ASTR 2050 Midterm Exam
Introduction to Astronomy and Astrophysics
Spring 1999 NAME: __Solution Key__

You have two hours to complete this exam. There are a total of *four* problems and you are to solve all of them. *Not all the problems are worth the same number of points.*

You may use your textbooks and class notes and handouts, or other books. You *may not* share these resources with another student during the test.

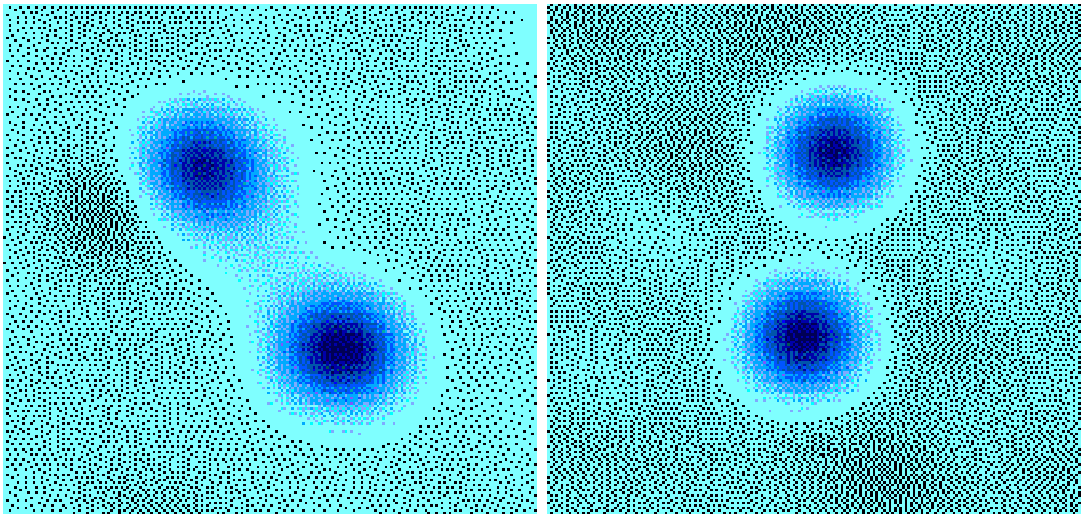
In general, if you can't get the one part of a problem, you can still do at least some of the remaining parts. Partial credit will be given if you can show that you know what needs to be done, even if you don't have all the numbers. In other words, *don't give up on a problem too quickly!*

Indicate any figures or tables you use in your calculations. Show all work!

GOOD LUCK!

Problem	Score	Worth
1.	________	30
2.	________	20
3.	________	25
4.	________	25
Total Score:	__________	100

Problem 1 (5+5+5+7+8=30 Points): The following high resolution image, discussed in class, shows that the "star" Capella is actually a binary star system. For this problem, assume that the two stars are identical, the orbit is circular, and we are viewing the orbit "face on" ($\sin i = 0$).



Images of Capella taken on the 13th (left) and 28th (right) September 1995. The separation between the stars is 55 milli-arcsec.

a) Unresolved, Capella is the sixth brightest star in the sky. *On what page of your textbook* can you find that the distance to this binary system is 14pc?

Page A-11, in Table A4-2 "The 25 brightest stars"

b) What is the distance between the two stars? Express your answer in terms of AU.

The distance a is the angular separation in radians times the distance to the stars:

$$a = 55\times10^{-3}\times\frac{1}{3600}\times\frac{\pi}{180}\times 14pc\times\frac{206265AU}{pc} = 0.77AU$$

You get the same answer (in AU) my multiplying the distance (pc) by the angle (")

c) Estimate the orbital period of this system from the images above. Assume that it has completed less than one half of a complete revolution between the two figures. Express your answer in years.

The stars rotate through an angle of $42°$ (I used a protractor) in 15 days, so the period is $P = (360 \div 42)\times(15 \div 365) = 0.35$ years.

d) What is the mass of each star, in terms of solar masses?

Using Eq.1-24 (which refers to Fig.1-14 for the definition of *a*), expressing things in the "natural" units of solar masses and years, and with $m_1 = m_2 = M$ for the mass, we get $M = (1/2)(a^3/P^2) = 1.86$ solar masses.

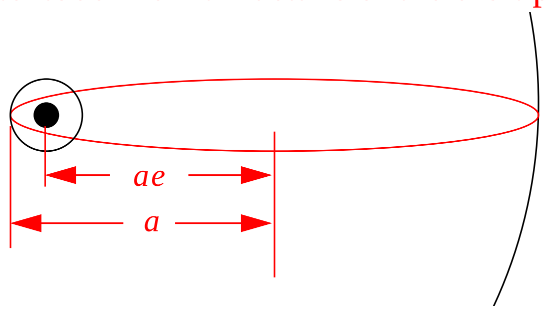
e) Estimate the aperture of the "telescope" used to make this image. Assume the image was taken with visible light. Is there a reason to suspect that there is something special about the telescope?

The stars are well separated so the resolution is much better than that. Say it is one tenth the separation, i.e. $\theta_{min} = 55\times10^{-4}$ arcsec. With $\theta_{min} = \lambda/d$ and a visible wavelength $\lambda = 550$ nm, the aperture $d = 20.6$ m. This is *not* the mirror diameter of a single telescope. (It is way too large). This observation was made using an optical interferometer with two well-separated telescopes.

Problem 2 (6+7+7=20 Points): An asteroid orbits the Sun within the orbits of Saturn and Venus, so that its aphelion and perhihelion are given by the (circular) orbital radii of those two planets. You can ignore any gravitational forces between the asteroid and the planets.

a) Sketch the orbit on the following diagram. The open circles are the planetary orbits, and the dark circle represents the sun. Label the various dimensions needed to describe the orbit.

See Figure 1-7 in the textbook for full details of the orbit parameters.



The perihelion is the radius of Venus' orbit and the aphelion is Saturn's orbit.

b) Calculate the eccentricity e of the orbit.

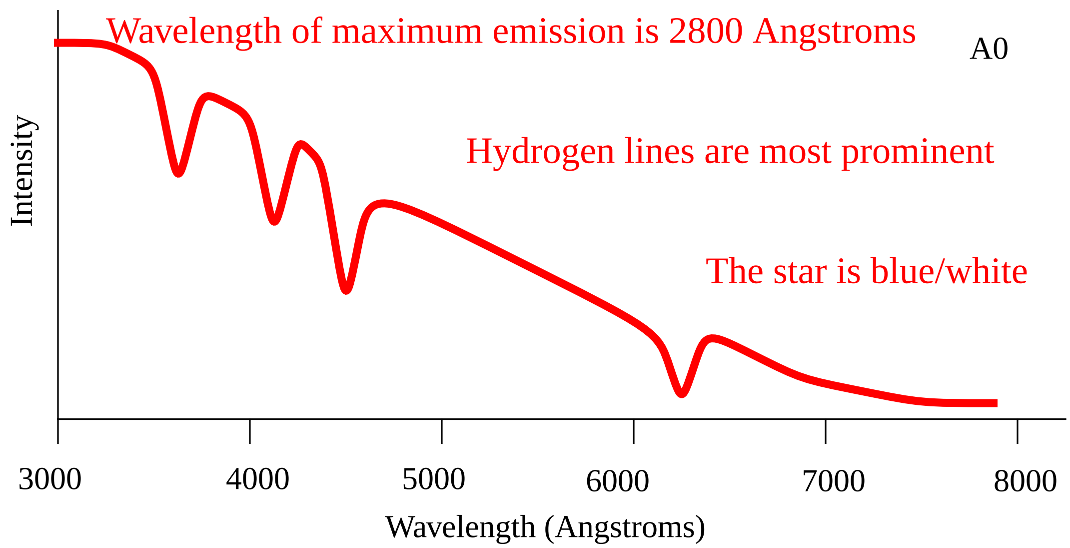
Using the data in Table A3-1 for the planet orbital radii, we have $a = (R_{Venus} + R_{Saturn})/2 = 5.13$ AU, and $a - ae = R_{Venus} = 0.72$ AU and so $1 - e = 0.72/5.13 = 0.14$ and finally $e = 0.86$. There are a number of other ways you can rearrange things to get the same answer, but I think this is the simplest.

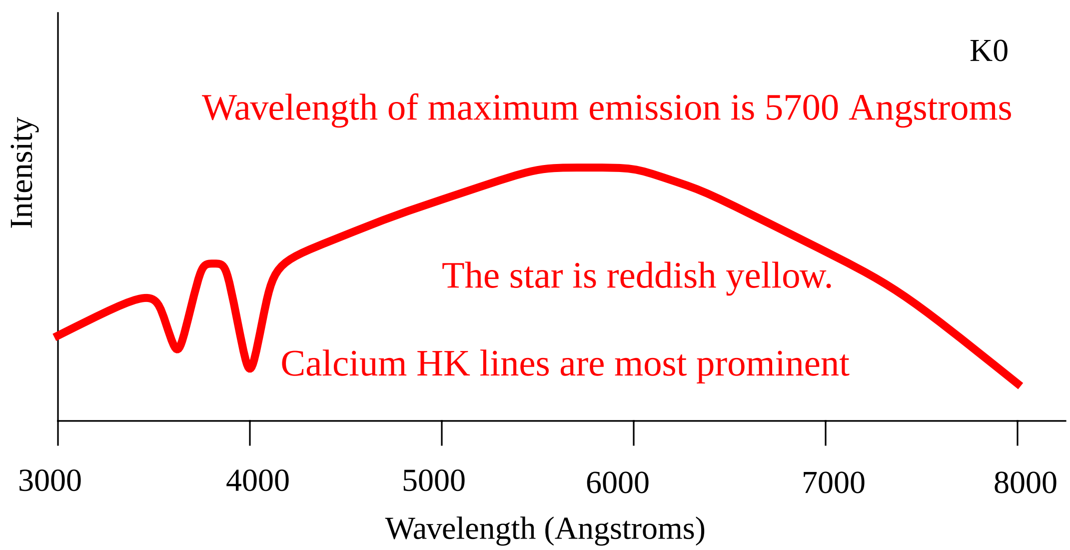
c) How long does it take the asteroid to complete one orbit around the Sun?

Just use Kepler's Third Law for the solar system as is, in natural units. That is, just take $P^2 = a^3$ and so $P = (5.13)^{3/2} = 11.6$ years

Problem 3 (25 Points): On the axes below, sketch two stellar spectra, one for a main sequence A0 star and the other for a main sequence K0 star. Use information such as the effective surface temperature to be specific, and as numerically accurate as possible, with regard to position (i.e. wavelength) of the important absorption lines and region of maximum overall flux. Indicate the atomic or molecular sources of the lines, as well as the color the star appears to the naked eye.

Table 13-1 gives you the description in terms of lines and star colors, and Fig 13-4 gives you the wavelengths of the important lines. (Figure 9-15 is also helpful, but be aware that these spectra have detection efficiency folded in.) You are also to use the Wien Displacement Law $\lambda_{max} = (2.898 \times 10^{-3} m)/T$ (K) to get the wavelength of maximum emission, where the surface temperature T can be found from either Table A4-3 or Fig 13-6. Putting all of this information together, you get something like...





Problem 4 (5+5+5+5+5=25 Points): Consider a yellow supergiant (i.e. G0Ia) star:

a) Use a figure in Chapter 13 of your textbook to estimate the absolute visual magnitude. Indicate which figure you use.

The best figure to use is Fig.13-11(B), but you can also use Figs.13-8(C) or 13-15 if you are careful. The answer is M_V=-7. (Note the typo at the bottom of Table A4-3.)

b) If the star has an apparent visual magnitude $V = 2$ what is its distance from us?

From Equation 11-6, and realizing that in the notation of visual magnitudes we have m=V and M=M_V, then $V - M_V = 2 - (-7) = 9 = 5\log d - 5$ in which case we have $d = 10^{(9+5)/5} = 631$ pc.

c) Estimate the effective surface temperature of the star. Once again, indicate any figures or tables you use to get this number.

Table A4-3 shows that T=5050K. Also, Figure 13-6 indicates a temperature near 6000K. Either answer is alright, but the former is better.

d) Determine the luminosity of this star, relative to the luminosity of the Sun. You can ignore the bolometric correction.

Equation 11-20b relates bolometric magnitude to luminosity. We will ignore the bolometric correction and equate bolometric magnitude to visual magnitude. Then $\log\frac{L}{L_{Sun}} = 1.9 - 0.4 \times (-7) = 4.7$ and so $L = 5.0 \times 10^4 L_{Sun}$.

e) Use the Stefan-Boltzmann law to calculate the radius of this star. (It is probably easiest to calculate the radius relative to the radius of the Sun.) Express your answer either in solar radii or AU.

The easy way to do this is to use the Stefan-Boltzmann Law in relative terms, that is $\frac{L}{L_{Sun}} = \left(\frac{R}{R_{Sun}}\right)^2\left(\frac{T}{T_{Sun}}\right)^4$ which comes from $L = 4\pi R^2 \sigma T^4$ and dividing. If we use the relative formula, then we can ignore the temperature term since the temperature is pretty close to the temperature of the sun, and so $R = \sqrt{5.0 \times 10^4} R_{Sun} = 223 R_{Sun}$. Using the original formula and looking up the value of σ from Table A7-2, you find $R = 295 R_{Sun}$ with T=5050K.