Gravitational and Inertial Mass

Physics Introduction. This lab is different than the ones that have come before. You've now learned a lot about how to make careful measurements and to assess measurement uncertainty. Your goal with this lab is to use what you've learned to test an important conjecture in physics, namely Einstein's Equivalence Principle.

Briefly stated, the Equivalence principle says that "inertial mass" is the same thing as "gravitational mass", something which is taken for granted in just about all elementary physics books. Inertial mass m_I is what appears in Newton's Second Law, that is

$$\sum \mathbf{F} = m_I \mathbf{a}$$

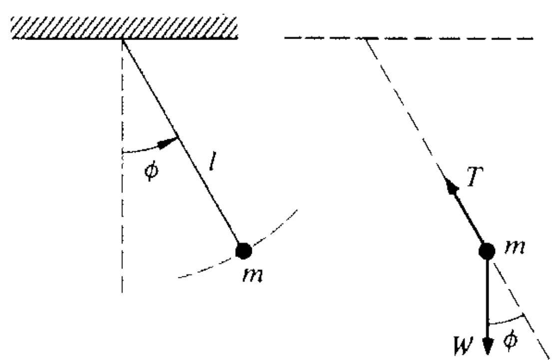
Gravitational mass m_G is what appears in Newton's law of gravity, that is

$$\mathbf{F} = -G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}} \qquad \text{or} \qquad F = m_G g$$

downward near the surface of the Earth. In other words, the downward acceleration from Earth's gravity is $a = (m_I/m_G)g$. The ratio m_I/m_G can, in principle. depend on how big is the mass or what material makes it up.

Your goal in this laboratory is to test the extent to which these two masses are the same, that is $m_I/m_G \stackrel{?}{=} 1$. You'll need to consider sources of uncertainty carefully, otherwise you are liable to discover that gravitational mass and inertial mass are not the same thing!

The experiment will make use of the pendulum. See your textbook, "The Simple Pendulum", a subheading in Section 6.6. You might also consult *The Pendulum: Rich physics from a simple system*, by Robert A. Nelson and M. G. Olson, Am.J.Phys. 54(1986)112.



The forces acting on the pendulum bob are the tension $\mathbf{T}$ and the gravitational force $\mathbf{W} = m_G\mathbf{g}$. The torque about the suspension point is zero from $\mathbf{T}$ and $lW\sin\phi$ from $\mathbf{W}$. Rotational inertia is $I = m_I l^2$ and the angular acceleration is $\alpha = \ddot{\phi}$. Therefore

$$\tau = I\alpha \qquad \Longrightarrow \qquad l m_G g \sin\phi = m_I l^2 \ddot{\phi}$$

Using the "small angle" approximation in which $\sin\phi \approx \phi$, the equation of motion is

$$\ddot{\phi} = -\left(\frac{m_G}{m_I}\right)\left(\frac{g}{l}\right)\phi$$

which is differential equation now well known to you. The solution is

$$\phi(t) = A\cos(\omega t + \alpha) \qquad \text{where} \qquad \omega = \sqrt{\left(\frac{m_G}{m_I}\right)\left(\frac{g}{l}\right)}$$

is the angular frequency of the pendulum.The pendulum period is

$$T = 2\pi\sqrt{\left(\frac{m_I}{m_G}\right)\left(\frac{l}{g}\right)} \tag{1}$$

Experimental Arrangement and Measurement. The experimental setup is simple. You will need to collect from the front of the room a ring stand to clamp to your desk, a length of string, and a mass for the pendulum bob. (Actually, you will be making measurements with several different bobs, but just pick one to start.) You should also pick up a meter stick for measuring the pendulum length. You will also need a stop watch for measuring the period. You do not need your computer to take data in this lab.

Measure the period of the pendulum carefully. Let the pendulum swing many times and measure the total time, then divide by the number of swings to get the period with a small uncertainty. Record the length of the string, and also the mass of the bob.

First confirm that your measurements are sound. See if you get a good value for g by assuming $m_I = m_G$ and using your values for l and T in Eq. 1. You might try to measure T as a function of l and determine the slope on a graph of T^2 *vs.* l, in order to get a particularly precise value of g. However, it will be hard to be very accurate, as there are many systematic errors. What is a tough one that you encounter right away? (You might take a look at the discussion of Kater's Pendulum, Example 6.12 in your textbook.)

Note that I've included a link on the website to a paper by N. Heckenberg which discusses in great detail, how you can make careful measurements of a pendulum and analyze the uncertainties.

Now try to test the Equivalence Principle directly. Measure the pendulum period as accurately as you can for different types and masses of bobs. Are the differences between different period measurements outside of your uncertainties? Maybe you can measure the period as a function of mass of the bob, and plot T versus m. Can you see a systematic dependence? If so, can you think of where it may come from, if not from a violation of the Equivalence Principle?

You can also try to put limits on the how different is the ratio m_I/m_G from unity. Rather than trying to extract it directly from Eq. 1, you can be more sensitive by studying the *dependence* on m_I/m_G as a function of the mass of the bob, or the material from which it is made. That is, see if the value of g which you extract depends on the mass in any systematic way. You may want to define the quantity

$$\Delta \equiv \frac{m_G - m_I}{m_G} = 1 - \frac{m_I}{m_G}$$

and study $\Delta(m)$ to see if it is not constant. (Here m is just the nominal mass of the bob.) You might fit your data to a straight line to find

$$\Delta(m) = a + bm$$

using a standard value for g. A nonzero value of a is just an indication of an inaccurate measurement of l or T, but the equivalence principle should imply that $b = 0$, to within uncertainties. It will be helpful to keep your data in a spreadsheet or some other application on your computer, so that you can manipulate it easily.

Of course, if you convince yourself that your measurement for Δ shows a nonzero value outside the limits of your uncertainties, then you've discovered that the equivalence principle is wrong! Go to Stockholm to accept your Nobel Prize.