Curve Fitting to a Trajectory

The point of this laboratory is to fit data to a model curve. Your data will be taken from a video clip of a ball flying freely through the air, given some initial velocity vector. In the end you should have a measurement of g, the acceleration due to gravity, along with some uncertainty for your measurement. That is, you should conclude by reporting $g \pm \delta g$. A second goal of this laboratory is to start you off using the data acquisition software that we'll use down the line. The software is called LoggerPro and is a commercial product that runs under Windows or MacOS.

Work in pairs if you like, but this laboratory can be done independently. You need to download three things: (1) LoggerPro, which you will use for other experiments; (2) the projectile video clip; and (3) the file Projectile.xmbl which sets you up to use LoggerPro for this exercise. Execute LoggerPro using Projectile.xmbl and you will advance the video one frame at a time. In each frame, use the cursor to find the position of the ball in two dimensions.

You will end up with three columns of numbers ("Time", "x", and "y") each with as many rows as there are frames, i.e. fourteen "data points". Store your data in whatever form you like, even if you just write it down in your lab book.

Now you can analyze the data. We derived equations for projectile motion in class. (See also Example 1.10 of your textbook.) These are

$$\begin{aligned} x(t) &= v_{x_0} t \\ y(t) &= v_{y_0} t - \frac{1}{2} g t^2 \\ \text{and} \qquad y &= \frac{v_{y_0}}{v_{x_0}} x - \frac{1}{2} \frac{g}{v_{x_0}^2} x^2 \end{aligned}$$

where we have arranged things so that $x = y = 0$ at $t = 0$. You can easily incorporate this constraint by subtracting the first of your fourteen data rows from the other thirteen.

Now plot your data as x versus t, y versus t, or y versus x. You can use Excel, Matlab, some other plotting program of your choice, or simply make a plot on the graph paper in your lab book. Try drawing curves through those points according to the equations above. A linear "curve" is simple. For a quadratic like $y = ax + bx^2$ you can pick two of your six (x, y) data points, insert them into the quadratic equation, and solve for a and b. Different pairs of data points will give you (slightly) different values of a and b, and you can use that to estimate your uncertainties in a and b. (Of course, you can do the same thing with y versus t data.)

Use your fitted values to determine v_{x_0}, v_{y_0}, and most importantly g. You will have to give some thought as to how to combine the uncertainties to come up with a value for δg.

It is possible to use "fitting programs" to come up with values of a and b, but I don't recommend using them right now. For one thing, you cannot get an uncertainty easily out of these programs. For another, I think you are better off trying it yourself first, before handing it over to a fitting program. Nevertheless, fitting programs are important tools in data analysis, and we will be talking about these some more.