PHYS4210 Electromagnetic Theory Spring 2009

Posted Problem for Homework Due Thursday 22 Jan 2009

(A) Use Gauss' Law and the relationship between the electric field and the electric potential $\phi(\mathbf{r})$ to show that

$$\boldsymbol{\nabla}^2\phi = -4\pi\rho(\mathbf{r}) \qquad \text{where} \qquad \boldsymbol{\nabla}^2 = \boldsymbol{\nabla}\cdot\boldsymbol{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

and $\rho(\mathbf{r})$ is the charge density distribution.

(B) Consider the function $f(\mathbf{r}) = 1/r = (x^2 + y^2 + z^2)^{-1/2}$. Show by direct (partial) differentiation that $\boldsymbol{\nabla}^2 f(\mathbf{r}) = 0$ so long as $\mathbf{r} \neq \mathbf{0}$.

(C) The electric potential for a point charge q located at $\mathbf{r} = 0$ is (see Schwartz Eq.2-4-7) $\phi(\mathbf{r}) = q/r$. Find an expression for the charge density $\rho(\mathbf{r})$ corresponding to this point charge at $\mathbf{r} = 0$. Make use of the so-called Dirac δ-function $\delta(\mathbf{r})$ which is defined to be zero at all points in space $\mathbf{r} \neq \mathbf{0}$, and "large enough" at $\mathbf{r} = \mathbf{0}$ so that

$$\int_{\text{all space}} \delta(\mathbf{r})dV = 1$$

You may also make use of the divergence theorem, problem 1-5(a) in Schwartz, and the "small sphere" trick that was used in class and that is discussed in Schwartz.