Lattices and Strings

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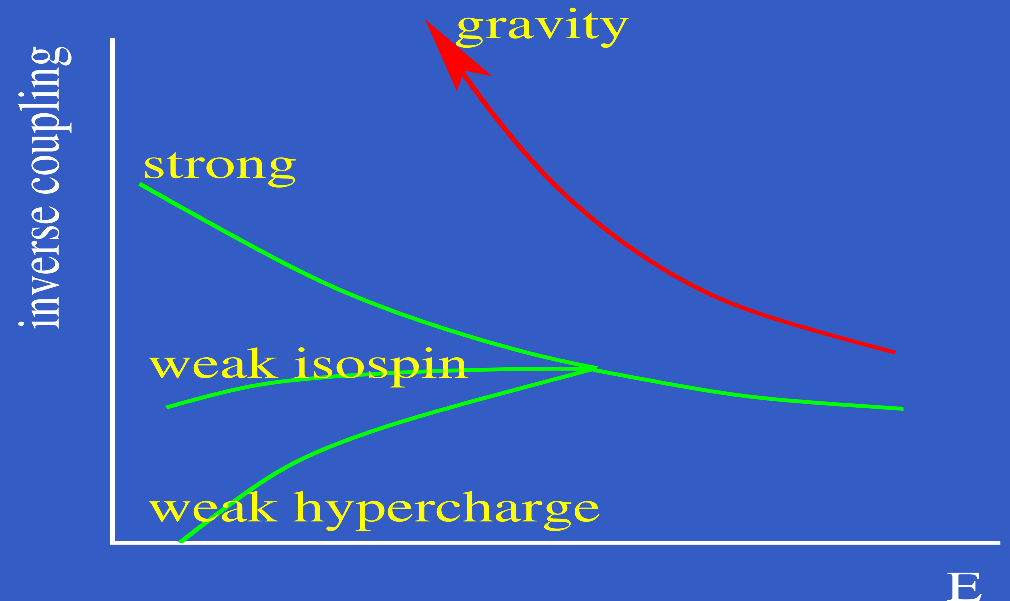
Why strings?

- Because we need a consistent theory of gravity.
- Einstein's gravity (general relativity) suffers from:
  - Cannot hide quantum ∞'s (nonrenormalizable).
  - No unification.



Why lattices?

Because we need a nonperturbative tool for strongly interacting particle physics systems:

- Quantum chromodynamics (QCD).
- Strong electroweak symmetry breaking (strong-EWSB).
- Other strong SB in extensions to standard model.
- Compositeness at the TeV scale.

Example of perturbative

Electron + positron collide to form new fermion-antifermion pair:

$$e^+e^- \longrightarrow \mu^+\mu^-$$

$$\text{probability} = c_1 e^4 + c_2 e^6 + \mathcal{O}(e^8)$$

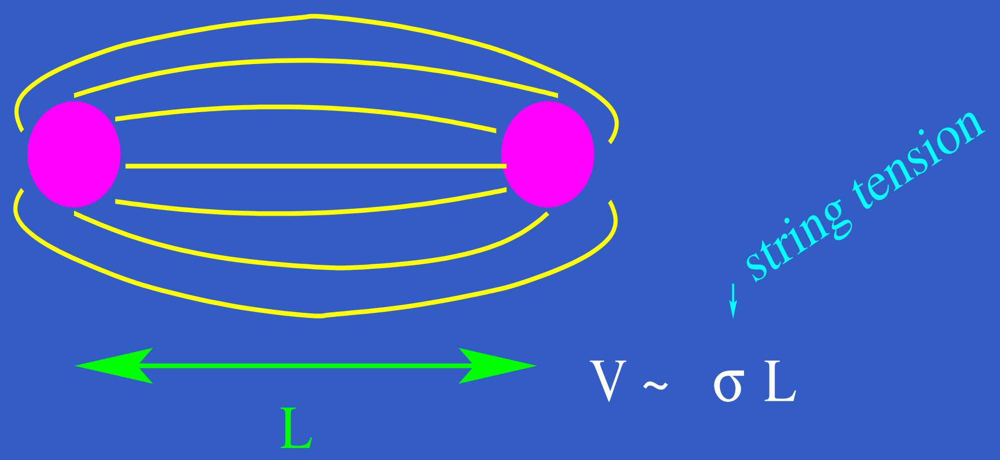
Works because $e^2 \ll 1$.

Example of nonperturbative

Static quark potential in strong coupling limit:

$$V(L) = \sigma L$$

$$\sigma = -\ln \frac{3}{2g^2} - \frac{81}{4g^8} + \mathcal{O}(g^{-12})$$



Why lattices?

- Essential singularities completely missed by perturbation series:

$$e^{-C/g^2} \not\sim c_0 + c_1 g^2 + c_2 g^4 + \cdots$$

  as $g \to 0$.

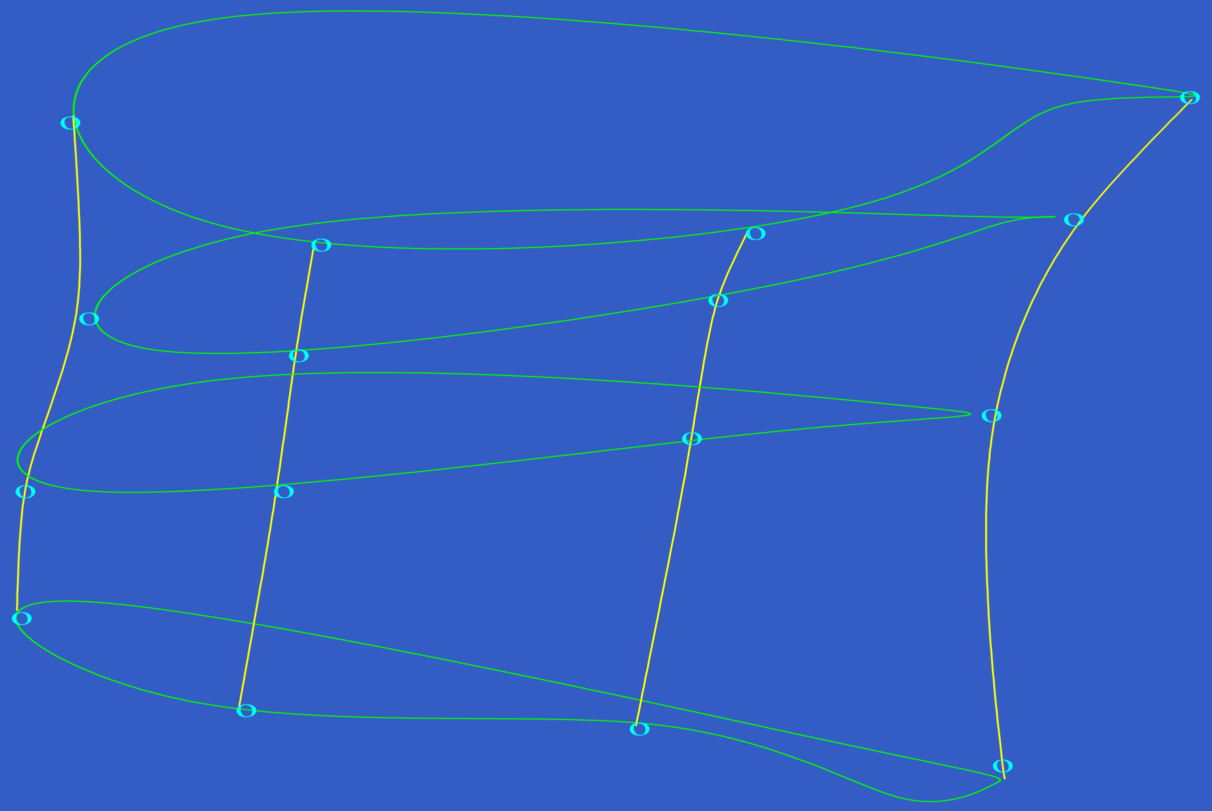
- Nonperturbative effects can be large:

$$e^{-C/g^2} \sim 1$$

  if $g^2 \sim C$.

What is a string?

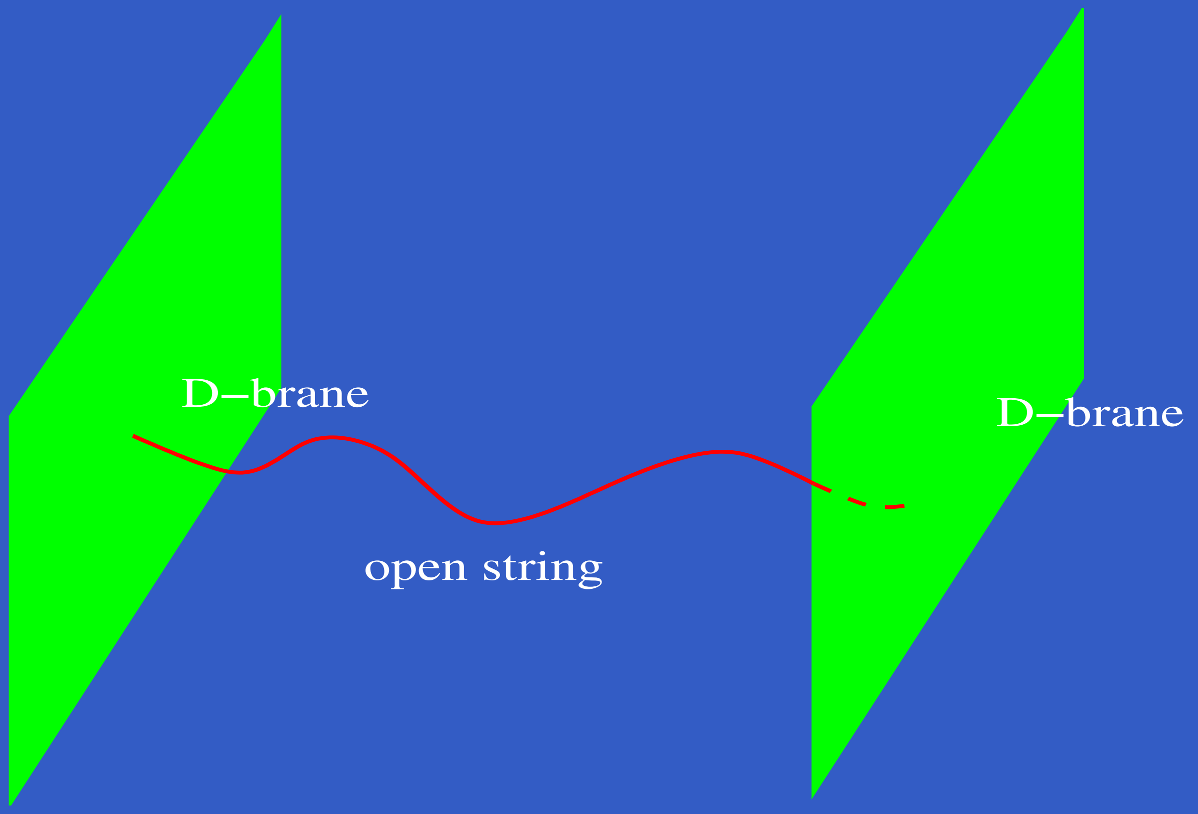
Closed string = loops tied together in time:



Yellow lines trace points on the strings (green) as they move in time.

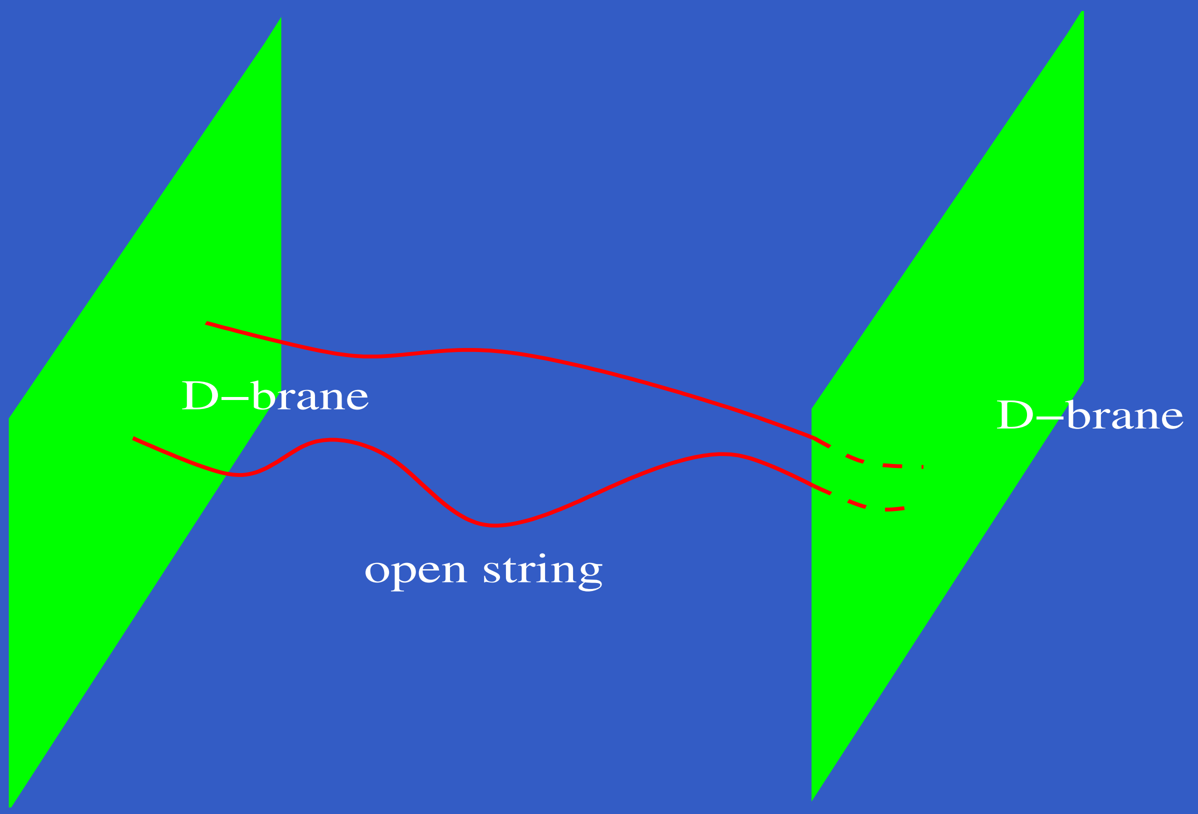
What is a string?

Open string = line ending on D-branes, sweeps out 2d sheet:



What is a string?

Ends can move on D-branes, w/ interesting results. Say for two strings:



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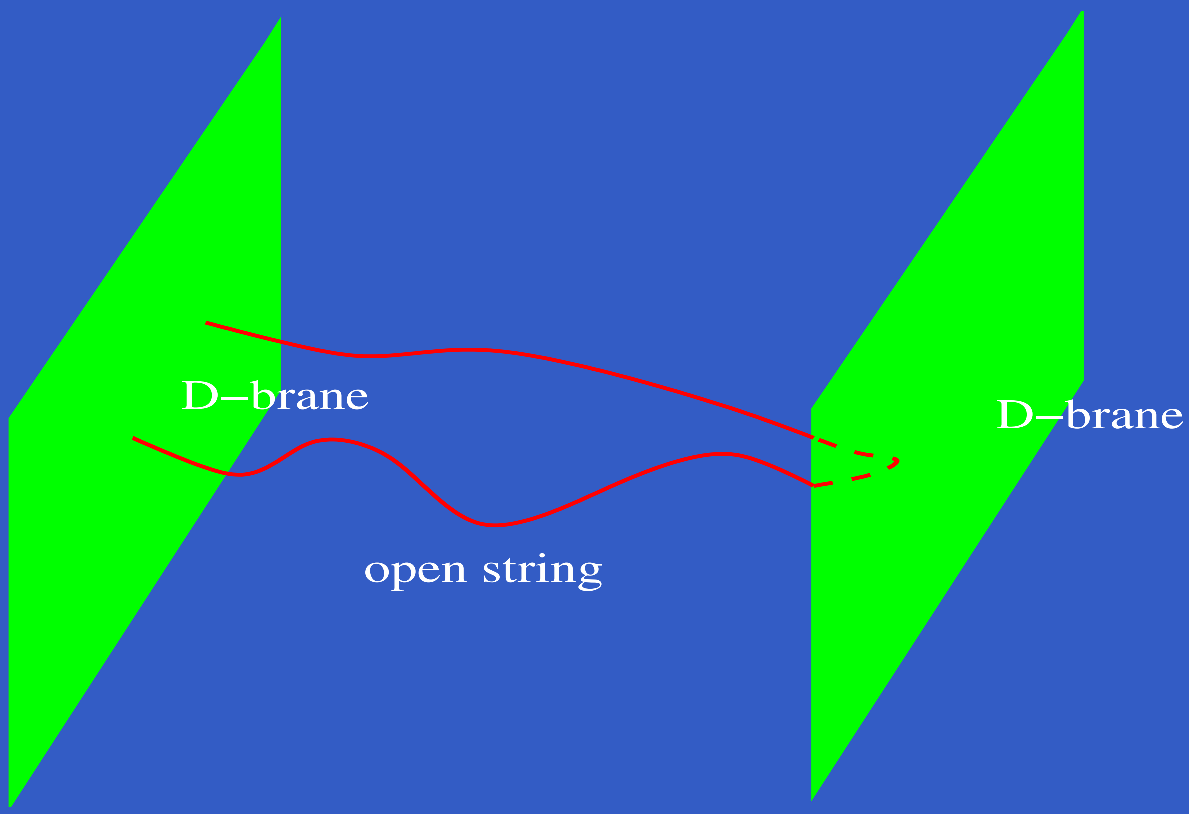

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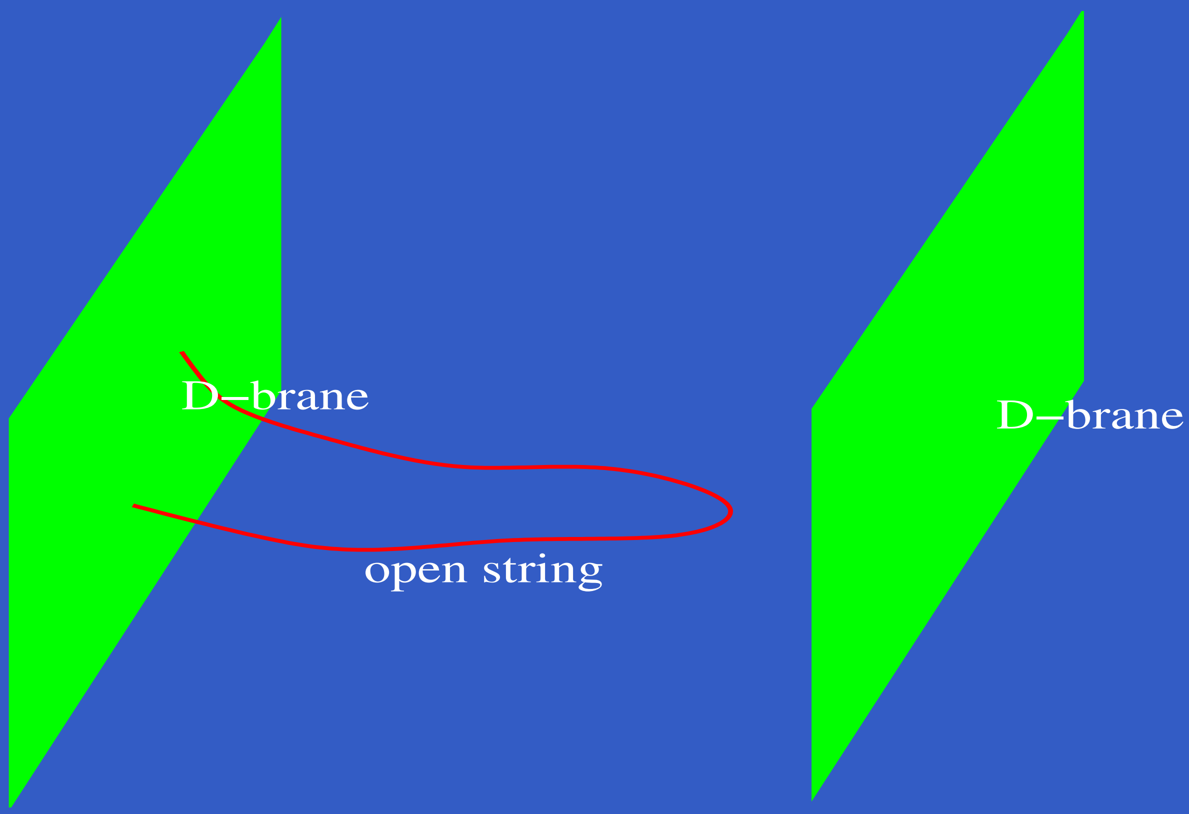

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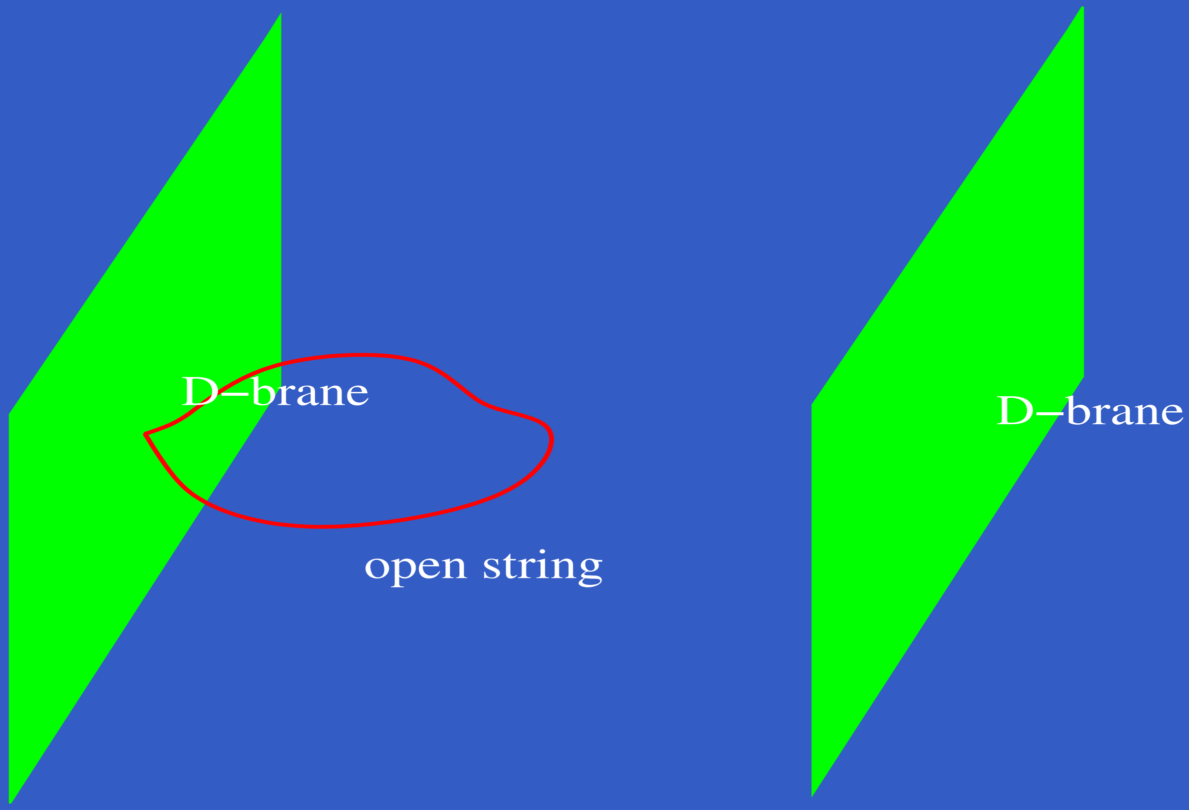

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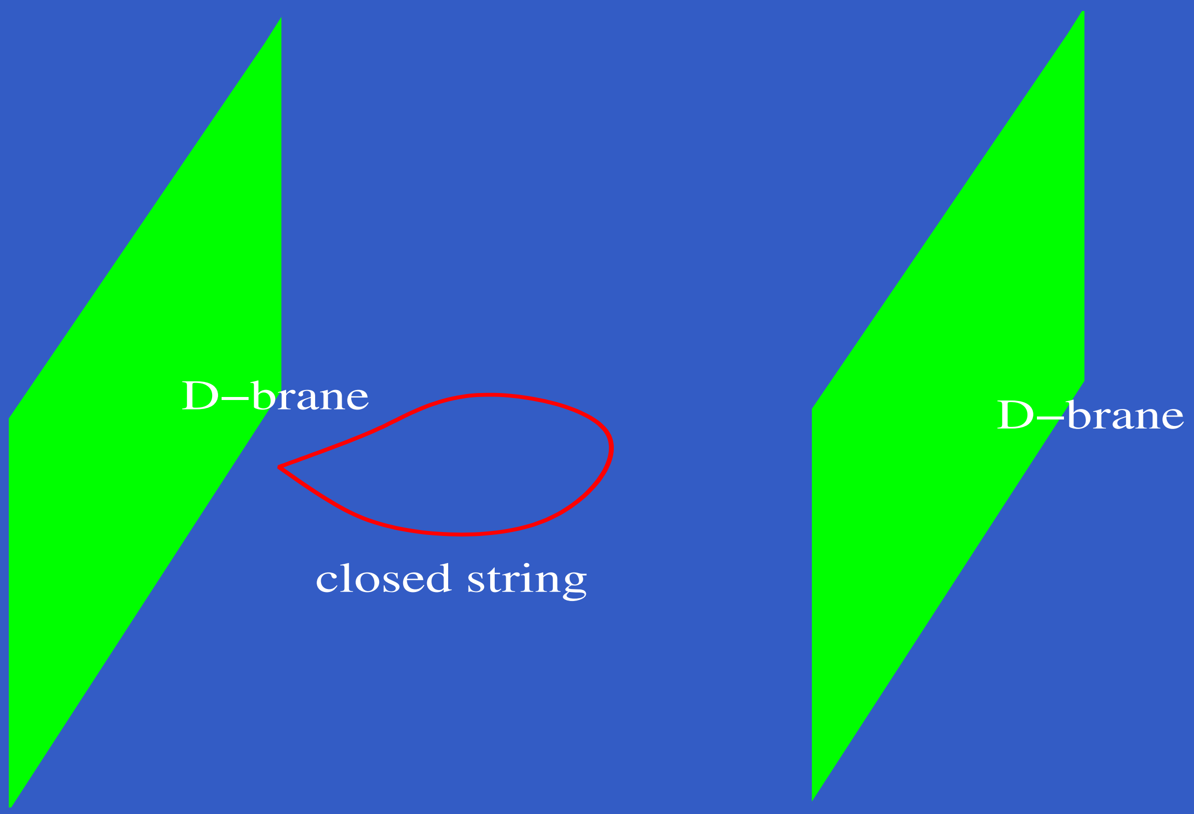


What is a string?

Something with an action/Lagrangian & least action principle:

$$
\begin{aligned}
S &= \int d\tau d\sigma \mathcal{L}[t(\tau,\sigma), \vec{x}(\tau,\sigma), \\
&\qquad \dot{t}(\tau,\sigma), \dot{\vec{x}}(\tau,\sigma), t'(\tau,\sigma), \vec{x}'(\tau,\sigma)] \\
\delta S &= \text{wiggle}(t, \vec{x}) = 0.
\end{aligned}
$$

What is S?

It is amazingly simple:

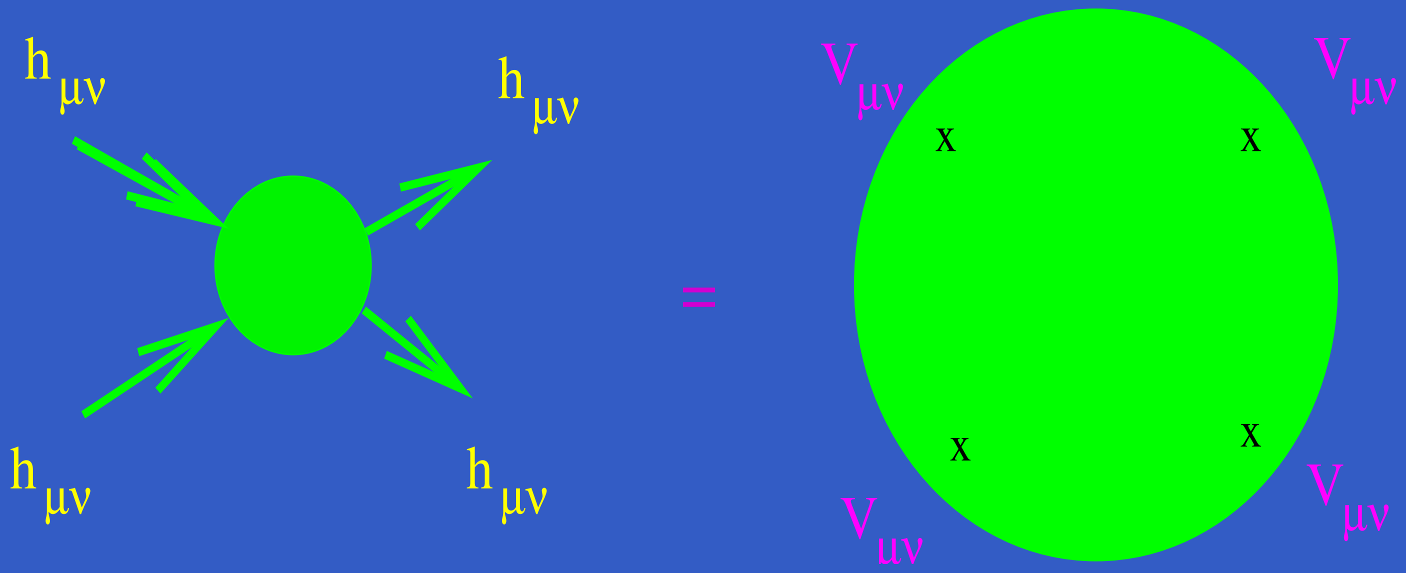
$S =$ Area of 2d surface.

What does this have to do with gravity!?

- There is a spin-2 excitation for the string.
- (Spin is the quantum—allowed discrete unit—of intrinsic angular momentum: $0, \frac{1}{2}\hbar, \hbar, \frac{3}{2}\hbar, \ldots$.)
- Its interactions are determined by the string action.

What does this have to do with gravity!?

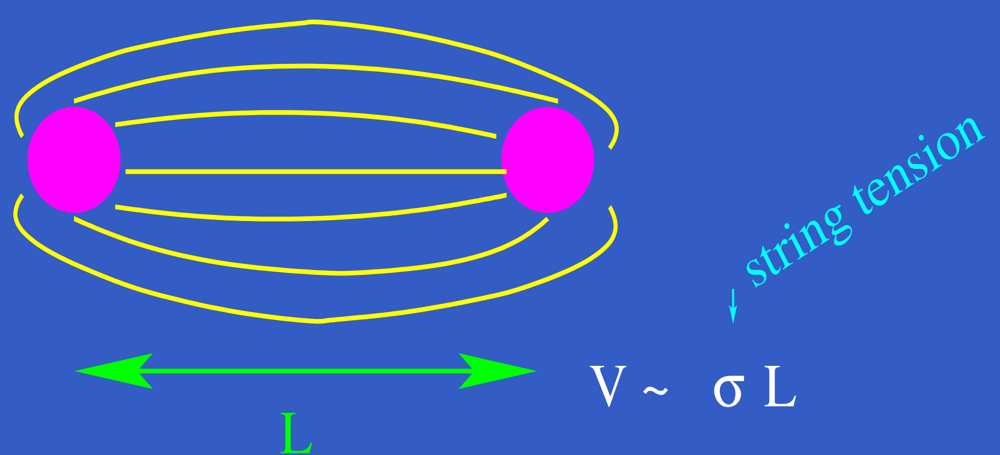
- Scattering of the low momentum modes of this spin-2 excitation is identical to scattering of the graviton in Einstein's gravity.



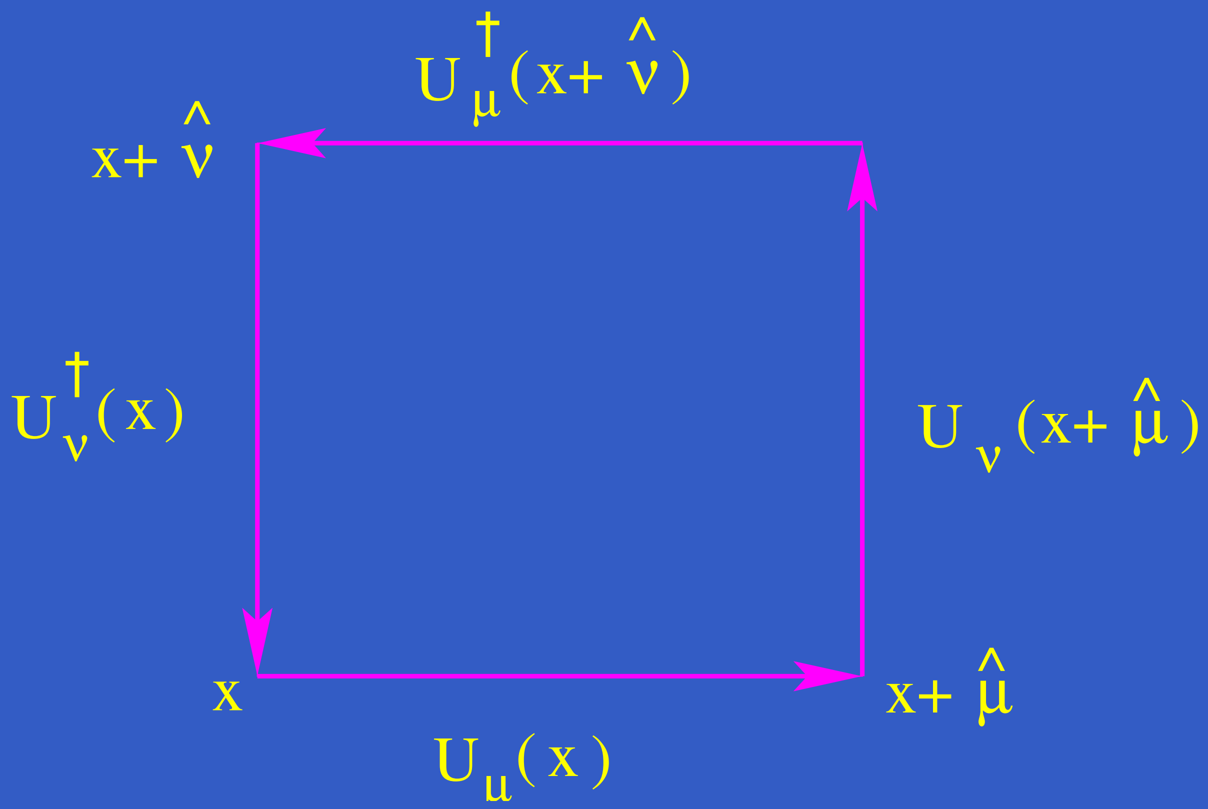
- Graviton, like photon, is quantum of field amplitude for given mode.

Confinement: another string

- QCD = theory of “color charge” (quarks, gluons).
- All states we observe are color neutral.
- Short distance: evidence of colored constituents.
- Color is “confined” within color neutral objects: mesons, baryons, glueballs, exotics...



Lattice Yang-Mills: exact gauge invariance [Wilson 74]



Link $U_\mu(x)$ points in $\hat{\mu}$ direction from site x.
The $\mu\nu$ plaquette:

$$U_{\mu\nu}(x) = U_\mu(x) U_\nu(x+\hat{\mu}) U^\dagger_\mu(x+\hat{\nu}) U^\dagger_\nu(x)$$

Continuum limit: $A_\mu = (\phi, \vec{A})$ hiding in U_μ

$$U_\mu(x) = \exp(iagA_\mu(x)), \quad A_\mu = \sum_{a=1}^{n^2-1} A_\mu^a T^a$$

$$\begin{aligned} U_{\mu\nu}(x) &= \exp[iag(A_\nu(x+\hat{\mu}) - A_\nu(x)) \\ &\quad - iag(A_\mu(x+\hat{\nu}) - A_\mu(x)) + \mathcal{O}(a^2)] \\ &= \exp[ia^2 g F_{\mu\nu}(x) + \mathcal{O}(a^4)] \end{aligned}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \sim (\vec{E}^a, \vec{B}^a).$$

Color electric $\vec{E}^a$, color magnetic $\vec{B}^a$.

Continuum limit

- Continuum action obtained from:

$$S = \sum_{x,\mu\nu} \frac{2n}{g^2}[1 - (1/n)\,\text{Re Tr}\,U_{\mu\nu}(x)] = -\sum_{x,\mu\nu} \frac{a^4}{4} F_{\mu\nu}(x) F_{\mu\nu}(x) + \mathcal{O}(a^8)$$

$$\sum_x a^4 \to \int d^4x$$

Gauge invariance

$$U_\mu(x) \to g^\dagger(x) U_\mu(x) g(x+\hat{\mu})$$

Since $U^\dagger U = 1$,

$$g(x) = U_\mu(x), \;\; g(x+\hat{\mu}) = 1 \;\; \Rightarrow \;\; U_\mu(x) \to 1.$$

- N^4 sites $\to N^4$ g's.
- $\mu = 1, 2, 3, 4 \to 4N^4$ U's.
- Can only fix 1/4 of links to 1.

E.g., $U_4(x) \equiv 1 \to A_4(x) = 0$ (temporal gauge).

Super-Yang-Mills

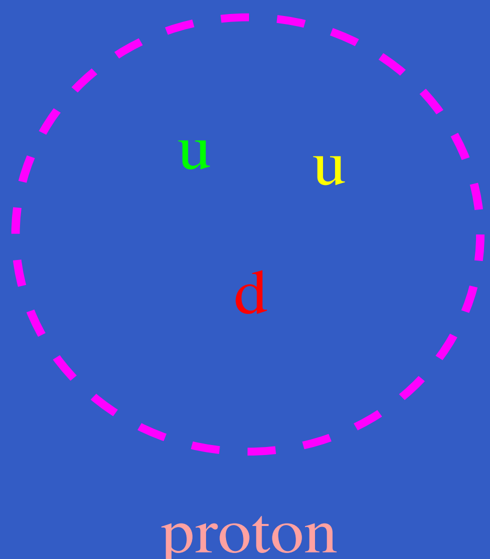
- Introduce fermions $\psi = \psi^a T^a$ too:

$$\Delta S = \frac{1}{g^2} \bar{\psi} \, \not{D}(U) \psi$$

- Exact gauge invariance $\Rightarrow m_A = 0$.
- Supersymmetry $\Rightarrow m_\psi = m_A$.
- Then any lattice formulation that protects $m_\psi = 0$ will give super-Yang-Mills.
- Domain-Wall Fermions: "expensive", use CCNI.
- Physics Goal: spectrum of bound states (glueballs, gluinoballs).

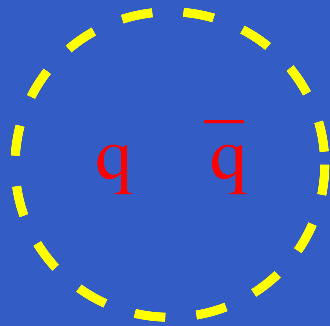
Strong chiral gauge theories

- One way to solve the "hierarchy problem" of the Standard Model [Weinberg, Susskind].
- Some standard Model particles are bound states (composites) of a strongly interacting theory.
- Similar to how proton is a composite of quarks.



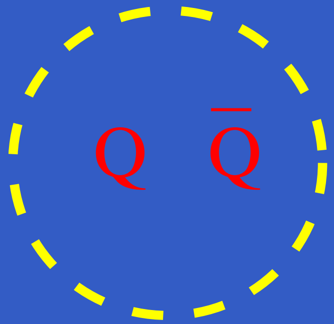
Higgs as a Nambu-Goldstone Boson

The pion is a better analogy.



pion (NGB)

q = u, d



Higgs (NGB)

Q = techniquark

Lattice as a tool

- Because the theory is strongly coupled, it is difficult to predict the spectrum and interactions of these states.
- Thus we would like to have the lattice as a tool.
- However, it is an unsolved problem how to formulate chiral gauge theories on the lattice.

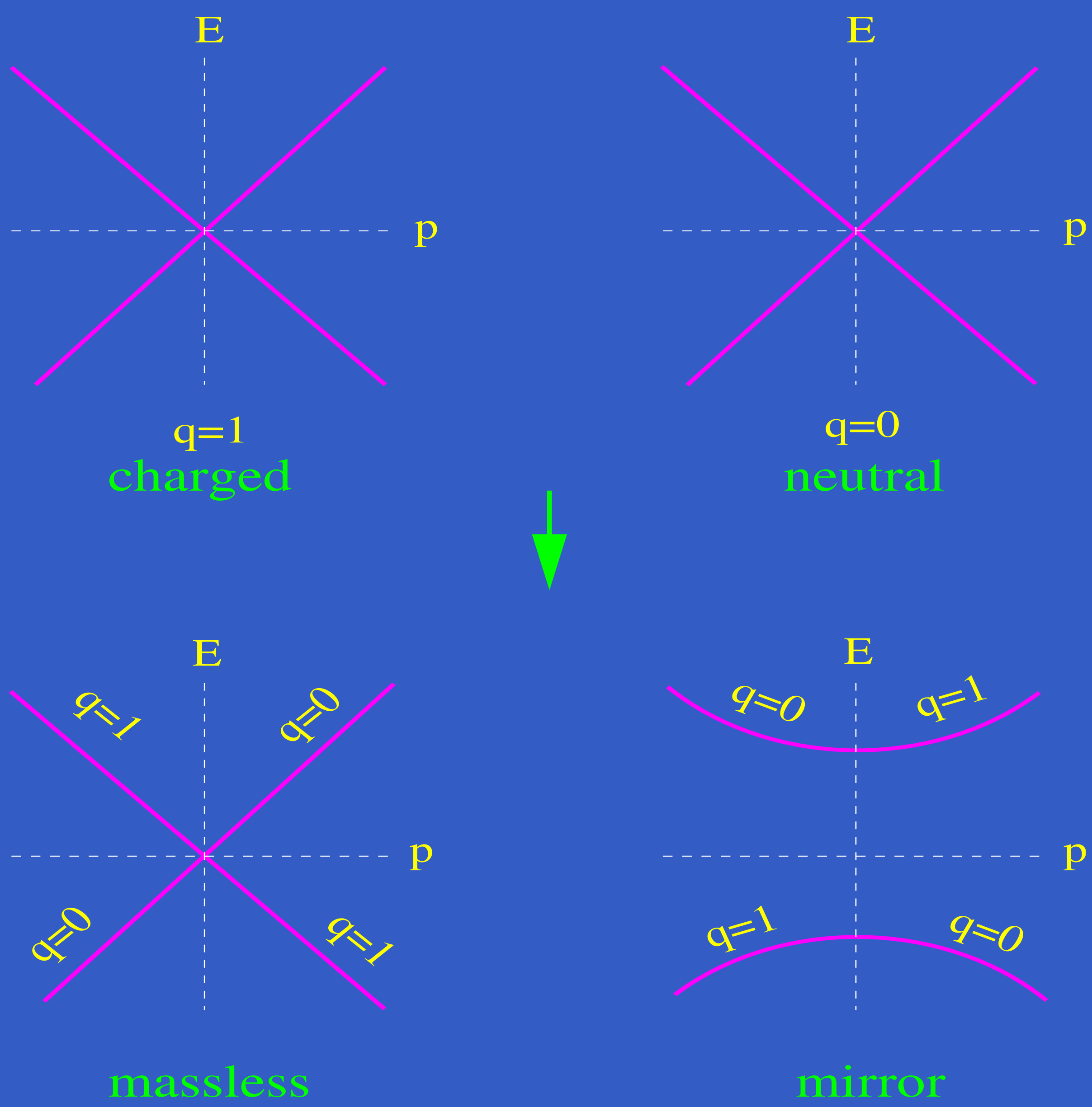
Splitting left and right

- In work with Erich Poppitz, I am currently studying a promising, recent proposal [Bhattacharya, Martin, Poppitz 06].
- This (1+1)-dimensional model has interacting fermions and bosons.
- A dispersion relation tells you how the energy of a state depends on momentum. E.g.:

$$E(p) = \pm\sqrt{p^2c^2 + m^2c^4}$$

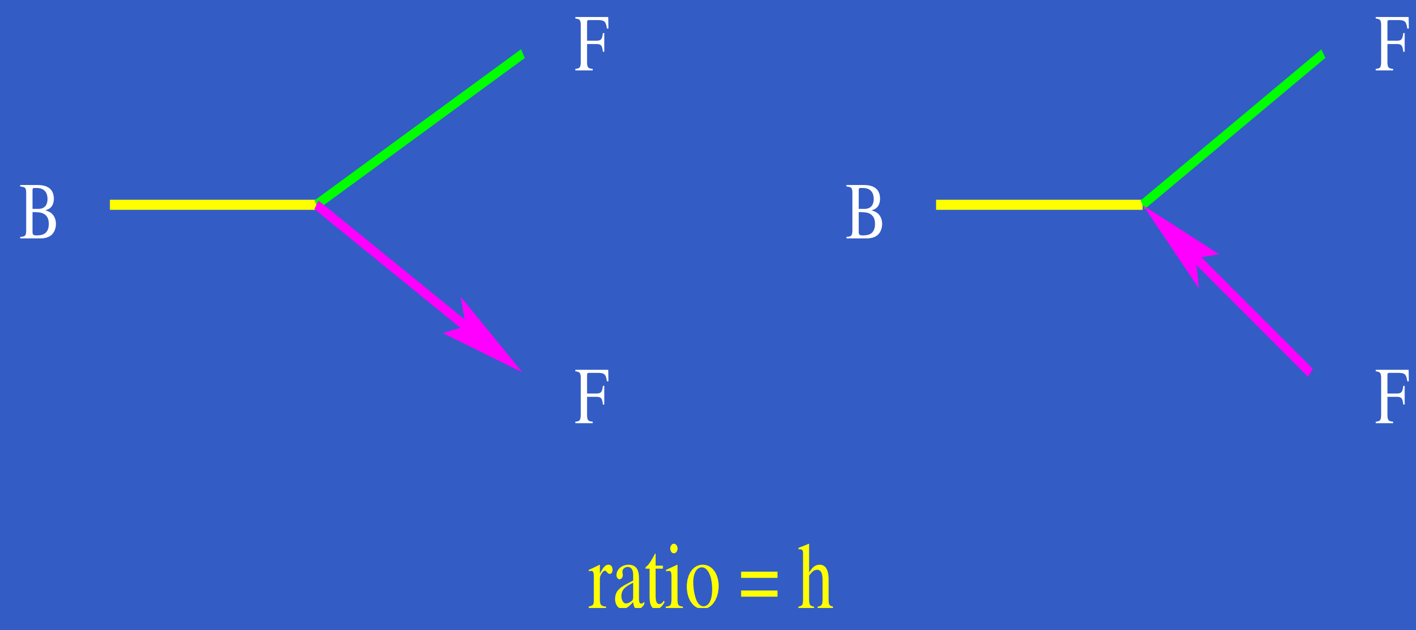
- For $m = 0$, there is a linear dispersion: $E \sim p$.
- For $m \neq 0$, the dispersion is hyperbolic: $E^2 - p^2c^2 = \text{const}$.

Effect of bosons on fermion modes

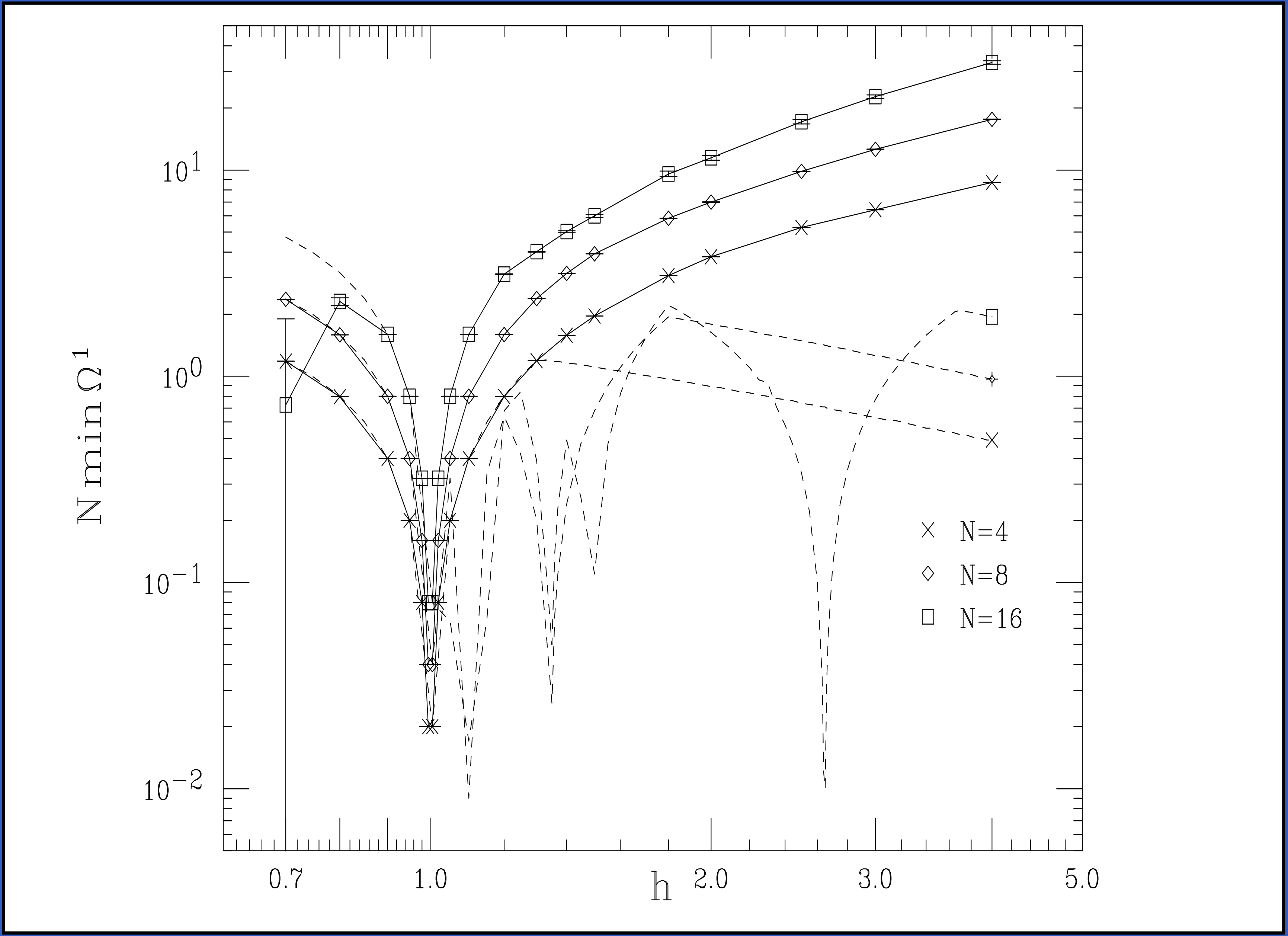


The mirror is massive

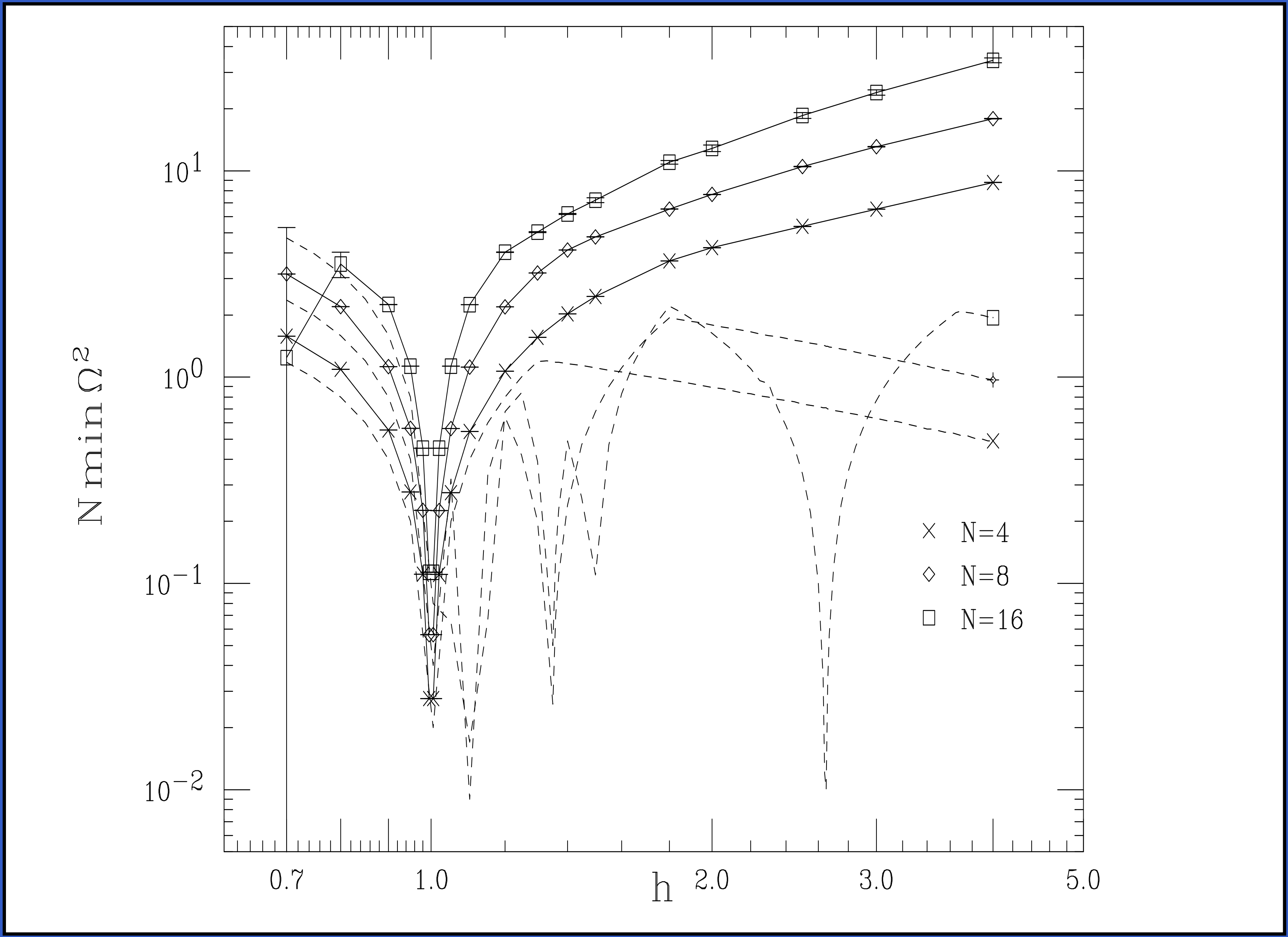
- We have confirmed this picture with Monte Carlo lattice simulations.
- The mirror fermions get masses $\mathcal{O}(a^{-1}) \to \infty$, as had been hoped for.
- There are two types of Yukawa couplings, and a relative strength h:



Neutral energy eigenvalue of mirror (magnitude)



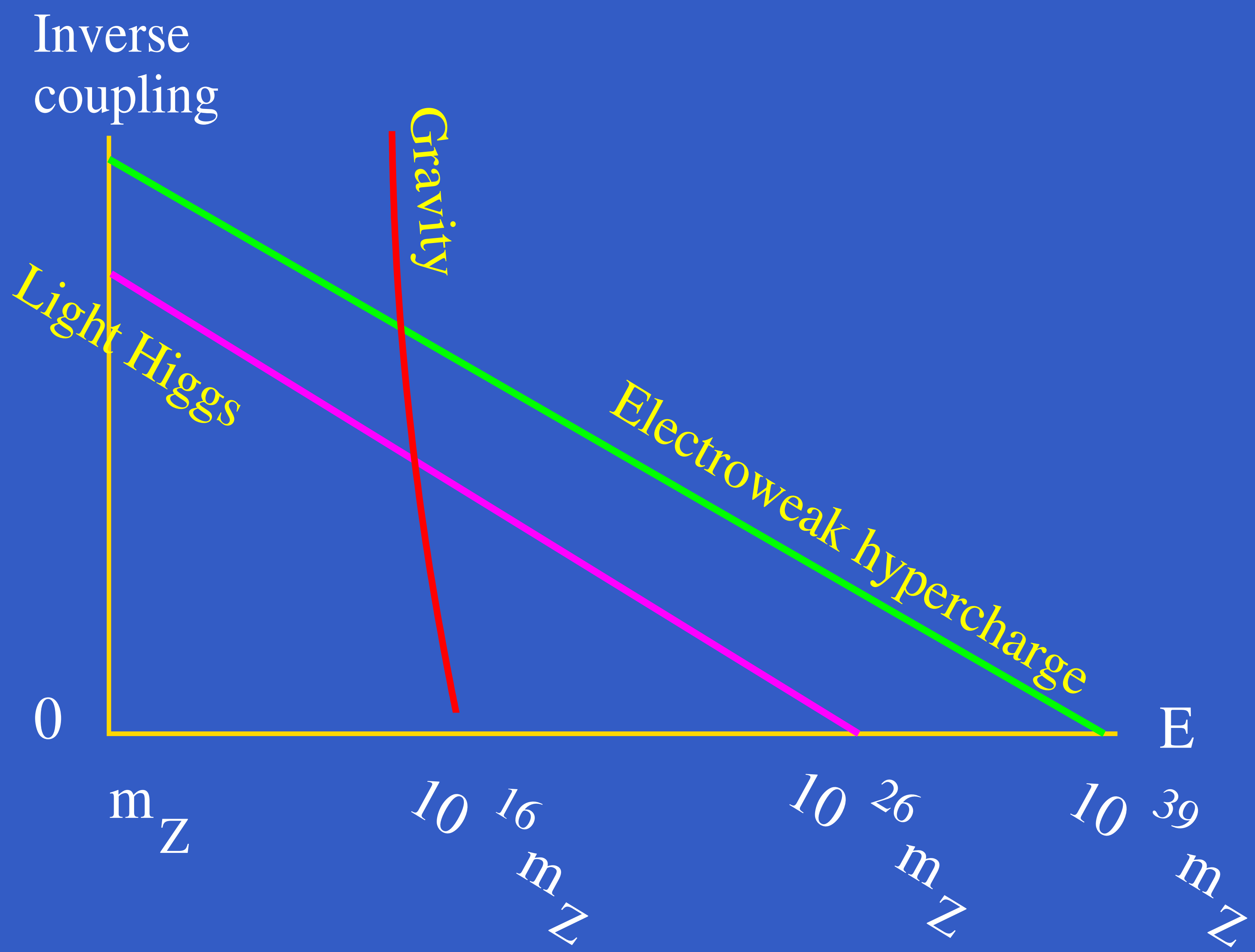
Charged energy eigenvalue of mirror (magnitude)



Outlook

- Our simulations are encouraging, and can be extended to (1+3)-dimensions.
- Important questions remain to be studied in this proposal; in progress.

Scales of mystery

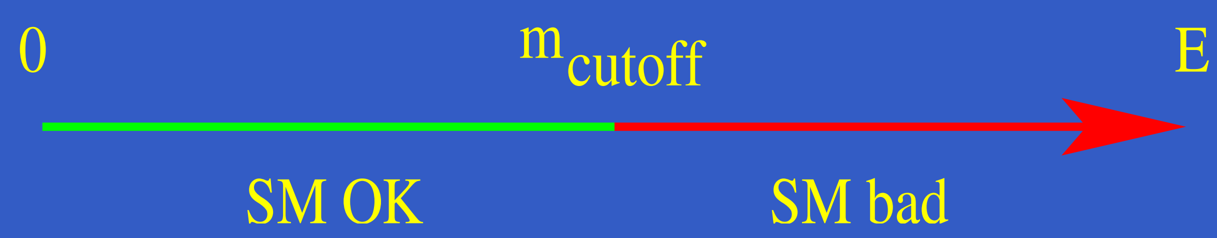


What does this mean?

- $\infty \rightarrow$ unphysical $\rightarrow$ approximation breaking down:

triviality problem

- Standard Model (SM) = "effective" or "cutoff" theory = finite range of validity:



$$m_{\text{cutoff}} \lesssim m_P = \text{unobtainable.}$$

If cutoff so big, who cares?

- The classical Higgs mass in the Standard Model is:

$$m_H^2 \sim m_Z^2.$$

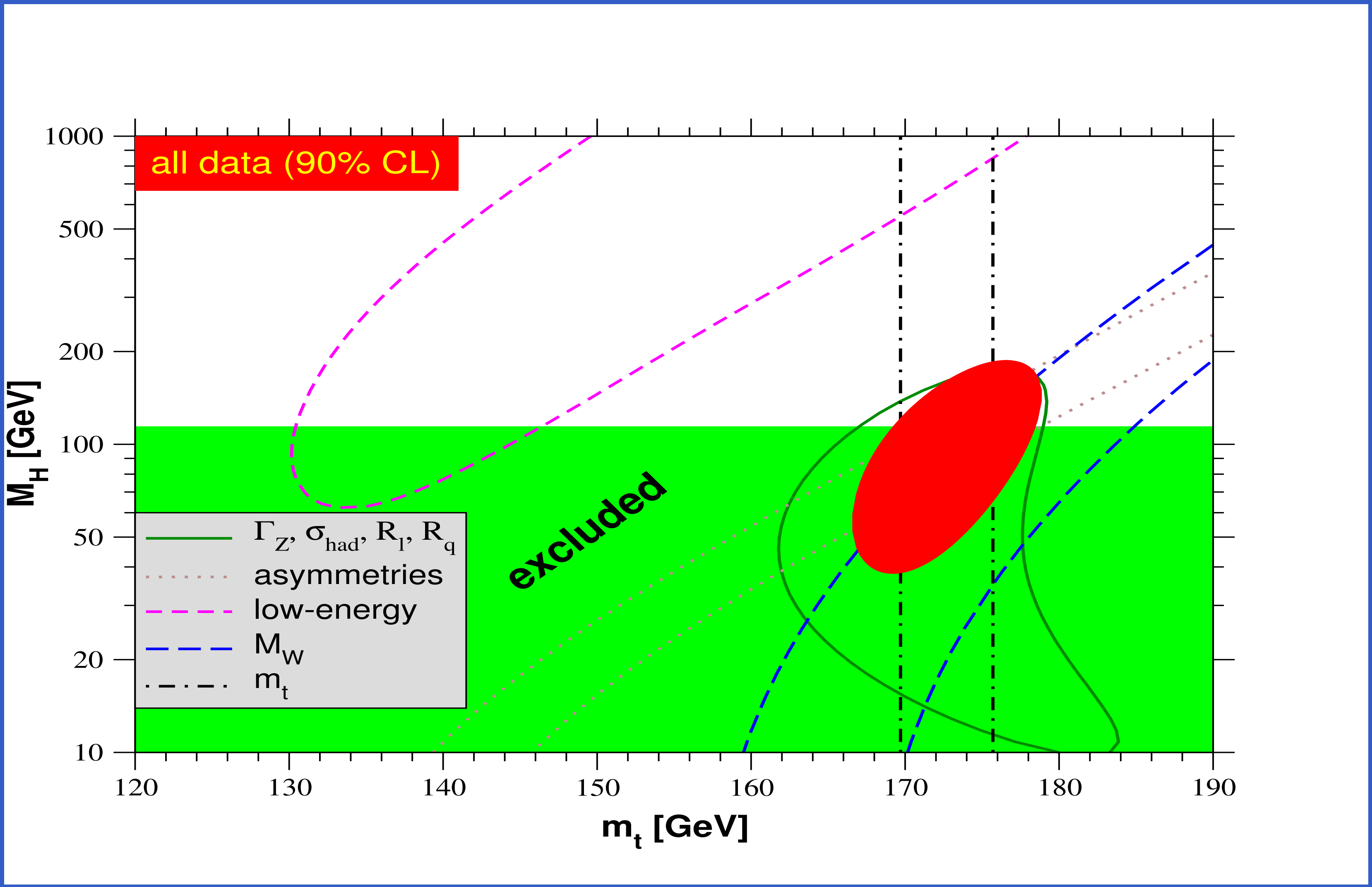
- The quantum corrections shift this mass by:

$$\Delta m_H^2 \sim m_{\text{cutoff}}^2.$$

- Physical mass = total:

$$m_H^2 + \Delta m_H^2 \sim m_{\text{cutoff}}^2 \lesssim m_P^2.$$

Experimental fit constraints



Particle Data Group, 2006

What gives?

- But the Standard Model quantum corrections predict a Higgs mass of the cutoff scale.
- What keeps the Higgs so light?
- This is the gauge hierarchy problem.

Keeping Δm_H^2 small

- All solutions in a nutshell:

  (1) $m_{\text{cutoff}} \lesssim$ 1000 GeV = 1 TeV.
  $\Rightarrow$ Beyond the Standard Model
  at our fingertips!

  (2) New particles and symmetries such that:

$$\Delta m_H^2 \lesssim m_H^2,$$

  in the Beyond the Standard Model theory.

Around the corner

- For $\Delta m_H^2 \lesssim m_H^2$ to work:

$$\text{mass(new particles)} \sim m_Z.$$

- We will be looking for these Beyond the Standard Model particles at the Large Hadron Collider.

LHC is coming...

- 225 MB of data will be recorded every second!
  (20000 GB = 4400 DVD’s per day, full luminosity)
- It will be an EXCITING period in physics.

Solutions to the hierarchy problem

- Cancellations: supersymmetry (SUSY).
- Fluffiness: Composite Higgs, transparent to high frequencies.
  - Old: Technicolor, ...
  - New: Warped extra dims. (via AdS/CFT), ...
- Hierarchy is a fake: Large extra dimensions.

BACKUP SLIDES

Supersymmetry

- It is a symmetry that transforms

  bosons $\leftrightarrow$ fermions.

- If the symmetry is exact, the masses of particles show a Bose-Fermi degeneracy:

$$m_B = m_F.$$

Superpartners

- Supersymmetry predicts superpartners:

  electron (fermion) $\Rightarrow$
  scalar electron (boson) = "selectron".

- Recall, however, that SUSY predicts:

$$m_B = m_F.$$

- Because we have not observed superpartners, SUSY must be spontaneously broken. E.g.,

$$m_{\text{selectron}} \gtrsim m_Z.$$

Tackling the nonperturbative

- Models for SUSY-breaking involve nonperturbative dynamics of SUSY gauge theories.
- Higgs effect at high scale
  $\longrightarrow$ frozen at weak coupling
  $\longrightarrow$ calculable models.
- Other scenarios:
  strongly interacting
  $\longrightarrow$ “noncalculable” models.

Gravity dual

- One approach:
  Find weakly coupled gravity dual.
- AdS/CFT correspondence:
  - Yang-Mills dynamics (=“CFT”)
  - takes on a gravitational meaning (=“AdS”).
- In recent work w/ T. Gherghetta (Minnesota), M. Gavella (Lausanne):
  - MSSM in bulk.
  - Deformed AdS$_5$.
- Susy-breaking from deformed metric, inspired by Type IIB supergravity solutions.
  [Borokhov, Gubser 02; Kuperstein, Sonnenschein 03]

RS1 [Randall, Sundrum 99]

- Slice of AdS$_5$:

$$ds_5^2 = A^2(z)\left(-dt^2 + d\vec{x}^2 + dz^2\right),$$
$$A^2(z) = \frac{1}{z^2}.$$

- 1+3 dim's: t, $\vec{x}$.
- 5th dim.: $1 \leq z \leq z_{\text{IR}}$.
- Warp factor: $A^2(z)$ (units of AdS curvature).
- UV brane: $z_{\text{UV}} = 1$.
- IR brane: $z_{\text{IR}} = m_P/$(0.1 to 10 TeV).

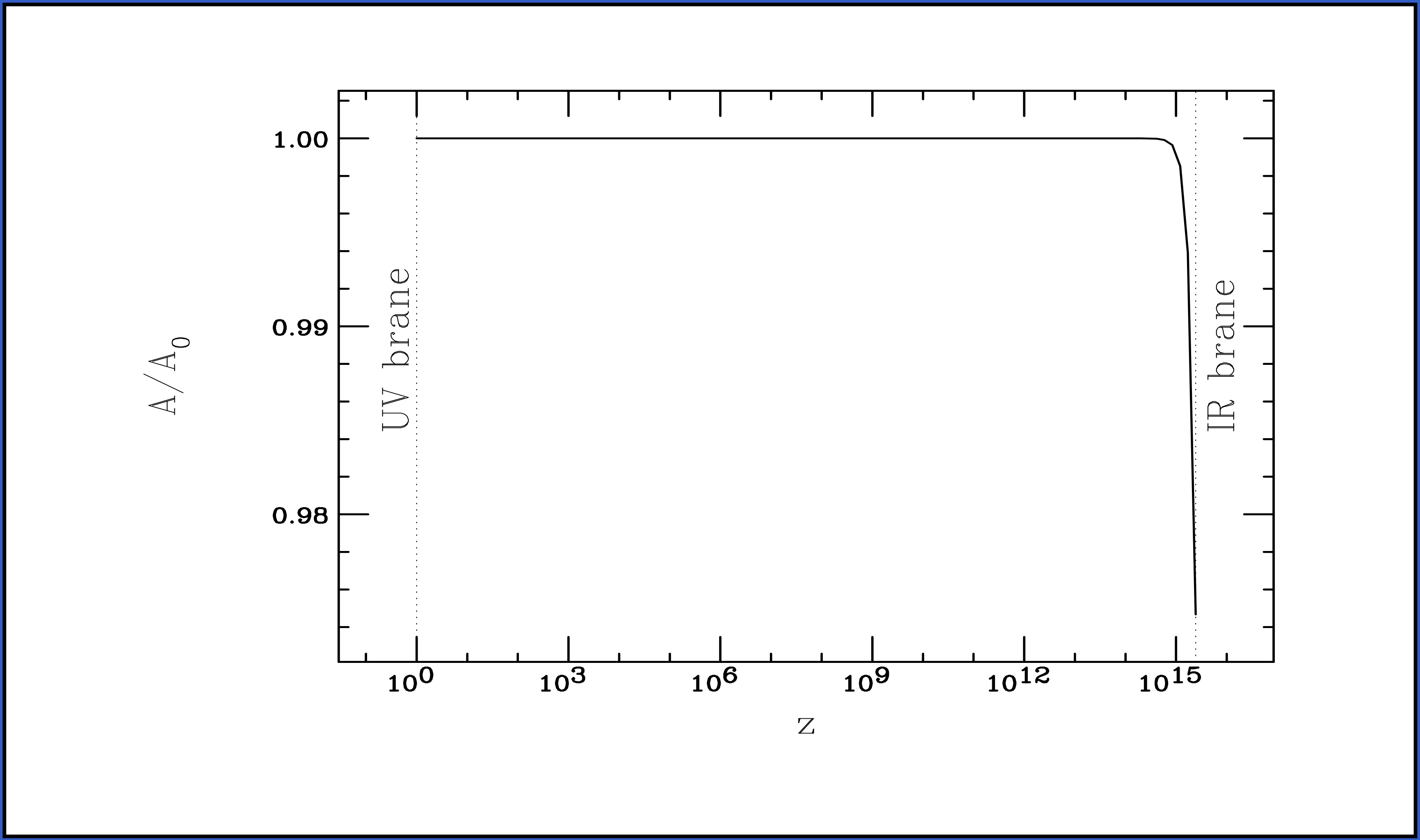
Deforming AdS$_5$

- Deform AdS according to the dim'l reduc. of Kup.-Sonn. soln.:

$$ds_5^2 = A^2(z)\left(-dt^2 + d\vec{x}^2 + dz^2\right),$$
$$A^2(z) = \frac{1}{z^2}\left(1 - \epsilon \cdot (z/z_{\mathrm{IR}})^4\right).$$

- Domain: $1 \leq z \leq z_{\mathrm{IR}}$.
- Dial 1: $\epsilon \sim 0.1$. [$\epsilon = 0.05$ in following.]
- Dial 2: $z_{\mathrm{IR}} = m_P/$(1-100 TeV).
- SUSY lim.: $\epsilon \to 0$.
- Interpretation: DSB in the $SU(N_c)$.

A few per cent violence in the IR: Will it matter?



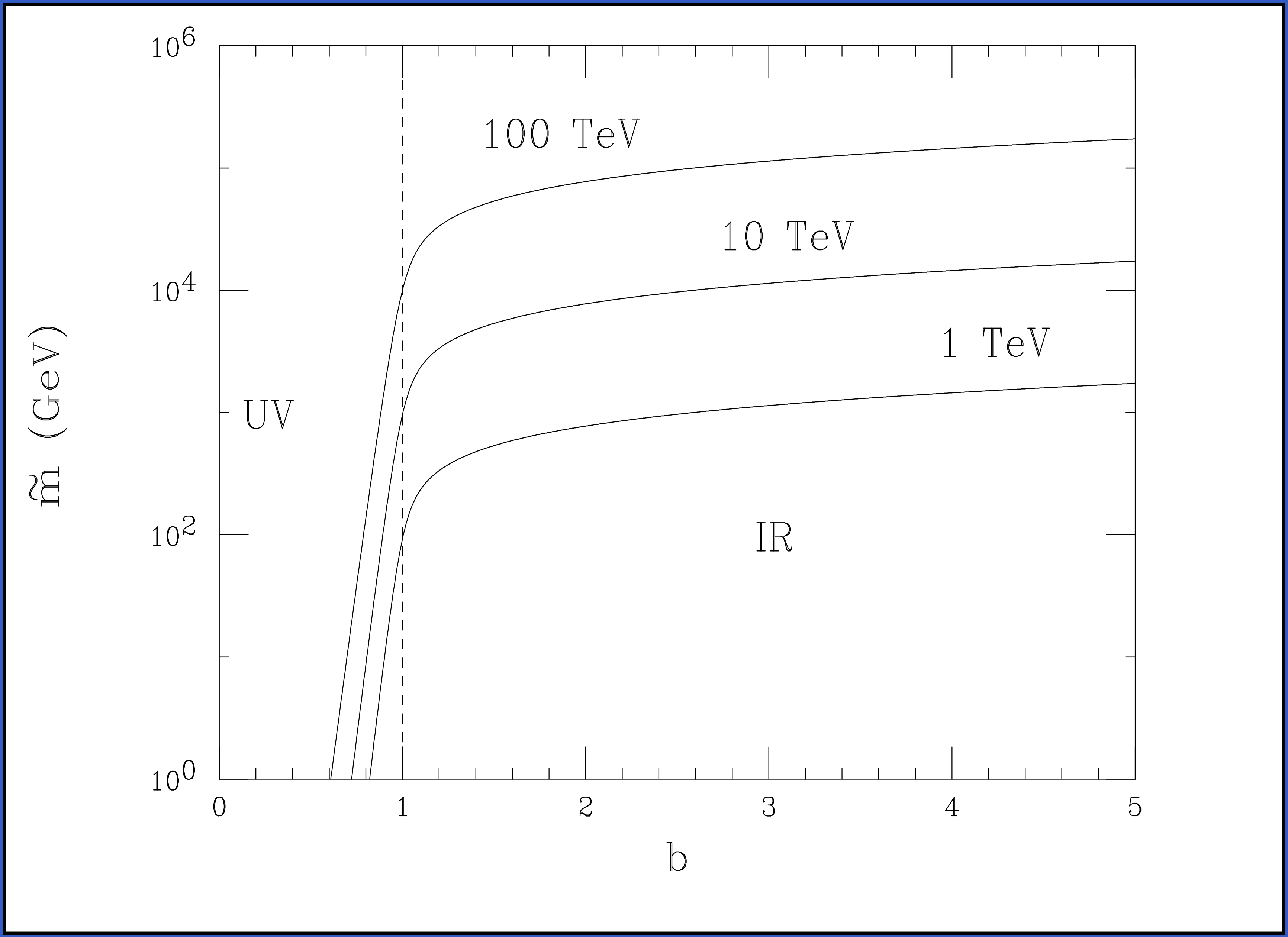
* Ratio deformed/undeformed warp factor. *
* Unchanged except very near IR brane. *

Scalars

- Solutions parameterized by boundary mass b.
- LO profile $\sim z^{b-1}$, unchanged.
- Zero modes lifted:

$$\begin{aligned}\tilde{m} &\approx \sqrt{\epsilon(b-1)(b+10)}\, z_{\mathrm{IR}}^{-1} \quad (b>1) \\ \tilde{m} &\approx \sqrt{\epsilon(1-b)(b+10)}\, (z_{\mathrm{IR}})^{b-2} \quad (0<b<1)\end{aligned}$$

Masses of quasi-zeromode scalars



WED WIP

- With students (Matt P., Saroosh S., Adam F.) studying neutrino masses in WED theories.
- Goal: better understand how “localization” can explain small masses and large/small mixings.
- Predictions of $\sin\theta_{13}$ would be tested at Prof. Napolitano’s Daya Bay experiment.