

Preliminary Examination Topics

Linear Algebra

- **Vector spaces; linear independence, basis and dimension**
- **Euclidean spaces; Schwarz inequality; orthogonal matrices**
- **Solving $Ax = b$ for square systems by Gaussian; $A = LU$ factorization**
- **Linear transformations; Range and null-space (kernel); Rank of a matrix**
- **Relations between ranges and kernels of A and its transpose (adjoint). [Solvability conditions for $Ax = b$ and relation to $Ax = 0$]**
- **Projections. Least squares solutions.**
- **Orthogonalization by Gram-Schmidt**
- **Properties of determinants.**
- **Changes of bases; similar matrices.**
- **Eigenvalues and eigenvectors (diagonalizing A , computing powers A^k and matrix exponentials).**
- **Symmetric, Hermitian, and unitary matrices. Positive-definite matrices**

Calculus (any Calculus textbook, such as *Larson* or *Stewart*, our Mathematical Analysis I or any equivalent are recommended)

Functions of one variable:

- limits, derivatives, differentials.
- Inverse functions;
- Elementary functions
- The mean-value theorem;
- Integrals, techniques of integration
- Improper integrals
- Parametric equations; polar coordinates

Infinite sequences and series

- Convergence of sequences
- Criteria of convergence (monotonic sequence, Cauchy criterion)
- Series; Convergence of series
- Power series
- Taylor series

Functions of several variables

- Partial derivative, tangent plane differential
- The chain rule
- Directional derivative
- Extremum problems; Lagrange multipliers

Multiple integrals

- Double and triple integrals; surface area and volume

- Change of variables in multiple integrals

Vector Calculus

- Vector Fields
- Line integrals of vector fields and work
- Conservative Vector Fields and Path Independence
- Green's Theorem
- Curl and Divergence
- Parametric surfaces and their areas. Surface integrals
- Stokes' theorem. The Divergence theorem