1 Review For Test Number 1

1.1 Consider the following mechanical problems...

1. Write down the equation of motion (the differential equation) describing the mass-spring system shown in figure 1. What properties of the differential equation can we use to guide us to a solution?

2. If air resistance and friction were to be investigated, how would the equation of motion change? What assumption(s) are used?

3. How is the natural frequency (ω_0) defined for the undamped harmonic oscillator?

4. How is the frequency defined with weak damping (like air resistance)? Does it decrease or increase?

5. What is the general form for the position as a function of time, x(t)? Is there more than one way to represent x(t)? Plot the velocity and acceleration of the mass-spring system.

6. How does a forcing function change the problem? Graph what you expect to see at large times?

7. Calculate the kinetic and potential energies for the undamped harmonic oscillator? What does this tell you about the power lost? Use a simple argument to defend your position.

1.2 Consider the following electrical problems...

1. Write down the equation of motion (the differential equation) that describes the electrical system shown in figure 2.

2. If a resistor network (see figure 3) was to be inserted into the electrical network between points A and B, how does the "equation of motion" change?

3. How is the natural frequency (ω_0) defined for the undamped electrical oscillator?

4. How is the frequency defined by adding a resistor network?

5. What parallels can be drawn from the mechanical and electrical harmonic oscillator systems?

6. How is resistance related to impedance? What is the impedance of a capacitor and an inductor?

7. What is the definition of a resonance? What "quality" factor, Q, determines the "sharpness" of a resonance? How is this quality factor defined?

8. What is the voltage at points B and C in figure 2?

1.3 Consider the following Mathematical problems...

1. Write Euler's formula with argument $\theta = \omega t$.

2. Using De Moivre's Theorem, write the expression for $(cos\,\theta - isin\,\theta)^4$.

3. Taylor expand the following expressions to 4th order...

   (a) $(1+2x)^2$ about the point $x_0 = 1$

   (b) $sin\,\theta$ about the point $\theta_0 = \pi/4$

   (c) $e^{\beta x}$ about the point $x_0 = 0$ here β is a constant

4. Prove the following relationships for complex numbers z_1 and z_2...

   (a) Show $z_1 + z_2 = (z_1^* + z_2^*)^*$

   (b) Show that z_1 can be represented as $re^{i\theta}$. What are r and θ in terms of x and y?

   (c) Show that $cos\,\theta$ and $sin\,\theta$ can be derived from the exponentials $e^{i\theta}$ and $e^{-i\theta}$.

5. Take the the all first and second derivatives of the following functions...

   (a) $f(t) = A_+ cos(n\omega t)$

   (b) $y(x) = x^2 + x + 1$

   (c) $F(x,y) = x^2 sin(xy)$

   (d) $y(x,t) = \sum_{n=1}^{\infty} B_n \left(\frac{2}{L}\right)^{\frac{1}{2}} sin(k_n x) sin(\omega_n t)$

   (e) $y(x) = x^x$ HINT: Write y(x) as an exponential first

1.4 Consider the following graphs...

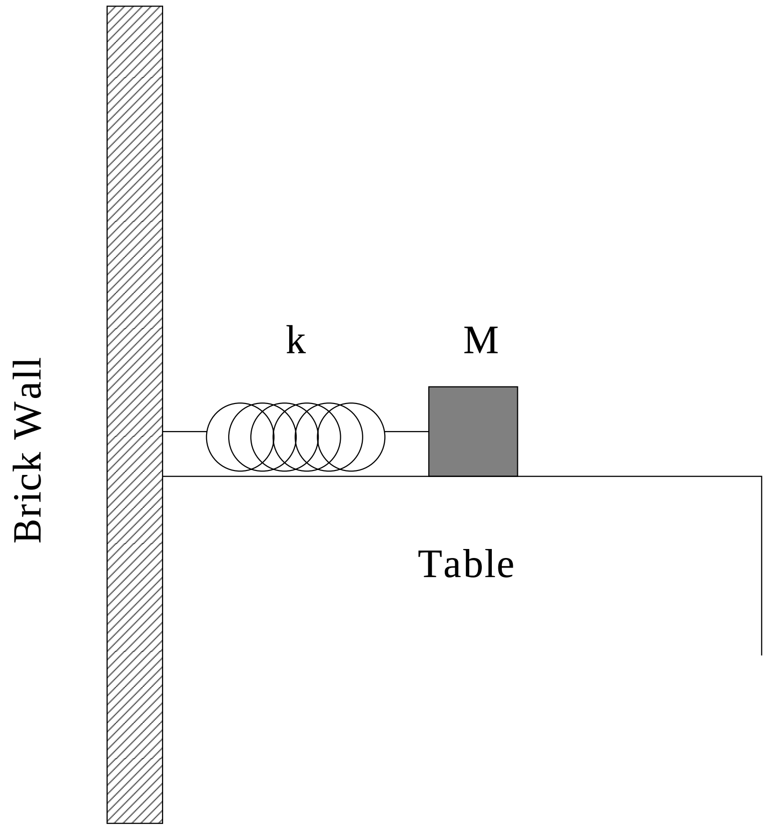
1. Make an odd extension of the graph in picture #1. What can be said about the Fourier coefficients?

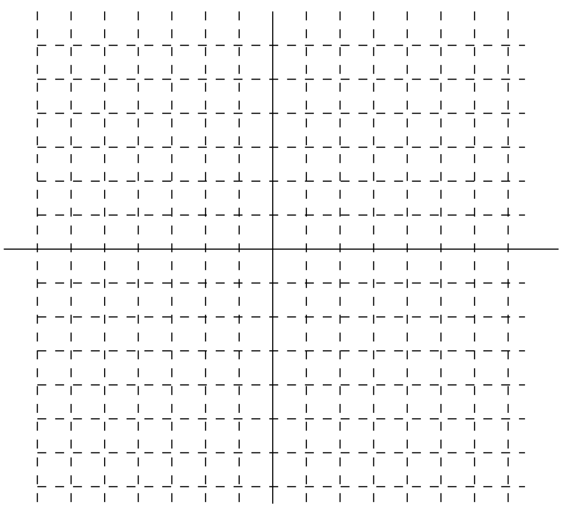
2. Make an even extension of the graph in picture #2. What can be said about the Fourier coefficients?

3. Make an even extension of the graph in picture #3. What can be said about the Fourier coefficients?

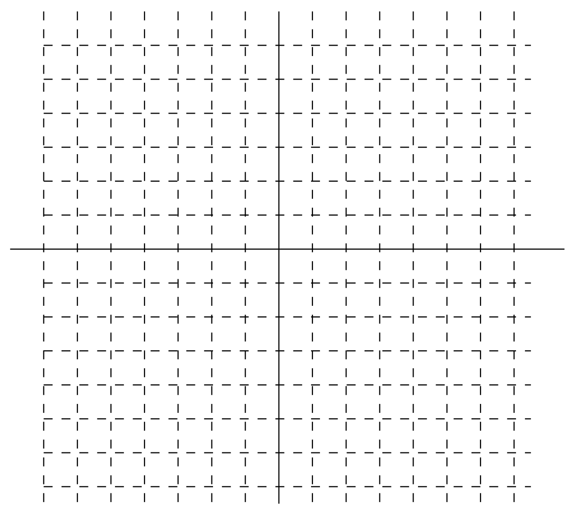
4. Make an odd extension of the graph in picture #4. What can be said about the Fourier coefficients?

Figure #1





Velocity



Acceleration

Figure #2

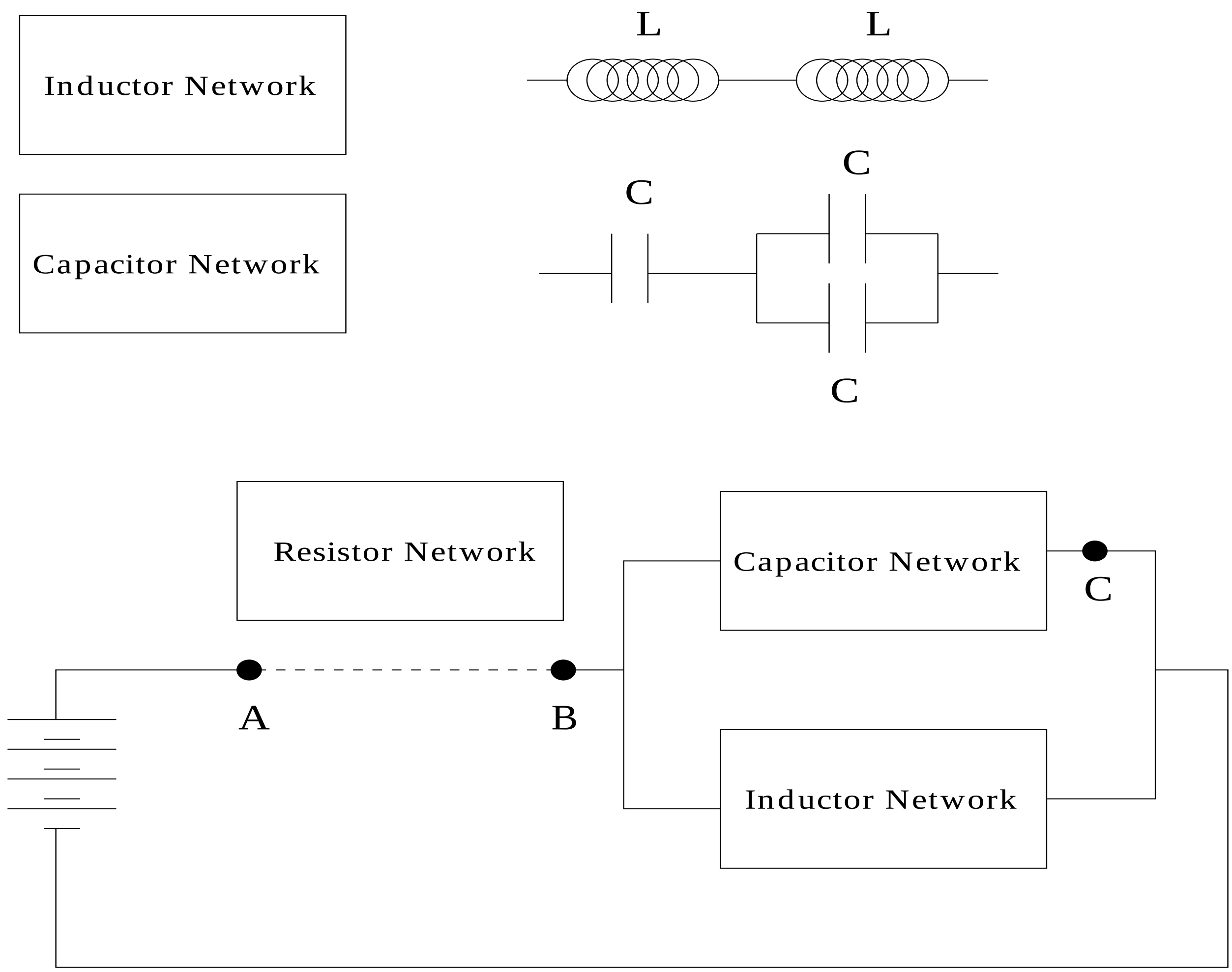
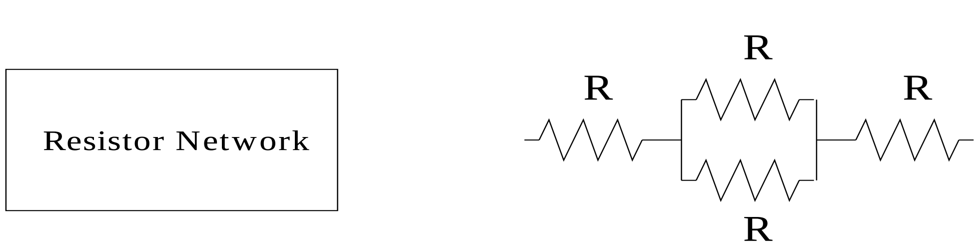
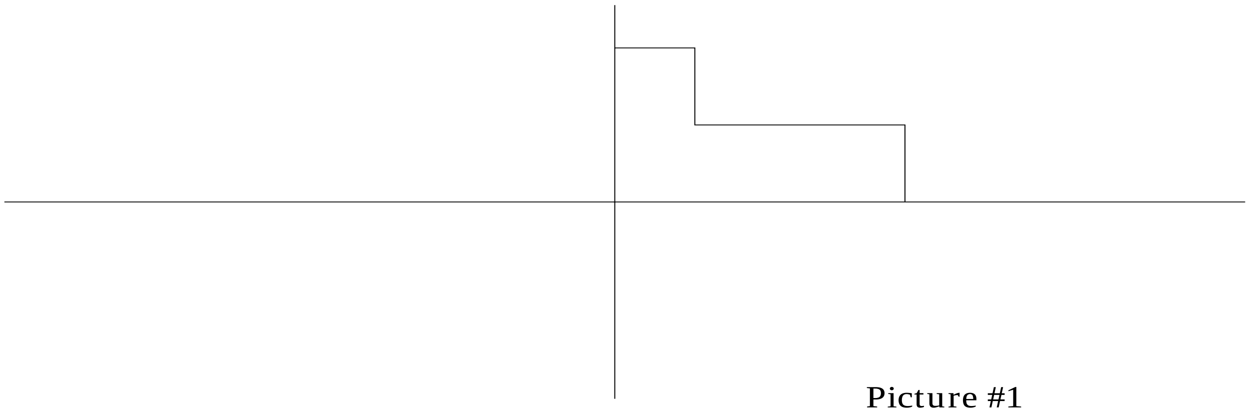
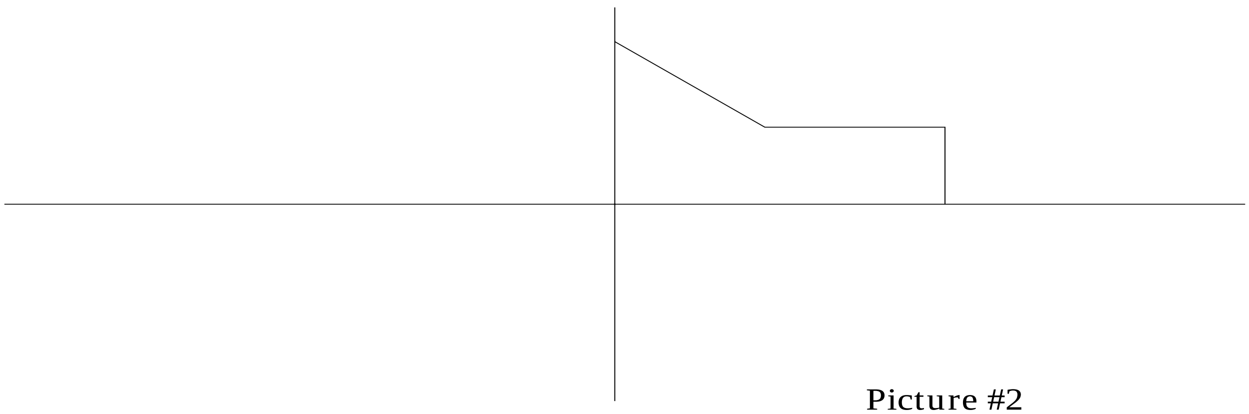


Figure #3

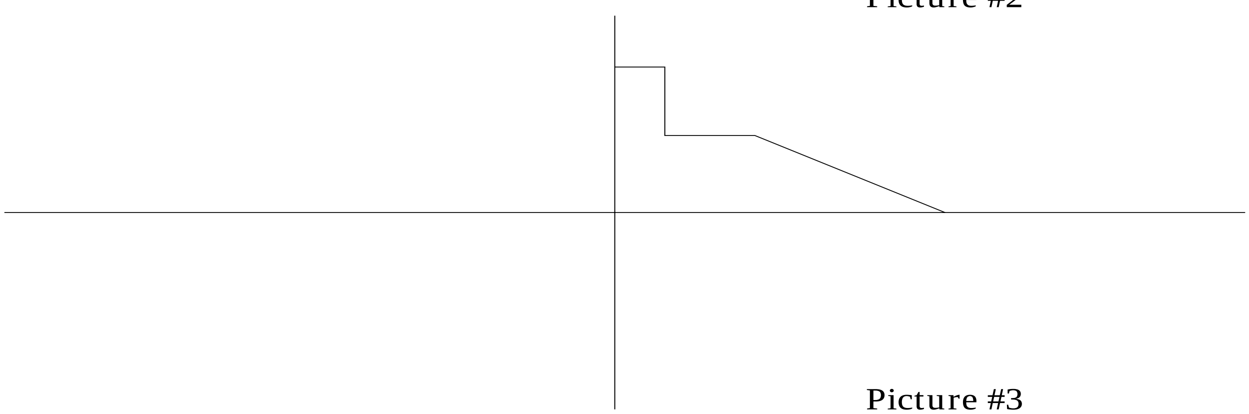




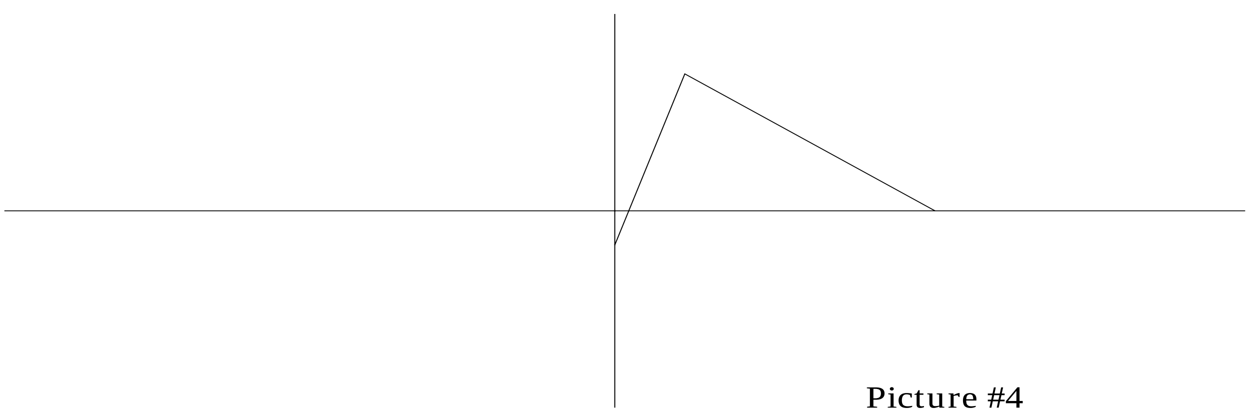
Picture #1



Picture #2



Picture #3



Picture #4