Exam #3
79205 Astronomy
Fall 1997

NAME: _____Solutions_____

You have two hours to complete this exam. There are a total of six problems and you are to solve all of them. *Not all the problems are worth the same number of points.*

You may use *Introductory Astronomy and Astrophysics* (Zeilik & Gregory), *Astronomy: The Evoloving Universe* (Zeilik), and class notes and handouts, or other books. You *may not* share these resources with another student during the test.

Indicate any figures or tables you use in your calculations. Show all Work!

GOOD LUCK!

Problem	Score	Worth
1.	________	12
2.	________	8
3.	________	20
4.	________	25
5.	________	15
6.	________	20

Total Score:	__________	100

Problem 1 (12 points): Five of the following celestial objects represent (or likely contain) a solar mass star at some point in its evolution. We associate the sixth object with an evolutionary stage of a much more massive star. Place a 1, 2, 3, 4, or 5 in age order (youngest to oldest) for the five objects associated with one solar mass, and place an "*X*" next to the sixth.

___**2**__	• The Sun	Main sequence star
___**5**__	• Sirius B	A white dwarf
___**3**__	• Betelgeuse	A red giant
___**X**__	• M1, the Crab Nebula	Supernova remnant
___**1**__	• M16, the Eagle Nebula	Star-formation region
___**4**__	• M57, the Ring Nebula	Planetary nebula

Problem2 (8 points): A B0 main sequence star has a mass of about 17 solar masses.

a (6 points) Estimate the main sequence lifetime of this star.

The luminosity of a main sequence star is proportional to M^4.
The available hydrogen fuel for the main sequence is proportional to M.
Therefore, the lifetime of a main sequence star is proportional to $1/M^3$.

(You could also use the equation on page 320 of your textbook which uses $L \propto M^{3.3}$ to come up with the lifetime proptional to $M^{-2.3}$.)

The sun has a lifetime of 10 Billion years.
Therefore, a B0 star has a lifetime of 10^{10} yr / 17^3 = 2 Million years

b (2 points) What do you expect to be the end-state of this star?

Very massive stars (like this one) will end in a supernova explosion. If it exists at all, the core will remain as either a neutron star or a black hole. Any combination of these answers is acceptible.

Problem 3 (20 points): A Type I supernova is observed in a distant galaxy. The supernova shows the Hα emission line at 682.6 nm and the Ca K absorption line is observed at 409.10 nm. What do you expect for the apparent magnitude of the supernova at maximum brightness?

You are told that $\lambda(H\alpha) = 682.6$ nm and $\lambda(\text{Ca K}) = 409.10$ nm.

In the lab, we have $\lambda_0(H\alpha) = 656.3$ nm (Text: Page 161 and Table 24-1)
and $\lambda_0(\text{Ca K}) = 393.4$ nm (Text: Fig.21-6; also Hubble lab)

For either of these two lines, we find

$$z = \frac{v}{c} = \frac{\lambda - \lambda_0}{\lambda_0} = 0.04$$

so that $v = 0.04c = 1.2\times10^4$ km/sec.

We get the distance d to the galaxy from Hubble's Law, $v = Hd$:

$$d = \frac{v}{H} = \frac{1.2\times10^4 \text{ km/sec}}{75 \text{ km/sec}\bullet\text{Mpc}} = 160 \text{ Mpc} = 1.6\times10^8 \text{ pc}$$

To get the apparent magnitude m , we also need the absolute magnitude M.
Table 18-3 in the textbook says that Type I Supernovae have $M = -20$. So from

$$m - M = 5\log d - 5 = 5\log(1.6\times10^8) - 5$$

we get $m = 16$. (Equivalent answers with different but valid values for the Hubble Constant are also acceptible.)

Note: This is much too faint to be seen with the naked eye, but no trouble at all for a moderate telescopes and a ~minute exposure time. Searches for Type I supernovae have proven to be a very useful way to measure the Hubble Constant.

Problem 4 (25 points): The point of this problem is to estimate the time it takes for a white dwarf to cool down. Assume this happens by radiating away its internal thermal energy like a spherical blackbody radiator with radius *R*.

a (4 points) Write an approximate expression for the internal thermal energy *E* of a white dwarf in terms of the total number of electrons *N* and its temperature *T*.

$$E = \frac{3}{2}NkT$$

b (6 points) Derive an expression for the time derivative of the temperature, *dT/dt*, in terms of *N*, *R*, and *T*, and fundamental constants. Be careful of the overall sign.

The rate at which the thermal energy *E* is lost ($-dE/dt$) must be the same as the rate $L=4\pi R^2\sigma T^4$ at which the white dwarf radiates its energy like a blackbody, so

$$-\frac{3}{2}Nk\frac{dT}{dt} = 4\pi R^2\sigma T^4$$

which is simply rearranged to

$$\frac{dT}{dt} = -\frac{8\pi R^2\sigma T^4}{3Nk}$$

c (10 points) The white dwarf starts out a high temperature T_i and ends up at a "cool" low temperature T_f, where $T_i \gg T_f$. Show that the time τ it takes for the star to cool to T_f is given to a good approximation by $\tau = \frac{Nk}{8\pi R^2\sigma T_f^3}$

The rate at which temperature changes depends on the temperature itself, so you have to integrate the expression above. This is most simply written as

$$-\int_{T_i}^{T_f}\frac{1}{T^4}dT = \int_0^{\tau}\frac{8\pi R^2\sigma}{3Nk}dt$$

The right side integration is trivial. The left side gives $1/3T^3$ evaluated from T_i to T_f which works out to be $1/3T_f^3$. Simple algebra gives you the desired formula.

d (5 points) How many years are needed for a one solar mass white dwarf with the radius of the Earth to cool to T_f=1500K?

The mass of a white dwarf is taken up by equal numbers of protons and neutrons. (It is mostly just ^{12}C.) The number of electrons equals the number of protons, so $N = \frac{1}{2}(M_{SUN}/m_H) = 6.0\times10^{56}$. The formula then gives $\tau = 8.9\times10^8$ years.

Problem 5 (15 points): Estimate the central temperature of the "gas giant" planet Neptune. Assume for simplicity that is made completely from pure un-ionized helium gas.

The assumption is that Neptune is big ball of gas, just like a star. We use the same results we had for hydrostatic equilibrium when we were building stars. Then, we found the central temperature after finding the central pressure. We could do the same here, but let's combine the equations first. It is easier.

Applying the ideal gas equation of state at the center of Neptune, we have

$$P_C = nkT_C = \frac{\langle\rho\rangle kT_C}{\mu m_H}$$

where we just follow Equations 16-4 to 16-7 in the textbook. We get the central pressure from hydrostatic equilibrium, for example on page 311 of the textbook:

$$P_C = \frac{GM\langle\rho\rangle}{R}$$

Combining these equations gives us an expression for the central temperature:

$$T_C = \frac{\mu m_H GM}{kR}$$

The mean molecular weight for pure neutral helium gas is μ=4. (There are no free electrons.) The other constants are in Appendices 3 and 7 of the textbook. The result is

$$T_C = 1.3\times10^5\ \text{K}$$

Problem 6 (20 points): Polaris, the North Star, is a population II (*W Virginis*) cepheid variable with a 3.97 day period and an apparent visual magnitude of +1.99. Assuming that population II cepheids are 1.5 magnitudes fainter than population I cepheids, find the distance to Polaris.

For a population I cepheid variable, the Period Luminosity relationship (textbook Eq.18-2) gives

$$M_V = -2.76(\log P - 1) - 4.16 = -3.05$$

However, a population II cepheid is 1.5 magnitudes fainter, so in fact

$$M_V = -1.55$$

We get the distance from

$$m - M = 1.99 + 1.55 = 5\log d - 5$$

and therefore

$$d = 10^{(3.54 + 5)/5} = 51 \text{ pc}$$