

NAME: \_\_\_\_\_

You have three hours to complete this exam. There are a total of *seven* problems. Solve as much of them as you can, but the point total is greater than 100, so “extra credit” is possible. *Not all the problems are worth the same number of points.*

You may use your textbooks and class notes and handouts, or other books. You *may not* share these resources with another student during the test.

In general, if you can't get the one part of a problem, you can still do at least some of the remaining parts. In other words, *don't give up on a problem too quickly!*

*Indicate any figures or tables you use in your calculations. Show all work!*

GOOD LUCK!

Problem	Score	Worth
1.	_____	20
2.	_____	20
3.	_____	15
4.	_____	20
5.	_____	15
6.	_____	10
7.	_____	20
Total Score:	_____	120

**Problem 1 ( 5+5+10=20 Points):** An *LCR* circuit or a one-dimensional mechanical system behaves according to  $y(t) = Ae^{-\gamma t} + Bte^{-\gamma t}$  where  $A$ ,  $B$ , and  $\gamma$  are constants and  $y(t)$  represents the charge on the capacitor (i.e.  $q$ ) for the circuit, or the position (i.e.  $x$ ) for the mass in the mechanical system.

**a)** Draw a picture of the system (mechanical or electrical, your choice) which might behave this way. Label all components clearly, using symbols to represent important dynamical quantities.

**b)** Write the differential equation that governs the diagram you drew and labeled in **(a)**.

**c)** Determine two necessary relationships among the component values so that this form for  $y(t)$  is in fact a solution of the differential equation.

**Problem 2 (8+7+5=20 points).** A nucleus of mass  $M$  is moving from left to right with speed  $v$ . It then emits a photon of energy  $E$  in the same direction as it is moving. The daughter nucleus of mass  $m$  that results from the decay, is at rest.



a) Find the speed  $v$  in terms of the masses  $M$  and  $m$ .

b) Show that the energy of the original nucleus is given by  $(m^2 + M^2)/(2m)$ .

c) Determine the photon energy  $E$  in terms of the masses  $M$  and  $m$ .

**Problem 3 (4+4+4+3=15 points):** In this course, we solved one particular differential equation over and over again, although with different symbols and values for different physical systems. In one arbitrary choice of symbols, this equation is

$$a \frac{d^2 w}{dz^2} + bw = 0$$

to be solved for the function  $w(z)$ , where neither  $a$  nor  $b$  is zero. Depending on the relative sign of  $a$  and  $b$  (that is  $ab > 0$  or  $ab < 0$ ) the solution has different algebraic forms.

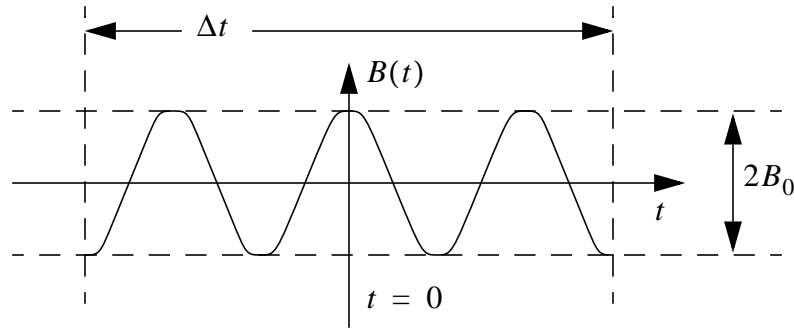
**a)** What is the form of the solution for  $ab > 0$ ? Indicate clearly any constants to be determined by “initial” or “boundary” conditions. If you use any new symbols, define them clearly.

**b)** Determine the complete solution for  $ab > 0$  and with  $w(0) = 1$  and  $w'(0) = 0$ . Eliminate any imaginary numbers from your solution.

**c)** What is the form of the solution for  $ab < 0$ ? Again, indicate clearly any constants to be determined by “initial” or “boundary” conditions, and if you use any new symbols, define them clearly.

**b)** Determine the complete solution for  $ab < 0$  and with  $w(0) = 1$  and  $w(z) \rightarrow 0$  as  $z \rightarrow \infty$ . Eliminate any imaginary numbers from your solution.

**Problem 4 (10+5+5=20 points):** The magnetic field  $B(t)$  of a single photon is represented as



That is, the photon “turns on” at a time  $t = -\frac{\Delta t}{2}$  and “turns off” at  $t = \frac{\Delta t}{2}$ . During the time that the photon is “on”, the magnetic field is given by  $B(t) = B_0 \cos(\omega_0 t)$ .

a) Show that the Fourier Transform of this photon pulse is given by

$$A(\omega) = \frac{B_0}{\sqrt{2\pi}} \left\{ \frac{\sin\left[(\omega - \omega_0)\frac{\Delta t}{2}\right]}{\omega - \omega_0} + \frac{\sin\left[(\omega + \omega_0)\frac{\Delta t}{2}\right]}{\omega + \omega_0} \right\}$$

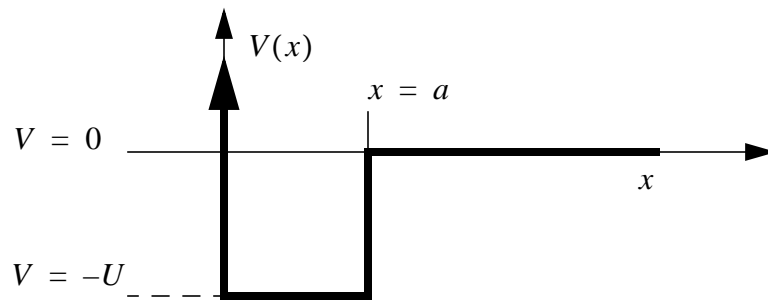
You might want to use the identity  $(\cos a)(\cos b) = [\cos(a - b) + \cos(a + b)]/2$ . Use the extra paper on the next page for your calculation.

b) For a visible light photon, a typical wavelength is  $5.5 \times 10^{-7}$  m. What is the value of  $\omega_0$  for this photon? Why does this mean that only the first term in the expression for  $A(\omega)$  is important?

c) Use the first term in the expression for  $A(\omega)$  to show that an “uncertainty principle” for this “photon” can be written as  $\Delta E \cdot \Delta t \approx 2h$  where  $E$  is the energy of the photon. (Use the distance between the two zero crossings of  $A(\omega)$ , one on either side of  $\omega_0$ , to define the “width” of the Fourier Transform.) What do you suppose is the smallest possible value of  $\Delta E \cdot \Delta t$  ?

## Extra Paper for Problem 4

**Problem 5 (2+3+5+5=15 points):** A particle of mass  $m$  sits in the following potential well:



There is an infinitely high potential wall at  $x = 0$  and the potential remains at zero for  $x \rightarrow \infty$ . The particle is in a bound, stationary quantum mechanical state.

**a)** Sketch a possible energy level on the diagram above.

**b)** Sketch a possible form for the time-independent wave function  $u(x)$  using the set of axes above. Be explicit about the dependence at zero, at  $x = a$ , and for  $x \rightarrow \infty$ .

**c)** Using *two* arbitrary constants, write down the form of the wave function in the regions  $0 < x < a$  and  $x > a$ . If you use any new variables, define them explicitly in terms of  $m$ ,  $U$ ,  $a$ , and any necessary fundamental constants.

**d)** Write the three equations you would need to solve for the two arbitrary constants in (c) and for the energy quantization condition. (*Don't try to solve these equations!*)

**Problem 6 (10 points):** A cube of side length  $a$  sits in the region  $x > 0$ ,  $y > 0$ , and  $z > 0$ , with one corner at the origin, i.e.  $x = y = z = 0$ . It is immersed in an electric field given by  $\vec{E} = Ax\hat{i} + By\hat{j} + Cz\hat{k}$  where  $A$ ,  $B$ , and  $C$  are constants. Find the total charge enclosed in the cube, in terms of  $A$ ,  $B$ , and  $C$ .

**Problem 7 (8+8+4=20 points):** A particle of mass  $m$  forms a pendulum by hanging on a string of length  $l$ . Neglect the mass of the string.

a) Write the Lagrangian function  $L$  for the pendulum in terms of the “coordinate”  $\theta$  which measures the angle the string makes from the vertical.

b) Show that the Euler-Lagrange equation reduces to  $\frac{d^2\theta}{dt^2} + \left(\frac{g}{l}\right)\sin\theta = 0$

c) Under what conditions does the equation in (b) reduce to the form of the equation in Problem3?