ENGR-1100 Introduction to Engineering Analysis

Lecture 27
**WEDGES AND FRICTIONAL FORCES ON FLAT BELTS**

**Today’s Objectives:**

Students will be able to:

a) Determine the forces on a wedge.

b) Determine tension in a belt.

**In-Class Activities:**

- Reading Quiz
- Applications
- Analysis of a Wedge
- Analysis of a Belt
- Concept Quiz
- Group Problem Solving
- Attention Quiz
APPLICATIONS

Wedges are used to adjust the elevation or provide stability for heavy objects such as this large steel pipe.

How can we determine the force required to pull the wedge out?

When there are no applied forces on the wedge, will it stay in place (i.e., be self-locking) or will it come out on its own? Under what physical conditions will it come out?
How can we decide if the belts will function properly, i.e., without slipping or breaking?

Belt drives are commonly used for transmitting the torque developed by a motor to a wheel attached to a pump, fan or blower.
How can you determine the tension in the cable pulling on the band?

Also from a design perspective, how are the belt tension, the applied force $P$ and the torque $M$, related?
ANALYSIS OF A WEDGE

A wedge is a simple machine in which a small force $P$ is used to lift a large weight $W$.

To determine the force required to push the wedge in or out, it is necessary to draw FBDs of the wedge and the object on top of it.

It is easier to start with a FBD of the wedge since you know the direction of its motion.

Note that:

a) the friction forces are always in the direction opposite to the motion, or impending motion, of the wedge;

b) the friction forces are along the contacting surfaces; and,

c) the normal forces are perpendicular to the contacting surfaces.
To determine the unknowns, we must apply EofE, \( \sum F_x = 0 \) and \( \sum F_y = 0 \), to the wedge and the object as well as the impending motion frictional equation, \( F = \mu_s N \).

Next, a FBD of the object on top of the wedge is drawn. Please note that:

a) at the contacting surfaces between the wedge and the object the forces are equal in magnitude and opposite in direction to those on the wedge; and, b) all other forces acting on the object should be shown.
ANALYSIS OF A WEDGE (continued)

Now of the two FBDs, which one should we start analyzing first?

We should start analyzing the FBD in which the number of unknowns are less than or equal to the number of E-of-E and frictional equations.
NOTE:
If the object is to be lowered, then the wedge needs to be pulled out. If the value of the force $P$ needed to remove the wedge is positive, then the wedge is **self-locking**, i.e., it will not come out on its own.
BELT ANALYSIS

Consider a flat belt passing over a fixed curved surface with the total angle of contact equal to $\beta$ radians.

If the belt slips or is just about to slip, then $T_2$ must be larger than $T_1$ and the motion resisting friction forces. Hence, $T_2$ must be greater than $T_1$.

Detailed analysis (please refer to your textbook) shows that $T_2 = T_1 e^{\mu \beta}$ where $\mu$ is the coefficient of static friction between the belt and the surface. Be sure to use radians when using this formula!!
EXAMPLE

**Given:** The crate weighs 300 lb and $\mu_s$ at all contacting surfaces is 0.3. Assume the wedges have negligible weight.

**Find:** The smallest force $P$ needed to pull out the wedge.

**Plan:**

1. Draw a FBD of the crate. Why do the crate first?
2. Draw a FBD of the wedge.
3. Apply the E-of-E to the crate.
4. Apply the E-of-E to wedge.
EXAMPLE (continued)

The FBDs of crate and wedge are shown in the figures. Applying the E-of-E to the crate, we get

\[ \sum F_X = N_B - 0.3N_C = 0 \]
\[ \sum F_Y = N_C - 300 + 0.3N_B = 0 \]

Solving the above two equations, we get

\[ N_B = 82.57 \text{ lb} = 82.6 \text{ lb}, \quad N_C = 275.3 \text{ lb} = 275 \text{ lb} \]
Applying the E-of-E to the wedge, we get

\[\sum F_y = N_D \cos 15^\circ + 0.3 N_D \sin 15^\circ - 275.2 = 0;\]

\[N_D = 263.7 \text{ lb} = 264 \text{ lb}\]

\[\sum F_x = 0.3(263.7) + 0.3(263.7)\cos 15^\circ - 0.3(263.7)\cos 15^\circ - P = 0;\]

\[P = 90.7 \text{ lb}\]
1. A wedge allows a ________ force P to lift a ________ weight W.
   A) (large, large)  B) (small, large)  C) (small, small)  D) (large, small)

2. Considering friction forces and the indicated motion of the belt, how are belt tensions $T_1$ and $T_2$ related?
   A) $T_1 > T_2$  B) $T_1 = T_2$  C) $T_1 < T_2$  D) $T_1 = T_2 e^\mu$
CONCEPT QUIZ

1. Determine the direction of the friction force on object B at the contact point between A and B.
   A) B) ←
   C) D) ←

2. The boy (hanging) in the picture weighs 100 lb and the woman weighs 150 lb. The coefficient of static friction between her shoes and the ground is 0.6. The boy will ______?
   A) Be lifted up   B) Slide down
   C) Not be lifted up   D) Not slide down
ATTENTION QUIZ

1. When determining the force $P$ needed to lift the block of weight $W$, it is easier to draw a FBD of ______ first.

A) The wedge  
B) The block  
C) The horizontal ground  
D) The vertical wall

2. In the analysis of frictional forces on a flat belt, $T_2 = T_1 e^{\mu \beta}$.

In this equation, $\beta$ equals ______.

A) Angle of contact in degrees  
B) Angle of contact in radians  
C) Coefficient of static friction  
D) Coefficient of kinetic friction
Given: Blocks A and B weigh 50 lb and 30 lb, respectively.

Find: The smallest weight of cylinder D which will cause the loss of static equilibrium.

Plan:
Plan:

1. Consider two cases: a) both blocks slide together, and, b) block B slides over the block A.
2. For each case, draw a FBD of the block(s).
3. For each case, apply the E-of-E to find the force needed to cause sliding.
4. Choose the smaller P value from the two cases.
5. Use belt friction theory to find the weight of block D.
Case a (both blocks sliding together):

\[ \sum F_Y = N - 80 = 0 \]

\[ N = 80 \text{ lb} \]

\[ \sum F_X = 0.4(80) - P = 0 \]

\[ P = 32 \text{ lb} \]
Case b (block B slides over A):

\[ \uparrow + \sum F_y = N \cos 20^\circ + 0.6 N \sin 20^\circ - 30 = 0 \]
\[ N = 26.20 \text{ lb} \]

\[ \rightarrow + \sum F_x = -P + 0.6 (26.2) \cos 20^\circ - 26.2 \sin 20^\circ = 0 \]
\[ P = 5.812 \text{ lb} \]

Case b has the lowest P (case a was 32 lb) and thus will occur first. Next, using a frictional force analysis of belt, we get

\[ W_D = P e^{\mu \beta} = 5.812 e^{0.5(0.5 \pi)} = 12.7 \text{ lb} \]

A Block D weighing 12.7 lb will cause the block B to slide over the block A.