x^2 + y^2 + z^2)^{-1/2} \). Show by direct (partial) differentiation that \( \nabla^2 f(r) = 0 \) so long as \( r \neq 0 \).

(C) The electric potential for a point charge \( q \) located at \( r = 0 \) is (see Schwartz Eq.2-4-7) \( \phi(r) = q/r \). Find an expression for the charge density \( \rho(r) \) corresponding to this point charge at \( r = 0 \). Make use of the so-called Dirac \( \delta \)-function \( \delta(r) \) which is defined to be zero at all points in space \( r \neq 0 \), and “large enough” at \( r = 0 \) so that

\[
\int_\text{all space} \delta(r) dV = 1
\]

You may also make use of the divergence theorem, problem 1-5(a) in Schwartz, and the “small sphere” trick that was used in class and that is discussed in Schwartz.