Problem 5 (25 points): An astronomer measures the n=4→3 transition of atomic hydrogen in some star-like object, at the same wavelength as the n=3→2 transition in the laboratory.

a (10 points) Is the object moving towards her or away from her? Explain your reasoning.

As can be read from the textbook Fig. 8-9, or calculated as shown below, the 4→3 transition has less energy and so a longer wavelength than the 3→2 transition. Therefore $\lambda = \lambda_{\text{OBSERVED}} < \lambda_{\text{ACTUAL}} = \lambda_0$ so the line is blueshifted, that is, shifted towards shorter wavlengths, so the object is moving *towards her.*

b (15 points) How fast is the object moving?

The hydrogen line wavelengths are given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

for a $n_1 \rightarrow n_2$ transition. Also, since the wavelength shift is rather large, it will be important to use the *relativistic* Doppler formula:

$$\frac{\lambda}{\lambda_0} = \left(\frac{1 - v/c}{1 + v/c}\right)^{1/2}$$

(If you used the nonrelativistic formula, you would have found a very large, or non-sensical, velocity, so that should have tipped you off to use the right formula.)

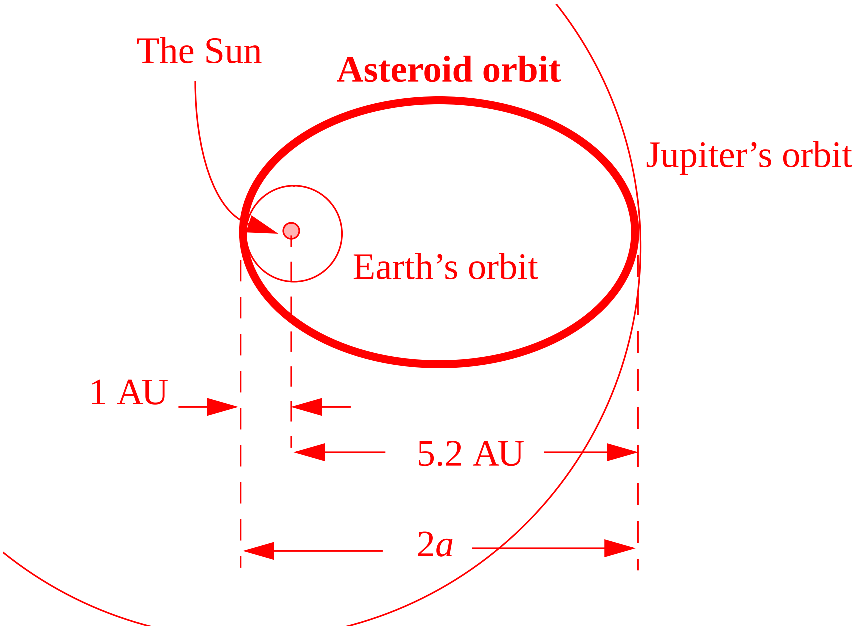
Combining these two gives

$$\frac{\frac{1}{3^2} - \frac{1}{4^2}}{\frac{1}{2^2} - \frac{1}{3^2}} = \left(\frac{1 - v/c}{1 + v/c}\right)^{1/2}$$

and a little bit of algebra gives you $v/c = 0.78$ or $v = 2.3 \times 10^8$ m/sec.

Problem 4 (15 points): A hypothetical asteroid executes an elliptical orbit that brings it as far away from the Sun as Jupiter and as close to the Sun as the Earth. How long does it take to complete one full orbit? You can assume that both the Earth and Jupiter travel around the Sun in circular orbits.

It is a good idea to start with a picture:



The "length" of the ellipse is twice the semimajor axis a, so

$$2a = 1\text{ AU} + 5.2\text{ AU}$$
$$a = 3.1\text{ AU}$$

and now we can use Kepler's Third Law in the simple form

$$P^2 = a^3$$

where a is in AU and the period P is in years, assuming we are orbiting around something with the mass of the sun. Hence.

$$P = 3.1^{3/2} = 5.46\text{ years}$$

Problem 3 (20 points): The sun radiates energy at the rate of 3.9×10^{26} J/sec. Assuming the sun is a uniform spherical mass, how much would the radius have to shrink each year if the radiated energy were strictly due to gravitational contraction?

This problem is just like our class discussion of energy from Jupiter, and your homework problem on energy from Saturn. Start with the approximate expression for the internal potential energy for a spherical, gravitating mass:

$$U \approx \frac{GM^2}{R}$$

and set the luminosity equal to the rate at which this internal energy changes:

$$L = \frac{dU}{dt} = \frac{GM^2}{R^2}\frac{dR}{dt}$$

where I'm being sloppy with signs, but remembering that the sun must *shrink* in order to release energy. Putting in the relevant values for the sun, you find

$$\frac{dR}{dt} = 7.1\times10^{-7} \text{ m/sec} = 22.3 \text{ m/year}$$

Later on we'll use this calculation to show that it is impossible for this to be the source of the sun's energy. (How big would the sun have been when the dinosaurs roamed the earth, for example?)

Problem 2 (20 points): Two stars orbit each other and spectral data indicates they are each about 1 kpc = 3.1×10^{19}m away. With a 1 m diameter aperture telescope, the stars can be resolved in blue light, but they cannot be resolved in red light. Estimate the distance between the two stars.

The object can be resolved if the wavelength is as short as ≈400 nm, but not if the wavelength is as long as ≈700 nm. To estimate the distance between the stars, based on diffraction limited resolution, pick a wavelength in the middle, say 550 nm.

The resolving angle is therefore

$$\theta_{\mathrm{RES}} = 1.22\frac{\lambda}{d} = 6.7\times10^{-7}$$

The actual angular separation between the two stars is

$$\theta_{\mathrm{SIZE}} = \frac{R}{D} = \frac{R}{3.1\times10^{19}\ \mathrm{m}}$$

Setting these two angles equal, you find

$$R = 2.1\times10^{13}\ \mathrm{m} = 140\ \mathrm{AU}$$

This is a perfectly reasonable separation for a binary star system.

NOTE: A lot of people made the mistake of using their calculator to get the sine or tangent of an angle, but left the calculator in "degrees" instead of radians. You should remember that this is never necessary for problems like this. If the angle is small and expressed in radians, its sine and tangent are both equal to the angle.

Problem 1 (20 points): Our sun is located about 8.5 kpc = 2.6×10^{20} m from the center of our galaxy, where most of the galactic mass (about 7×10^{11} solar masses) is concentrated. Assuming the sun executes circular orbits about the galactic center, how long does it take to make a round trip?

This is a straightforward application of our work on circular orbits. Starting with F=ma for an object moving in a circle under the influence of gravity from a much larger object, we have

$$\frac{GMm}{R^2} = m\frac{v^2}{R} = m\frac{1}{R}\left(\frac{2\pi R}{T}\right)^2$$

which leads to the equation

$$T = 2\pi\left(\frac{R^3}{GM}\right)^{1/2}$$

where $M=7\times10^{11}\times1.99\times10^{30}$ kg and R is given above. The other numbers can be looked up in the textbook appendices. You find

$$T = 2.7\times10^{15} \text{ sec} = 86 \text{ Myr}$$

We'll be returning to this when we talk about the structure of our Galaxy, including spiral arm structure and the evidence for "dark matter".

Exam #1
79205 Astronomy
Fall 1997 NAME: ___Solution Key ___

You have two hours to complete this exam. There are a total of five problems and you are to solve all of them. *Not all the problems are worth the same number of points.*

You may use *Introductory Astronomy and Astrophysics* (Zeilik & Gregory), *Astronomy: The Evoloving Universe* (Zeilik), and class notes and handouts, or other books. You *may not* share these resources with another student during the test.

Indicate any figures or tables you use in your calculations. Show all Work!

GOOD LUCK!

Problem	Score	Worth
1.	________	20
2.	________	20
3.	________	20
4.	________	15
5.	________	25
Total Score:	___________	100