PHYS4210 Electromagnetic Theory Spring 2009

Posted Problem for Homework Due Thursday 19 March 2009

This problem is adapted from Schwartz Problem 6-7, but includes suggestions for the solution, and asks a different question regarding interpreting the solution.

A pair of equal and opposite charges with electric dipole moment $\mathbf{p}$ rotate with angular frequency ω about an axis perpendicular to the line joining them. Show that the energy radiated per unit time can be approximated by the expression

$$\frac{dU}{dt} = -\frac{2}{3}\frac{p^2\omega^4}{c^3}$$

in the event that the separation is much less than the wavelength of the radiation. Compare this with the energy radiated from a single, linearly oscillating dipole, as calculated in class.

Solve this problem with superposition, instead of working from scratch with the retarded potentials. In class, we solved the problem of a dipole $\mathbf{p}(t) = \hat{\mathbf{k}} p_0 \cos\omega t$. For this problem, let the dipole rotate in the xy plane.

a) Write the time dependent dipole moment $\mathbf{p}(t)$ in terms of its (fixed) magnitude p, the angular frequency ω, and the unit vectors in the xy plane. Argue that this dipole moment is the superposition of two linear dipoles $\mathbf{p}_x(t)$ and $\mathbf{p}_y(t)$, oriented in different directions.

b) For the problem we did in class, we found that $\mathbf{E}(\mathbf{r},t) = f(\mathbf{r},t)\hat{\boldsymbol{\theta}}$ and $\mathbf{B}(\mathbf{r},t) = f(\mathbf{r},t)\hat{\boldsymbol{\psi}}$ where the function $f(\mathbf{r},t)$ are the same for each field. Use the unit vectors in spherical coordinates in terms of those in cartesian coordinates, to show that

$$\begin{aligned}
\mathbf{E}(\mathbf{r},t) &= \mathbf{B}(\mathbf{r},t) \times \hat{\mathbf{r}} \\
\mathbf{B}(\mathbf{r},t) &= \frac{k^2}{r}\hat{\mathbf{r}} \times \mathbf{p}\left(t - \frac{r}{c}\right)
\end{aligned}$$

where $k \equiv \omega/c = 2\pi/\lambda$ is the wavenumber. Argue that this result is valid, regardless of the orientation direction of the linear dipole.

c) Use this result to write the Poynting vector $\mathbf{S}$ for the radiating dipole in terms of the magnitude of the magnetic field alone, plus appropriate constants and unit vectors.

d) Superimpose the time dependent magnetic fields from $\mathbf{p}_x(t)$ and $\mathbf{p}_y(t)$, take the time average, and integrate the result over a sphere of radius r to calculate the desired result. Don't forget to comment on the answer, in relation to the result we got in class.