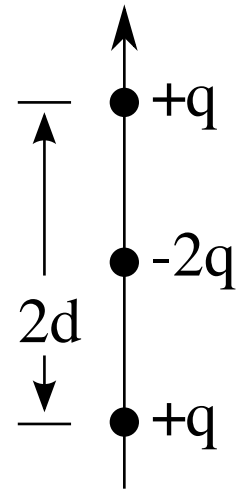


PHYS4210 Electromagnetic Theory Spring 2009

Posted Problem for Homework Due Thursday 9 April 2009

The figure on the right shows an arrangement of three charges. Two of them, with magnitude $+q$ each, are separated by a distance $2d$. The third has magnitude $-2q$ and is situated on the line between the first two, and midway between them.



- (a) What is the total charge of this collection of charges?
- (b) What is the dipole moment of this collection of charges? Recall that the dipole moment, a vector, is defined to be

$$\mathbf{p}_0 \equiv \int \mathbf{r}' \rho(\mathbf{r}') dV'$$

for a continuous charge distribution $\rho(\mathbf{r})$. Write this dipole moment as the sum of two nonzero dipole moments.

- (c) What is the quadrupole moment for this collection of charges? Recall that

$$Q_0 \equiv \frac{1}{2} \int [3z'^2 - r'^2] \rho(\mathbf{r}') dV'$$

is the quadrupole moment for a continuous charge distribution. It is all that we need to calculate the electric quadrupole tensor for a charge distribution with cylindrical symmetry about the z -axis.

- (d) Let the two dipole moments you found in (b) oscillate coherently in time, that is the same time dependence, namely $e^{i\omega t}$. Find the radiated electric field, magnetic field, and power per unit solid angle of this dipole pair to lowest nonzero order. Recall from your textbook

$$\mathbf{E}_1 = k^2 \frac{e^{-i(kr-\omega t)}}{r} [\mathbf{p}_0 - (\mathbf{p}_0 \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}] \quad \mathbf{B}_1 = \hat{\mathbf{n}} \times \mathbf{E}_1 \quad \langle \mathbf{S} \rangle_{\text{time av}} = \frac{c}{8\pi} \text{Re}(\mathbf{E} \times \mathbf{B}^*)$$

for the radiated fields and Poynting vector for an oscillating dipole. Be careful to make sure that the two dipoles have different origins. Show that the radiated power per unit solid angle agrees with Eq. (8-3-26) in your textbook, namely

$$\left\langle \frac{d^2 u}{dt d\Omega} \right\rangle_{\text{time av}} = \frac{ck^6 Q_0^2}{32\pi} \cos^2 \theta \sin^2 \theta$$