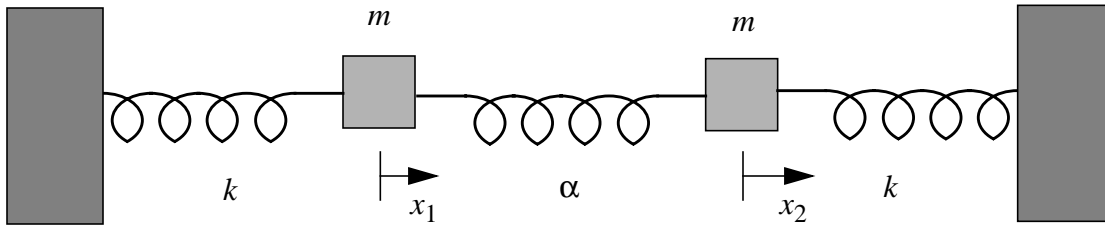


Extra Credit Problem I

This problem is worth two full homework assignments. It will count as extra credit towards your final course grade. It is due Friday, Sept. 11, in the grader's box in the Physics Department.

Consider the following system of coupled oscillators:



The two masses m are equal, as are the spring constants k of the two outer springs. However, the spring constant α of the middle spring is arbitrary. As in class, the positions of the two masses are specified by x_1 and x_2 . When $x_1 = x_2 = 0$ there are no forces on the masses from the springs.

- a) Write down the equations of motion, and show that the two normal modes have angular frequency ω where $\omega^2 = \omega_0^2$ and $\omega^2 = \omega_1^2 = \omega_0^2 + \beta^2$ with $\omega_0^2 = k/m$ and $\beta^2 = (2\alpha)/m$.
- b) As we did in class, show that the two equations of motion decouple when written in terms of $x = (x_1 + x_2)/2$ and $y = x_1 - x_2$.
- c) Assume that oscillations start from rest with $x_1 = x_0$ and $x_2 = 0$. Show that

$$x_1 = x_0 \cos\left(\frac{\omega_0 + \omega_1}{2}t\right) \cos\left(\frac{\omega_1 - \omega_0}{2}t\right)$$

$$x_2 = x_0 \sin\left(\frac{\omega_0 + \omega_1}{2}t\right) \sin\left(\frac{\omega_1 - \omega_0}{2}t\right)$$

- d) Show that the first two terms of the Taylor expansion (about $x=0$) for the function $f(x) = (1 + x)^\delta$ are $1 + \delta x$

- e) Show that for the case of “weak coupling”, i.e. $\alpha \ll k$, the oscillations are given by

$$x_1(t) = x_0 \cos(\omega_0 t) \cos\left(\frac{\beta^2}{4\omega_0}t\right) \quad x_2(t) = x_0 \sin(\omega_0 t) \sin\left(\frac{\beta^2}{4\omega_0}t\right)$$

- f) Make a plot of x_1 and x_2 for $0 < t < (200\pi)/\omega_0$ and using $\beta/\omega_0 = 0.2$.